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TOPOI – *Towards a Historical Epistemology of Space*

Matthias Schemmel

Medieval Representations of Change and Their Early Modern Application

TOPOI – TOWARDS A HISTORICAL EPISTEMOLOGY OF SPACE

The TOPOI project cluster of excellence brings together researchers who investigate the formation and transformation of space and knowledge in ancient civilizations and their later developments. The present preprint series presents the work of members and fellows of the research group *Historical Epistemology of Space* which is part of the TOPOI cluster. The group is based on a cooperation between the Humboldt University and the Max Planck Institute for the History of Science in Berlin and commenced work in September 2008.

Max Planck Institute for the History of Science

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Matthias Schemmel

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1 Symbolic means of knowledge representation and early modern derivations of the law of fall

This paper discusses the epistemic role of symbolic means of knowledge representation by way of a concrete historical example: the early modern use of medieval representations of change to describe functional relations between quantities, in particular relating to the motion of fall.

The means for the external, or material, representation of knowledge in general play a double, on the face of it paradoxical role in the history of knowledge. On the one hand they serve the communication of knowledge and its transmission from one generation to the next and thereby constitute a precondition for the stability and longevity of knowledge structures. On the other hand, they serve as tools for thinking, and in consequence of their being manipulated and the results of these manipulations being reflected upon, knowledge structures change. An obvious example displaying both aspects is numeric notation. While it is a vehicle for the transmission of arithmetic knowledge, its use may lead to changes of the number concept itself, for instance, when negative results of arithmetic procedures are themselves considered to be numbers, thus leading to an extension of the number concept.

In the present study we will further explore this double character, mainly discussing two means of knowledge representation: (1) proportions involving the kinematic quantities of the velocity, or speed, of a mobile, the space traversed by it, and time elapsed during the motion; and (2) representations of the change of qualities and motions by geometric figures, or diagrams. We refer to these as symbolic means of knowledge representations, relating to the general and non-intuitive aspects of how they represent meaning. But this is not meant to exclude other aspects of the way they represent meaning. In fact, as we shall see, the iconic quality of diagrams as geometric representations of relations between quantities plays a central role in explaining conceptual change.[1]

The widest early modern application of medieval representations of change, which was also the most far-reaching for the development of modern science, occurred in the context of the analysis of the motion of fall. Let us therefore begin with some historiographic remarks on the discovery of the law of fall. This will provide the background against which we can appreciate what can be gained for an understanding of the early

[1]The iconic quality of diagrams and its central role in reasoning has been emphazised by Charles Sanders Peirce, for instance, when he states that

> [...] since a diagram, though it will ordinarily have Symbolide Features, as well as features approaching the nature of Indices, is nevertheless in the main an Icon of the forms of relations in the constitution of its Object, the appropriateness of it for the representation of necessary inference is easily seen. (Peirce *Collected Papers* 4.531, see Peirce 1967, 415)

Similar views on the role of diagrams in reasoning have recently been taken up again; see, in particular, Stjernfelt 2007. Another example for the central role of geometric representations in conceptual development is provided by the widespread construction of projectile trajectories by early modern mathematicians, see Büttner et al. 2003.

modern conceptual transformations by paying close attention to the symbolic means of knowledge representation.

The law of fall is among the first laws of classical physics to have been formulated. Its discovery, traditionally ascribed to Galileo Galilei (1564–1642), has even been termed "the starting point of the modern era in physics."[2] The law states that, certain assumptions being made, the space a body traverses in its falling motion increases as the square of the time elapsed.

Let us quickly recall the modern derivation of the law, which makes use of key concepts of classical physics. For the derivation of the law of fall in classical physics, it is assumed that the motion of fall takes place in a vacuum and that there is only the gravitational force acting on a falling body. It is further assumed that the distance of fall is so small that the force can be considered constant over that distance. Then, by the proportionality of moving force and acceleration (Newton's second law of motion), the body moves with uniform acceleration, as shown in the uppermost diagram of Figure 1.

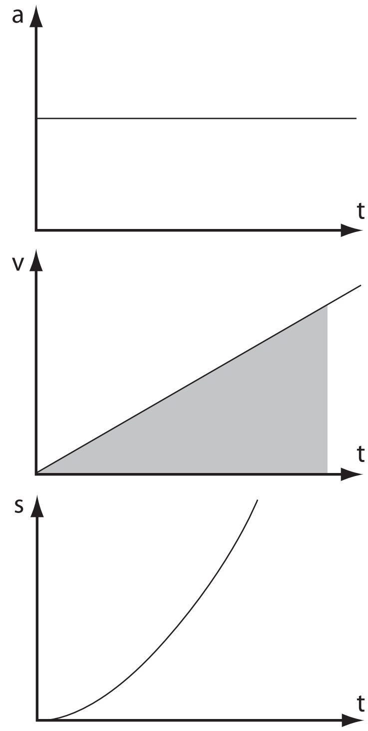

Figure 1: Derivation of the law of fall from the assumption of a constant force of gravity in classical mechanics

In order to derive the change of velocity over time, we integrate the acceleration function with respect to time. The result is a velocity increasing uniformly with time, as shown in the diagram in the middle of Figure 1. (All constants of integration are set to zero.) The integration of the velocity function with respect to time, in turn,

[2]Drake 1973, 90, in the context of an analysis of the notes on Galileo's manuscript page Biblioteca Nazionale, MS Gal 72, f. 152r.

produces the space traversed as a function of time. The graph of the space function turns out to be a parabola, as illustrated in the diagram at the bottom of Figure 1. In other words space s is proportional to the square of the time t. This is the law of fall.

The quadratic relation of space and time can be visualized by noticing that the integral of the velocity function is represented by the area under the graph of the function. This area, which increases quadratically with time and which is indicated by a shaded area in the middle diagram in Figure 1, thus represents the space traversed.

This derivation relies on modern physical and mathematical concepts which only developed in early modern times. In particular, it relies on the concept of integration and the idea that space traversed, velocity, and acceleration are functions derived from one another with respect to time. But the calculus did not exist at the beginning of the seventeenth century, when Galileo and others were occupied with the motion of fall.

Yet, around that time, the law of fall was formulated and mathematically derived independently more than once. Best known is Galileo's formulation of the law, first documented in 1604.[3] An independent formulation of the law resulted from the correspondence between René Descartes (1596–1650) and Isaac Beeckman (1588–1637), and is found in the latter's notebook from 1618 (Beeckman 1939, I, 262).

Alexandre Koyré in his *Études Galiléennes* has called this parallel discovery the most striking incidence of its kind in the entire history of science, because it was a coincidence not only in "truth" but also in "error" (Koyré 1978, 65). By "truth" Koyré referred to the correct law of fall, by "error" he referred to the incorrect assumption of a proportionality between velocity of fall and space traversed rather than time elapsed, as in classical physics. This incorrect assumption is indeed found in both Galileo's and Descartes' early work on the motion of fall.

Besides Galileo and Descartes-Beeckman, another early modern mathematician and philosopher shall be discussed here: Thomas Harriot (1560–1621), an English contemporary of Galileo. Harriot's work is left to us almost exclusively in form of his working notes and was unknown to Koyré. Had he been aware of it, he could have discerned the same coincidence in "truth" and "error" once again in Harriot's early work: the formulation of the correct law of fall and the incorrect assumption of a proportionality between velocity of fall and space traversed.

These historical findings give rise to several historiographic questions. First:

- How can the similarities between the independent early modern derivations of the law of fall be explained?

While the parallel achievements are striking in themselves, the fact that the similarities also concern the shortcomings of the derivations is clearly in need of explanation. So, what were the origins of the common difficulties in deriving the law of fall? This question becomes even more pressing when Pierre Duhem's claims are taken into account. Duhem claimed that the law of fall had not been first formulated in early modern times, but that its key ingredients were already part of the late scholastic body of knowledge.[4] But if the early modern scientists were simply reviving what was already known in the

[3] Galileo to Paolo Sarpi, October 16, 1604, edited in Galileo 1968, 115–116.

[4] "[. . .] on connaissait deux des lois essentielles de la chute des corps [viz. the proportionality between time and speed of fall and the so-called Mean Degree Theorem, which will be explained below and

Middle Ages, where did their new difficulties come from? This brings us to the second historiographic question:

- What was new about the early modern derivations of the law of fall?

As we shall see, neither the physical concepts such as those of velocity and force nor the means of knowledge representation were new at the beginning of the early modern scientists' work. Both concepts and means were still firmly grounded in the Aristotelian and medieval traditions. And this immediately leads to a third historiographic question: the question of how the modern concepts of motion came about:

- How could the modern concepts of motion emerge from work that still employed medieval concepts?

In this contribution it will not be attempted to give comprehensive answers to these questions. The aim is rather to demonstrate, by way of example, what the study of the symbolic means of knowledge representation available in a given culture at a given time in history can contribute to answering such questions.

In the following, the three historiographic questions will be discussed successively from the angle of the role of the symbolic means of knowledge representation. The question of how the similarities between the independent early modern derivations of the law of fall can be explained will lead us to the consideration of symbolic means as embodiments of shared knowledge (Section 2). The question of what was new about the early modern derivations of the law of fall will be approached by focusing on the consequences of the fact that the old means were applied to new purposes (Section 3). As concerns the question of how the modern concepts of motion could have emerged from work that still employed medieval concepts, we will argue that the diagrammatic representations of motion served as catalysts for conceptual development (Section 4). The conclusion then summarizes our results on the role of symbolic means of knowledge representation in early modern derivations of the law of fall (Section 5).

2 Symbolic means as shared knowledge

As explained in the beginning, symbolic means of knowledge representation serve the social reproduction of knowledge and at the same time constitute tools for thinking. They thus mediate between thinking, which takes place in the minds of individuals, and the socially shared knowledge they embody. When individuals approach a given problem or attempt to conceptualize a given phenomenon, they rely on the means of knowledge representation available to them. The set of symbolic means available in a given culture at a given time in history thus defines a space of possible solutions or of possible conceptualizations. The individual solutions and conceptualizations are actual variations within this space of possibilities.

Accordingly, it is impossible, for instance, that the early modern mathematicians quantified motion by means of modern calculus which developed only later. And it

with the help of which the quadratic relation of space traversed and time elapsed can be derived]; en faveur de ces lois, Galilée pourra bien apporter de nouveaux arguments, tirés soit du raisonnement, soit de l'expérience; mais, du moins, il n'aura pas à les inventer." Duhem 1913, 562.

would have been equally impossible for them to use some kind of approximation to it, e.g. by means of infinite series. While in hindsight earlier means may be described by relating them to later ones, for addressing the question of the epistemic role of symbolic means in the history of knowledge, their function and scope have to be described as what they were in their time.[5]

What were the symbolic means to represent functional relations between physical quantities and to conceptualize motion in early modern times? The prevailing concept of motion at that time was provided by the Aristotelian tradition. In particular, the relation between the quantities: velocity of the moving body, space traversed and time elapsed was given by the Aristotelian definition of "the quicker" and its medieval specification.

According to Aristotle, the quicker of two bodies is the one that traverses more space in the same time. Alternatively, the quicker may be defined as the one that traverses the same space in less time.[6] In the Middle Ages, this definition was specified in terms of proportions which were the main tool for representing functional relations. Thus, if the times of motion of two bodies are equal, their velocities are proportional to the spaces traversed. And if the spaces traversed by two bodies are equal, their velocities are inversely proportional to the times:

$$t_1 = t_2 \;\Rightarrow\; v_1 : v_2 :: s_1 : s_2$$
$$s_1 = s_2 \;\Rightarrow\; v_1 : v_2 :: t_2 : t_1.$$

While these statements are compatible with classical mechanics only in the case of uniform motions, no such restriction was prescribed by the Aristotelian tradition. In the sequel these statements will be referred to as the *Aristotelian proportions on motion.* Their application is found in the work on motion of Descartes, Beeckman, Galileo, and Harriot.

Although the Aristotelian proportions were not restricted to uniform motion, they were not sufficient to quantify non-uniform motion. To achieve such a quantification, the early modern philosopher-mathematicians mainly had one other means at their disposal: a particular sort of diagram of change. From the time of their earliest occupation with the relation of time elapsed and space traversed in the motion of fall, Descartes, Galileo, and Harriot all used a particular kind of triangular diagram. And in all three cases the terminology employed clearly reveals the diagrams' origins in a medieval tradition. A brief deliniation of this tradition is in order.

The diagrams emerged in the context of a scholastic discourse on the *intension and remission of forms* (*intensio et remissio formarum*), which itself goes back to Aristotle and which constituted a central topic in the scholastic literature.[7] It concerned the

[5]On the concept of *shared knowledge* in the history of early modern mechanics, see Büttner et al. 2001 and Schemmel 2006.

[6]Both statements are found in a more involved discussion of "the quicker," Aristoteles *Physics* VI, 232a23–232b20.

[7]See Maier 1968, 3–109. A much shorter presentation of the prehistory of the late medieval treatment of qualities and motions is given in Clagett 1950. The present account is mainly based on these two works.

change of a wide variety of qualities, such as sensory qualities like heat ore whiteness, theological qualities like the grace of god, and psychological-moral qualities like virtues.

The history of the discourse can be roughly divided into three phases. During the earliest phase, in the thirteenth century, the discussion was mostly of philosophic-ontological character. It dealt with the question of how it is to be understood that qualities can increase or decrease within a concrete subject. What was changing when, for instance, something became hotter or whiter, or when a person became more righteous? Was one dealing with a change of the quality itself, or rather with a change of the concrete subject that was informed by the quality?

The second phase of the history of the doctrine of intension and remission, beginning in the early fourteenth century, is marked by mathematization. The focus of the discussion shifted from the ontological question of the meaning of change to its quantitative treatment.[8] The quantitative method flourished in Oxford, where the so-called school of *calculatores* developed at Merton College.[9] The *calculatores* performed detailed quantitative studies on motion and change and thereby created a shared conceptual framework whose influence went far beyond Merton College. Through application of the same scheme to qualities as well as to local motion a new kinematic vocabulary emerged, which was particularly suited for the description of non-uniform motion.

In the middle of the fourteenth century, then, a diagrammatic method was built upon the philosophic-mathematical conceptual framework. This marked the beginning of the third phase. The emergence of the diagrammatic method is usually associated with the

[8] Anneliese Maier interprets this as a process that takes place following changes in arithmetics which occurred already in the thirteenth century under the decisive influence of Jordanus of Nemore (Maier 1952, p. 258). This process led to the quantitative treatment of all possible qualities, even such qualities that were not only impossible to measure at that time, but for which we would today principally deny the possibility of quantitative measurement, such as the quality of benevolence.

[9] The most prominent Oxford *calculatores* were Thomas Bradwardine, William Heytesbury (Hentisberus), Richard Swineshead (Suisset) and John Dumbleton. Very little is known about the biographies of these authors and it has been questioned whether they actually formed a 'school' (Weisheipl 1959, 439–441). We give a brief outline of the central texts of this group and the dates when their authors are documented to have been at Oxford (Clagett 1959, 199–205). Thomas Bradwardine's *Tractatus de proportionibus* of 1328 is arguably the earliest text of the Merton Group dealing with the quantitative description of motion. A later text, also touching the subject, *De continuo*, was possibly written by Bradwardine as well. Bradwardine left Merton College in the 1330s and died in 1349. Wilhelm Heytesbury's *Regule solvendi sophismata* of 1335 is another early text of its kind. It is the first text we know in which the *Mean Degree Theorem* occurs, which will be explained below. Another text, *Probationes conclusionum*, was probably written by Heytesbury. It contains proofs of several theorems of the *Regule solvendi sophismata*, among them the earliest proof of the Mean Degree Theorem. Heytesbury was a member of Merton College already in 1330. In 1338 he obtained a scholarship from that college. Later, in 1371, he became Chancellor of the University of Oxford. Of Richard Swineshead one can safely say that he was active at Merton College between 1344 and 1355. His *Liber calculationum* was obviously written after Bradwardine's *Tractatus de proportionibus* of 1328. Swineshead is often judged to be the most subtle of the logicians of the Merton Group and in later references he is often simply called *Calculator*. John Dumbleton is noted in the Merton College registers in 1331(?), 1338, 1344, and 1347–8. His contribution to the logico-physical literature under discussion is the *Summa de logicis et naturalibus*. Furthermore, there is a text of an unknown author, the *Tractatus de sex inconvenientibus*, written by an adept of the Merton Group, who was possibly himself at Oxford. The text builds upon Bradwardine's *Tractatus de proportionibus* as well as on Heytesbury's *Regule solvendi sophismata* and deals with definitions of velocity for the different categories of motion (i.e., change).

name Nicolas Oresme (ca. 1320–1382), who called it the *doctrine of the configuration of qualities and motions*.[10] The diagrammatic method was of great influence and many of the earlier writings were later richly illustrated with Oresme-type diagrams.

In order to understand how an Oresme-type diagram was understood in the Middle Ages it is important to note that in the scholastic tradition, a quality or a motion was considered in two respects: its extension and its intension (or intensity). The basic idea of the geometrical representations was then to depict the extensive and the intensive dimensions by orthogonal straight lines (see Figure 2).

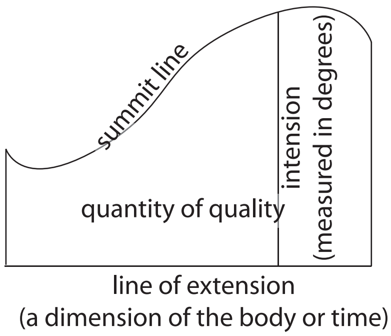

Figure 2: An Oresme-type diagram and its related terminology

In the medieval tradition, the extension is usually represented by a horizontal line. It is either a dimension of the body, or it is the time over which the variation of the quality or motion takes place. Thus, the line of extension may for instance represent the length of an iron rod along which the heat varies, or it may be the time over which the heat at a given point on the rod varies.[11] The intension of a quality or motion at any point of the extension is represented by a vertical line. The ratio of the lengths of any two lines represents the ratio of the intensities at the corresponding points of the extension. The intension is measured in degrees. This, of course, does not refer to a physical measurement. All processes are considered only *secundum imaginationem*. By drawing the line of extension, the lines of intension at its end points, and the *summit line* (*linea summitatis*), which is composed of the upper end points of all possible lines of intension, one obtains a closed figure which represents the variation of the quality or motion over the extension. The area of the figures represents the 'quantity of the quality'.

In the case of local motion, the extension is again either given by a dimension of the moving body or by the time in which the motion takes place (see Figure 3). The intension is measured in degrees of motion or degrees of velocity. And the area represents the 'quantity of motion' or the 'total velocity'. This area was further identified with

[10]Oresme presented this method in two works, the *Tractatus de configurationibus qualitatum et motuum*, which was probably written around 1350, and the *Questiones super geometriam Euclidis* of unclear production date (Maier 1952, 270, 289). Both texts have been edited and translated into English in Clagett 1968.

[11]Oresme also considers extensions of higher dimension, viz. surfaces and solids (see, for instance, Clagett 1968, 530), but he develops his figures only for the case of a one-dimensional extension.

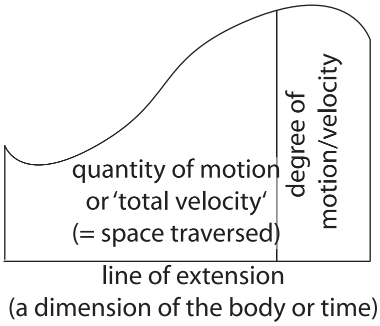

Figure 3: An Oresme-type diagram of motion and its related terminology

the space traversed in that motion. The reason for this simply lies in the Aristotelian definition of velocity as space traversed with the implicit qualification: in a given time.[12]

Using the diagrammatic method, many theorems of the Oxford *calculatores* become very clear. Thus, the *Mean Degree Theorem*, a version of which figures prominently in Galileo's final derivation of the law of fall and which is first found in a text by Heytesbury from 1335, the *Regule solvendi sophismata*, is illustrated by Oresme in a similar way as shown in Figure 4 and formulated as follows:

> Every quality, if it is uniformly difform, is of the same quantity as would be the quality of the same or equal subject that is uniform according to the degree of the middle point of the same subject. (Clagett 1968, 409)

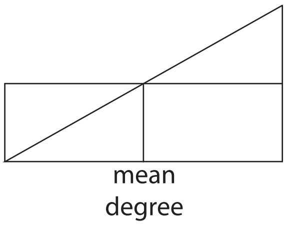

Figure 4: Oresme-type representation of the Mean Degree Theorem (similar to Clagett 1968, 409)

The *uniform* quality is represented by the rectangle. It is called uniform since its intension does not vary over the extension. The *uniformly difform* quality is the one represented by a triangle. It is not uniform, since the intension varies over the extension, but its difformity is uniform, since the intensity varies uniformly, so that the summit line is still a straight line. The equality of the quantity of the uniform and the uniformly difform qualities is proven by the equality of the rectangle's and the triangle's areas.

[12]See, e.g., Clagett 1968, 414. On this aspect of the medieval concept of velocity, see, e.g., Maier 1952, 286, 316, and 338–340. Compare also Clagett 1968, 278, where Oresme states that the velocity of motion in a straight line is measured (*attenditur*) by space traversed.

Applying this statement to local motion, we obtain a reduction of uniformly accelerated motion to uniform motion. The mean degree theorem was crucial in the treatment of uniformly accelerated motion, since it allowed it to be replaced by the much simpler case of uniform motion.

Furthermore, Oresme formulates ratios of quantities of a quality as they depend on the ratios of sections on the line of extension. In particular, he formulates the following statement, which may be called the *Squared Number Law*:

> [. . .] in the case of every subject uniformly difform [in quality] to no degree, the ratio of the whole quality to the quality of a part terminated at no degree is as the square of the ratio of the whole subject to that part [of the subject]. (Clagett 1968, 557)

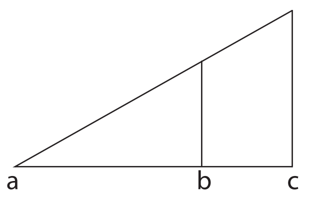

Figure 5: Oresme-type representation of a uniformly difform quality to no degree

The uniformly difform quality is again represented by a triangle (see Figure 5). The triangle's left acute corner at 'a' is the termination "at no degree." Now, the triangle's area over the line 'ac' to the area over 'ab' is as the square of 'ac' is to the square of 'ab'. If applied to motion, this law describes the relation of space traversed and time elapsed as in the law of fall (in which case 'a' would now be the starting rather than the end point of the motion). Oresme did not apply his scheme to the motion of fall, however.

Note the difference between the modern derivation of the times-squared law according to classical mechanics, illustrated on the left in Figure 6, and the medieval derivation illustrated on the right. In the modern case, the uniform increase of velocity is related to a uniform acceleration which makes it possible to establish a dynamical argument involving the constant force of gravity. While in the Middle Ages many dynamical arguments were made in connection with the motion of fall, a relation to the doctrine of the diagrams was not usually made. And while in the modern case the interpretation of the area under the graph of the velocity function as space traversed follows from an integration, in the medieval doctrine the interpretation of area as space is derived from its interpretation as total velocity: According to the Aristotelian definition of velocity as space traversed (in a given time), velocity is measured by space.

Summing up, we can state that in early modern times the means to describe non-uniform motion were limited. All protagonists under consideration mainly employed two such means: The Aristotelian proportions and the medieval diagrams of change. These shared means may explain many of the common traits of early modern work on motion, as will become clear in the following. We have also seen that within Oresme's

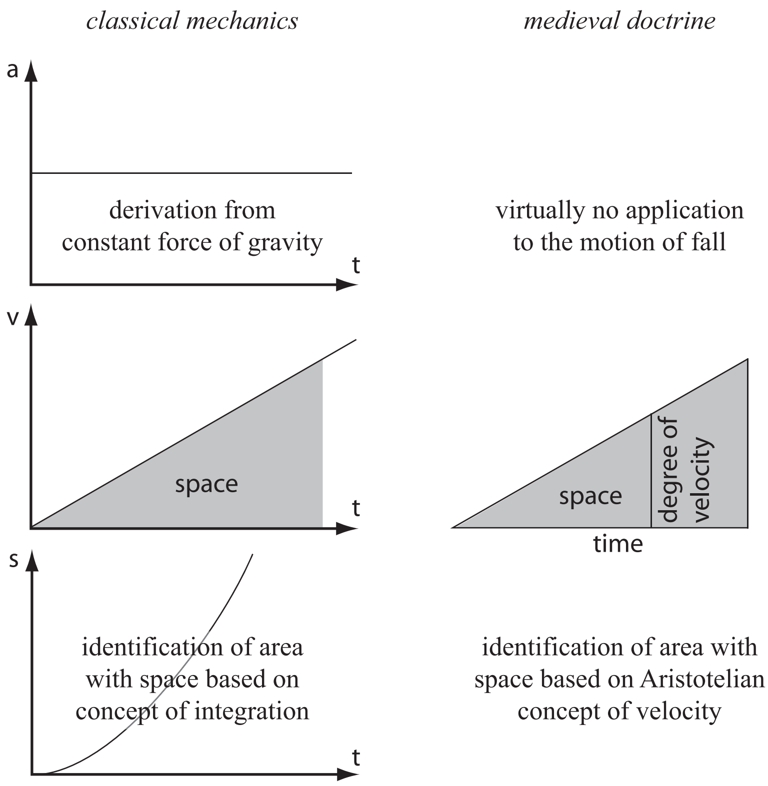

Figure 6: Comparison of the derivation of the times-squared law in classical mechanics and in the medieval tradition

scheme the correct law of fall could be derived. This was the basis for Duhem's claim that in early modern times a derivation of that law did not have to be newly invented. But why then did this derivation present problems to the early modern mathematicians? This leads us to the question of the next section: What was new about the early modern derivations of the law of fall?

3 Old means for new purposes

The most obvious difference between the medieval use of the diagrams of change and their early modern use is the context of their application. In medieval times, the diagrams were part of a general theory of change, often employed in the context of logico-mathematical analysis of sophisms.[13]

The modern question concerning the relation between distance and time of fall did not come up in a discourse that was more concerned with deepening the logico-mathematical techniques of disputation than with natural knowledge and its practical dimensions. The disparity between the scholastic and the modern question becomes most obvious exactly at those places where the scholastic tradition does connect its conceptual scheme to the motion of fall. In most of these rare cases, the motion of fall simply serves as an example for one or the other kind of motion. Thus, the motion of a falling stone is understood as an example of uniform motion—*quoad subiectum*, since all parts of the stone fall with equal degree of velocity![14] And as an example for uniformly difform motion over time the motion of fall competes in the late scholastic texts with the motion of Socrates who continually increases his pace.[15]

The main problems addressed by means of these diagrams in early modern times, by contrast, were not part of the scholastic tradition. They originated in a different tradition, that of practical mathematics. The new context of application of the medieval means was thus a direct consequence of the increasing entanglement of practical and theoretical knowledge traditions in the work of early modern mathematicians, which, it has been argued, lies at the roots of the early modern Scientific Revolution (see, e.g., Zilsel 2000, 7 and *passim*). Thus, the question of the functional relation of space traversed and time elapsed in the motion of fall was clearly related to concerns with the mathematical treatment of technology-related phenomena like projectile motion, or the effect of machines that work by striking.[16] This new purpose to which the old

[13]For instance, when diagrams were used in later printed editions of Heytesbury's *Regule solvendi sophismata* (see, e.g., Heytesbury 1494). For a discussion of this particular work and its physico-mathematical content in the context of the scholastic tradition of *sophismata*, see Wilson 1956.

[14]For instance, John of Holland and Gaetano of Thiene, see Wallace 1968, 386, 390.

[15]The example of Socrates is mentioned, for instance, by John of Holland and John Dullaert, see Wallace 1968, 386, 394.

[16]The practical context of Galileo's occupation with the motion of fall, for instance, is indicated when he argues for the (fallacious) proportion between distance and speed of fall by stating that

> [t]his principle appears to me very natural, and one that corresponds to all the experiences that we see in the instruments and machines that work by striking, in which the percussent works so much greater effect, the greater the height from which it falls [...] (Biblioteca Nazionale, MS Gal 72, f. 128)

or when he points out that

means were put had an impact on the way the means were understood. The difficulties in deriving the law of fall by means of the diagrams mainly resulted from the new interpretations given to them. Let us begin with Descartes and Beeckman and see how they understood the diagrams.[17]

Beeckman had developed a theory of motion according to which "in a vacuum what is once in motion will always be in motion" (Beeckman 1939, I, 263, Descartes 1964–74, X, 60).[18] He now asked Descartes to derive for him mathematically the relation of space and time in the motion of fall, "supposing that there is a vacuum between the Earth and the falling stone" (Beeckman 1939, I, 263, Descartes 1964–74, X, 60).

Descartes develops his answer starting from dynamical considerations and arrives at the diagram shown in Figure 7. From his deliberations, the following interpretation of the diagram's elements can be inferred. The line of extension (which he draws vertically) represents time. The horizontal lines represent the increasing force causing the motion. The area of the figure represents motion. The identification of extension with time and area with motion fully complies with the medieval tradition, within which, as we have seen, the law of fall can easily be derived.

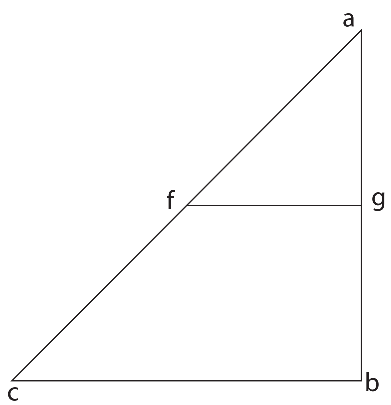

Figure 7: Descartes' diagram of motion, 1618 (Descartes 1964–74, X, 77)

Descartes' formulation of his result is somewhat ambiguous, however. He states:

> The part 'gb' which is half will be covered by the stone three times more quickly than the other half 'ag', because it will be drawn by the Earth with

> [. . .] the naturally falling body and the violent projectile pass through the same proportions of velocity. (Galileo to Paolo Sarpi, October 16, 1604; Galileo 1968, X, 115–116)

For recent work on the practical context of Galileo's scientific work, see Valleriani 2010. Harriot's manuscripts equally bespeak the close connection of practical and theoretical concerns in his work on motion, see Schemmel 2008, in particular 25–37.

[17] A systematic analysis of Descartes' and Beeckman's work on the motion of fall is given in Damerow et al. 2004, 24–69. The broader context of the use of medieval diagrams of change in the seventeenth-century transformation of mechanics is presented in Bertoloni Meli 2006.

[18] This and the following translations from Beeckman, Descartes, and Galileo are taken from Damerow et al. 2004.

> three times the force. The space 'fgbc' is three times the space 'afg', as is easily proved. (Descartes 1964–74, X, 77)

How is this statement to be understood in terms of space and time? Beeckman and Descartes interpreted it in different ways. Let us first look at Beeckman's interpretation, which is documented in his *Journal* of 1618 (Beeckman 1939, I, 260–265, see also Descartes 1964–74, X, 58–61).[19]

Beeckman identifies the area of a diagram with space traversed. The line of extension he again understands as representing time. His interpretation of the crucial elements of the diagram thus fully coincides with the one given by the medieval tradition. Accordingly, he arrives at the correct law of fall, which he formulates as follows:

> The space through which it [the heavy body] falls in two hours doubles the proportion of time [...] (Beeckman 1939, I, 262)

in other words: space increases as the square of the time.

But Descartes concretized his own solution in a different way. He formulated his interpretation in a diary note and repeated it later in letters to Marin Mersenne (1588–1648). From Descartes' explanations of his solution to Beeckman's problem, the following interpretation of the elements of the figure emerges: The area of the figure is interpreted as the velocity, or the "swiftness" (*celeritas*) of the motion as in the medieval tradition. But now Descartes does not interpret the vertical line of extension as the time of fall as Beeckman did and as would have been natural in the framework of the medieval tradition—and as he himself had done in his letter to Beeckman—but rather as the space traversed! Descartes now interprets the diagram as showing that the falling body traverses the space 'gb' three times more quickly than the space 'ag'.

Descartes' interpretation employs the Aristotelian proportions on motion. The segments of the line of extension represent equal intervals of space. The corresponding areas represent the velocities. Now, according to the Aristotelian proportions, for equal spaces the times are in inverse proportion to the velocities. Since the velocities are as 1 to 3, the times are as 1 to 1/3.

If we were to continue Descartes' argument, the times to traverse successive equal intervals of space would be to each other as 1, 1/3, 1/5, 1/7, ... Accordingly, the times needed to traverse spaces beginning at point 'a' (Figure 7) and increasing according to the sequence of natural numbers 1, 2, 3, 4, ..., would be to each other as 1, 1 + 1/3, 1 + 1/3 + 1/5, ... This is in contradiction with the law of fall, according to which these times should be to each other as the square roots of the corresponding spaces.[20]

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[19] Following Descartes' derivation, Beeckman approximates the triangle by squares and then considers more and more squares of lesser and lesser size to demonstrate that the arrangement of squares approaches the triangle 'ACB'. Beeckman's and Descartes' interpretations are systematically compared in Damerow et al. 2004, 29–31.

[20] From his correspondence with Mersenne it is known that Descartes considered another way of extending the study of the motion of fall beyond the initial first two units of space (Descartes to Mersenne, November 13, 1629, Descartes 1964–74, I, 69–74). Descartes' considerations had yielded the ratio 1 to 1/3 for the times in which the first two units of space are traversed. So if the first unit was traversed in 9 units of time, the second was traversed in 3 and the first two units taken together in $9 + 3 = 12$ units of time. But since this ratio of 1 to 1/3 had, of course, to be independent of the units chosen

The contradiction seems not to have been clear to Descartes and Beeckman, however. Nowhere in their notes and correspondence do we find any evidence for an awareness that there are two incompatible interpretations of the diagrams, one in which the extension represents time, and one in which it represents space. On the contrary, there is evidence that Descartes believed the two interpretations to be equivalent.

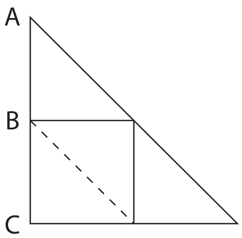

Figure 8: Descartes' diagram of motion, 1634 (Descartes 1964–74, I, 304)

In a 1634 letter to Mersenne, after having seen Galileo's *Dialogo* in which Galileo formulates the correct law of fall, Descartes states that he had formulated that law before. Referring to Figure 8 he explains that

> [. . .] the spaces traversed by heavy bodies when they fall are to one another as the squares of the times which they employ to descend, i.e. [. . .] if a ball employs three moments to descend from A to B, it will employ but one to continue from B to C, etc. (Descartes to Mersenne, August 18, 1634, Descartes 1964–74, I, 304)

While the first part of the sentence is a reiteration of the correct law of fall, the second part is a reiteration of Descartes' fallacious theorem, which is incompatible with it. Where does this confusion come from? Are we dealing here with an individual blunder of Descartes?

Actually, we find the same conflation of temporal and spatial interpretations of extension in Galileo's and Harriot's work on motion. Galileo had arrived at the insight into the law of fall most probably through considerations and experiments on projectile motion (see Renn et al. 2001). In a letter to Paolo Sarpi from 1604 he formulates the

it should equally apply to double units of space; i.e., the time needed to traverse the first two units of space to the time needed to traverse the third and fourth unit of space could again be assumed to be as 1 to 1/3. In other words, if the first two intervals of space were traversed in 12 units of time, the third and fourth should be traversed in $12/3 = 4$ units of time, and the first four units of space accordingly in $12 + 4 = 16$ units of time. The sequence of 9, 12, and 16 units of time in which the first, the first two, and the first four units of space must, according to this argument, respectively be traversed is what is found in Descartes' correspondence (Descartes 1964–74, I, 72–73). These numbers are again in contradiction with the law of fall as they also fail to be to each other as the square roots of the corresponding spaces. In addition they are incompatible with the above continuation of Descartes' argument which yielded a sequence $1, 1 + 1/3, 1 + 1/3 + 1/5, \ldots$ for the units of times. Descartes could have discovered the contradictions by exploring the numerical examples further and comparing the results (Damerow et al. 2004, 60–64). As will be explained below, a systematic exploration of the different laws and the discovery of the contradictions between them and the squared times law of fall can be discerned in Harriot's manuscripts.

law and claims to have found a "completely indubitable principle to put as an axiom" from which to derive the law (Galileo to Sarpi, October 16, 1604; Galileo 1968, X, 115–116). This principle was the proportionality between the velocity and the space traversed, which is in fact incompatible with the law of fall.

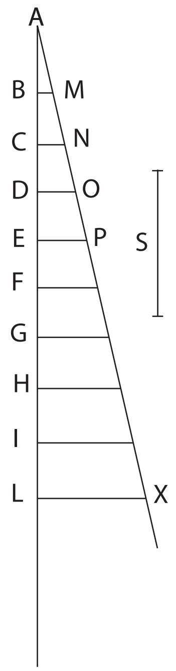

Figure 9: Galileo's diagram of motion, ca. 1604, (Biblioteca Nazionale, MS Gal 72, f. 85v)

In Galileo's manuscript notes on motion there are different attempts to derive the law of fall from this erroneous principle, all making use of diagrams of change. Let us consider one of these diagrams, reproduced in Figure 9.[21] Galileo interprets the extension, represented by the vertical line 'AL', as space traversed, as we have seen it in Descartes. The horizontal lines he calls "velocities," in other places he also refers to them as "degrees of velocity" or "moments of velocity," in accordance with the medieval terminology. The area of the figure is made up of the "velocities in all points" of the extension. Galileo then applies the Aristotelian proportions on motion to the diagram in order to relate the spaces represented by segments of the line of extension with the times. His derivation is ill-formed, however, so that he succeeds in deriving the correct

[21] The folio page the diagram is taken from (Biblioteca Nazionale, MS Gal 72, f. 85v) contains a complete (albeit fallacious) derivation of the law of fall from the assumption of a proportionality between velocity and space. Another example is found on MS Gal 72, f. 179v, which contains the beginning of a derivation. On this folio, Galileo writes about "distances" on the line of extension, which he clearly conceives of as representing space, and about "moments of velocity," which he claims to be in double proportion to the distances (Damerow et al. 2004, 168). Obviously, by "moments of velocity" he refers to what he had called "degrees of velocity" in his letter to Sarpi, and he probably thought of the set of all moments of velocity over a certain distance as being represented by the corresponding area of the diagram.

law of fall from the incorrect assumption of a proportionality between velocity and space.[22]

The same ambiguity that we have discerned in the work of Descartes, Beeckman, and Galileo is also found in the work of Thomas Harriot.[23] Harriot, in his early application of the diagrams, seems not to have been aware of the incompatibility of the temporal and spatial interpretations of extension. On the one hand, he made use of the diagrams in their medieval interpretation and derived from them the correct law of fall, as was the case for Beeckman. On the other hand, there are notes in which he clearly interprets the line of extension as representing space.

As an example, consider the diagram reproduced in Figure 10. In the accompanying text, Harriot refers to the horizontal lines as degrees of motion, in accordance with the medieval tradition. When he interprets the line of extension (which is vertical) as representing time, he interprets the area as space, just as in the medieval tradition. When he interprets the line of extension as representing space, he interprets the area as time, apparently in plain analogy to the medieval interpretation. There is evidence that Harriot initially believed both interpretations to be compatible with the law of fall.[24]

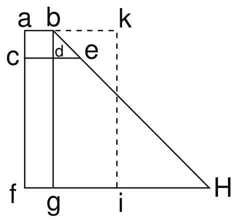

Figure 10: Harriot's diagram of motion, ca. 1600 (British Library, Add MS 6789, f. 30r)

Summing up, the common deviations from the medieval interpretations of the diagrams of change in the work of Descartes, Galileo, and Harriot can be understood as resulting from a change in the context of application of the diagrams. The canonical interpretation of the line of extension of a diagram as time elapsed was very natural within the medieval doctrine which was generally concerned with change. Change, after all, always takes place in time. For the early modern considerations on the motion of fall without regard to a general theory of change, in contrast, no such obvious argument for a restriction to a temporal extension was available. If the incompatibility of temporal and spatial interpretations of the diagrams had been perceived by the early modern scientists, any commitment to either interpretation would have had to be counted as

[22] At this point in time, Galileo had not yet explored the correct assumption of a proportionality between velocity and time; accordingly it must be assumed that he was unaware of the fact that it constituted a real alternative to his assumption of a proportionality between velocity and space.

[23] For a detailed discussion of Harriot's extensive work using diagrams of motion, see Schemmel 2008.

[24] Schemmel 2008, 65–69. Note furthermore the dotted line 'bKi'. This line denotes the uniform motion of mean degree and thus documents Harriot's knowledge of the Mean Degree Theorem. While this theorem applies in the case of a temporal extension, it does not apply in the case of a spatial extension, a fact that Harriot initially ignored (Schemmel 2008, 69–70).

premature in view of the task to determine the relation of space and time in the motion of fall. But as the cases of Descartes, Harriot, and others suggest, initially the two possibilities were not even conceived of as mutually exclusive alternatives but rather as two ways of looking at the same thing. As long as the quantitative consequences of the spatial and temporal interpretations had not been explored and compared to each other, it did not occur to the early modern mathematicians that they were incompatible.[25]

Thus the new context of application of the diagrams changed the way they were understood. The possible meanings of a diagram's elements were less canonized and partly contradicted each other. In view of the fact that the law of fall could be neatly derived within Oresme's scheme, this may seem to have been a step backwards. Actually, however, as shall be argured in the next section, it was a precondition for conceptual development, a development which eventually led to the new kinematic concepts of classical mechanics.

4 Diagrams as catalysts for conceptual development

It was only by exploring the implications of the diagrams' different interpretations that the early modern scientists discovered that the spatial and the temporal interpretations implied two distinct kinds of motion. It was further only by such exploration of the diagrams' implications that they arrived at well-founded arguments for identifying a diagram's area with space.

Galileo, for instance, tried to derive by means of the diagrams not only the law of fall, but also other theorems, such as his so-called *Double Distance Rule* for which he employed the diagram reproduced in Figure 11.[26] The Double Distance Rule is closely related to the Mean Degree Theorem. It states that the distance traversed by a uniform motion lasting for a certain time is double the distance traversed by a uniformly accelerated motion lasting for the same time, starting from zero (degree of) velocity and ending with the (degree of) velocity of the uniform motion. The rectangle 'abcd' represents a uniform motion which has a degree of velocity 'ad' and traverses the space 'ab'. The triangle 'abc' represents an accelerated motion. The Double Distance Rule now states that, in a given time, the space traversed by the uniform motion is double the space traversed by the accelerated motion.

In these notes on the Double Distance Rule, Galileo again employs a spatial interpretation of extension. The area of a diagram he conceives of as representing "all velocities." Comparing the triangular diagram and the rectangular one of double area, he argues that the rectangular diagram represents a motion that traverses the space 'ab' "twice as fast," or—by the Aristotelian proportions—twice the distance in the same time (Biblioteca Nazionale, MS Gal 72, f. 163v).

But despite the fact that he derived both the law of fall and the Double Distance Rule on the basis of a spatial interpretation of extension, he later recognized that, taken

[25]Another interesting case is Leonardo da Vinci, whose manuscripts contain what is probably the earliest known application of Oresme-type diagrams to the motion of fall. Leonardo describes the extension as representing time but assumes space traversed to be proportional to time elapsed. See the translation and discussion in Clagget 1959, 572–575.

[26]The early development of Galileo's theory of motion is analyzed in detail in Büttner 2009.

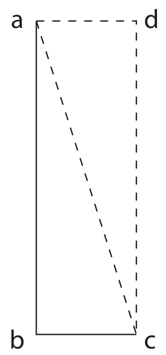

Figure 11: Galileo's diagram for his derivation of the *Double Distance Rule*, ca. 1604 (Biblioteca Nazionale, MS Gal 72, f. 163v)

together, the two theorems contradicted his assumption of a proportionality between velocity and space. In this context, he also discovered that by assuming a proportionality between velocity and time one could avoid the contradiction.[27] This is how Galileo arrived at the insight that a temporal interpretation of extension must be used as a starting point for deriving the law of fall.

According to this insight, in his mature work on mechanics, the *Discorsi* of 1638, Galileo presents a proof of the law of fall on the basis of a temporal interpretation of extension. A crucial step in this derivation is a proof of the Mean Degree Theorem which employs the diagram shown in Figure 12. The line of extension is now interpreted as time.[28]

Galileo thus arrived at a greater explorative depth than was achieved in the medieval tradition. He used the temporal interpretation of extension not simply because this was

[27]On the manuscript page Biblioteca Nazionale, MS Gal 72, f. 91v, Galileo uses the Double Distance Rule to prove that the degree of velocity increases in proportion with time elapsed, an assertion that contradicts his earlier assertion that it increases in proportion with space traversed. He probably first encountered a contradiction between law of fall, Double Distance Rule and proportionality of velocity with space traversed on MS Gal 72, f. 152r. On this folio he also considers two ways of representing motion uniformly difform with respect to time: (1) a triangular diagram with temporal extension, thus representing motion uniformly difform over time, and (2) a diagram with spatial extension representing the same motion which therefore takes on a parabolic shape. See the discussion in Damerow et al. 2004, 182–197. Similar comparisons of representations of motion uniformly difform over time with diagrams of either temporal or spatial extension are found in Harriot's manuscripts, e.g., on British Library, Add MS 6789, f. 53r, see Schemmel 2008, 87–88.

[28]The horizontal lines are referred to as "degrees of velocity." So far the interpretation coincides with the medieval one. The area of the diagram, however, is explicitly *not* identified with the space traversed. That space is rather represented by the separate line 'CD'. Rather than directly comparing the areas of the rectangle 'GFBA' and the triangle 'AEB', as Oresme did when proving the Mean Degree Theorem, Galileo compares two infinite sets of degrees of velocity, the one of the triangular diagram and the one of the rectangular diagram. He argues that there are equally many moments of speed consumed in both motions, since the (triangular) deficit 'GIA' of the accelerated motion is made up by the (triangular) surplus 'IEF' in the second half of the motion. He then applies the Aristotelian proportions in order to introduce the spaces traversed in both motions and to derive their equality. The problematical point of this "proof" is the matching of two infinite sets (compare Galluzzi 1979, 354; Blay 1998, 73–75; Damerow et al. 2004, 241).

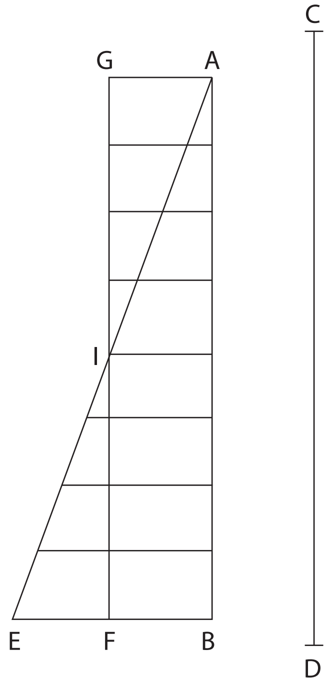

Figure 12: Galileo's diagram for the derivation of his version of the Mean Degree Theorem, 1638 (Galileo 1968, VIII, 208)

the canonical way to describe change, but because he had arrived at the insight that the spatial interpretation contradicted his other findings.

But despite this insight into the correct interpretation of extension, Galileo's final proof of the law of fall fails to be rigorous. This can most easily be seen by noting that his derivation of the Mean Degree Theorem could equally be applied to a triangular diagram with spatial extension, in which case it would lead to an incorrect result.[29] Galileo eliminates the spatial interpretation only *post hoc* by arguing that it implies the motion to be instantaneous. And even this *post-hoc* exclusion of the spatial interpretation is invalid: Actually, motion uniformly difform with respect to space implies a logarithmic relation between space traversed and time elapsed and therefore takes place only in the case of a finite initial velocity.[30] Paradoxically, Galileo's argument is wrong exactly in

[29]Galileo's derivation of the Mean Degree Theorem is sketched in footnote 28 on page 20.

[30]Be v_0 the initial velocity and c some constant having the dimension of reciprocal time, then the relation between elapsed time t and traversed space s is

$$t = \frac{1}{c} \ln \frac{v_0 + cs}{v_0}.$$

The velocity as a function of time is

$$v(t) = v_0 e^{ct}.$$

From this it can be read off that $v(t)$ is different from zero for any finite time only if $v_0 \neq 0$. For a derivation and discussion of these formulae, see Schemmel 2008, 57–58.

presupposing the validity of the Mean Degree Theorem for a triangular diagram with spatial extension, for which it does not hold.[31]

Galileo's derivation was improved in 1673, when Christiaan Huygens (1629–1695) performed "more accurately the demonstration which he [Galileo] gave in a less perfect form" (Huygens 1986, 40), now providing a rigorous derivation of the law of fall from the assumption of the proportionality between velocity and time, using an approximation of uniformly accelerated motion by an increasing number of uniform motions continuing for ever shorter times.

Harriot had also initially believed in the applicability of the Mean Degree Theorem to both diagrams of temporal and of spatial extension. This is indicated by the dotted line 'bKi' in the diagram mentioned before (see footnote 24 on page 18). By exploring the consequences of this and other assumptions about the quantitative relations implied by the diagrams, Harriot's understanding of the diagrams in terms of mechanical quantities gradually changed.

Harriot's manuscripts document a much more systematic exploration of the diagrams than can be found in the work of Galileo or Descartes. Accordingly, they provide a unique source for reconstructing the horizon of possibilities of early modern applications of the medieval representations of change. In this contribution we can only briefly discuss some of Harriot's results.[32]

In his early notes, Harriot had either interpreted extension as time, in which case he interpreted area as space, or he had interpreted extension as space in which case he interpreted area as time. In notes he took between 1600 and 1606, Harriot carefully examined the case of uniform motion and encountered contradictions within the latter interpretation.

In these notes, Harriot considered six propositions concerning uniform motion formulated in a late scholastic work, the *Liber de triplici motu* by the Portuguese Alvarus Thomas, published in 1509 (Alvarus 1509, 73v–74v). The propositions include the Aristotelian proportions on motion—now explicitly applied to uniform motion—but also statements about motions that differ in the space they traverse as well as in the times they last for, statements that in all cases but one involve compund ratios of times and velocities. In the notes reproduced in Figure 13, Harriot illustrates Thomas' first two propositions with two diagrams each: one with temporal extension on the left and one with a spatial extension on the right. In the first case Harriot identifies the area of the figure with space; in the second, with time, as he had done in his earlier notes. In all diagrams, the vertical lines represent the degrees of velocity, the horizontal lines the extensions. Only lines of extension representing space are drawn out, all other lines are drawn as dashed lines.

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[31]This claim concerning Galileo's failure has earlier been made (Cohen 1956) and been disputed (Drake 1970). In any case, in his argument, Galileo makes use of the statement that "[w]hen speeds have the same ratio as the spaces passed or to be passed, those spaces come to be passed in equal times [. . .]" (Galileo 1968, VIII, 203). This is the reverse of the first of the Aristotelian proportions on motion. It applies to uniform motion and would apply to motion uniformly difform over space only if this kind of motion could be reduced to uniform motion, e.g., by means of the Mean Degree Theorem.

[32]For a more comprehensive discussion, the reader is referred to Schemmel 2008, Chapters 4–6.

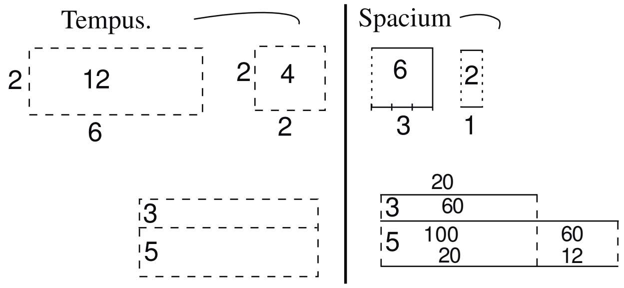

Figure 13: Harriot's diagrams illustrating propositions on uniform motion, having either temporal (left) or spatial (right) extensions, ca. 1602 (British Library, Add MS 6789, f. 51r)

In his first proposition, Thomas states that if the velocities of two uniform motions are equal, the ratio of the spaces equals that of the times. As a numerical example, Harriot considers two motions of a velocity of two degrees lasting for six and two units of time, respectively. He is able to illustrate the example consistently with both types of diagram (see the first row in Figure 13): $12/4 = 6/2$ (left) as well as $3/1 = 6/2$ (right).

In his second proposition, Thomas states that if the velocities of two uniform motions are unequal, but the times in which the motions take place are equal, the ratio of the spaces traversed equals that of the velocities. Harriot again considers a numerical example (see the second row in Figure 13). He is able to illustrate it consistently in the case of a diagram with temporal extension. In the case of a diagram with spatial extension, however, he encounters a contradiction. When he makes the areas of the rectangles representing the times of the two motions equal, as is required by the premise of the proposition, the resulting ratios of spaces and velocities are unequal, in contradiction to the proposition. (In the example, the areas of the diagrams of both motions are 160 units of time, but while the ratio of velocities is $8/5$, the ratio of spaces is $20/32$.)

It is important to note that in his analysis of uniform motion, Harriot identified the *velocity* figuring in Thomas' propositions with the *degree of velocity* of a diagram, and not with the *quantity of velocity* represented by a diagram's area, as Galileo and Descartes had done for the velocity figuring in the Aristotelian proportions. In this way he eventually achieved a consistent representation of uniform motions by means of rectangular diagrams of either temporal or spatial extension.

He could show that in the case of a temporal extension the identification of area with space traversed was fully consistent with Thomas' propositions on uniform motion. He thus arrived at a justification of the interpretation of a diagram's area as space traversed: a justification that went beyond the medieval conventional identification of area with space. He could further conclude that in the case of a spatial extension, motions of equal duration were represented by similar rectangles. All these results were, however, restricted to uniform motions.

How could Harriot get from uniform to uniformly accelerated motion? An obvious means would have been the Mean Degree Theorem. But while working with numerical sequences representing space and time, Harriot discovered that in the case of a spatial extension the application of the theorem resulted in contradictions (Schemmel 2008, 83–86). In fact, the Mean Degree Theorem presupposed what had to be derived: the interpretation of a diagram's area in the case of non-uniform motion.

To get from uniform to non-uniform motion without employing the Mean Degree Theorem, Harriot took recourse to approximative methods. The drawing reproduced in Figure 14 shows the approximation of a diagram with spatial extension (drawn horizontally) by an ever greater number of uniform motions lasting for ever shorter times. The horizontal line 'cd' represents the space traversed from right to left. Vertical lines represent degrees. All rectangles employed within one step of approximation are similar, i.e. they represent motions of equal duration, as Harriot had learned from his analysis of Thomas' propositions.

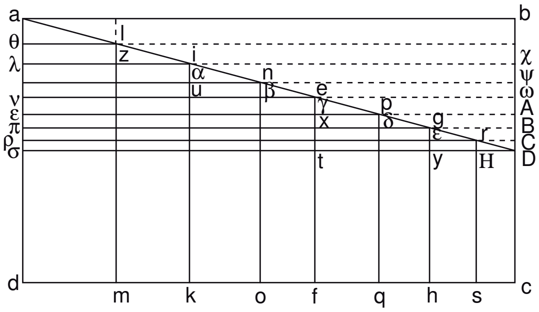

Figure 14: Harriot's approximation of motion uniformly difform over space by an ever greater number of uniform motions lasting for ever shorter times, ca. 1602 (British Library, Add MS 6788, f. 123v)

In the first approximation Harriot considers the two equitemporal uniform motions 'νef' and 'tDc'; in the second approximation the four equitemporal motions 'λik', 'uef', 'xgh', and 'yDc'; and so on. This sequence, which approaches the time of the difform motion from above, Harriot calculated up to the twelfth approximation. Then he calculated the tenth and the twelfth element of the sequence approaching the time from below. From the mean values, he estimated the remaining error and arrived at a determination of the time for motion uniformly difform over space which is accurate up to the sixth digit. The relation between time and space for this kind of motion is logarithmic and in fact Harriot's value represents a numerical approximation to the logarithmic value.[33]

There are indications in Harriot's manuscripts that he employed similar methods also for approaching motion accelerated uniformly with respect to time rather than to space.

[33]Interestingly, however, when attempting to extend his result to motions traversing different times or spaces, Harriot assumes that the ratio of the spaces traversed in different times by one and the same motion equals the ratio of the times, an application of the Aristotelian proportions to non-uniform motions which is incorrect from the perspective of classical physics.

In any case, Harriot's method is similar to Huygens' later approximation of uniformly accelerated motion by uniform motions mentioned above. We see here the emergence of an infinitesimal concept of velocity—the velocity of a uniform motion of ever smaller extension—which is radically different from the Aristotelian global concept of velocity which still plays such a central role in Galileo's work.

Summing up, we have seen that the conceptual development was driven by the ambiguities and contradictions emerging in the interpretation of the diagrams in their combination with the Aristotelian proportions on motion. It was the iconic character of the diagrams of motion, their graphic representation of uniformly increasing motion, that made them catalysts of conceptual development. On one hand, owing to the geometrical relations that exist between their elements, the diagrams implied specific relations between physical quantities which were not obvious from the beginning. On the other hand, they represented general aspects of uniformly increasing motion, quite independently from any rigid interpretation of their elements. Thus their appearance could remain virtually unaltered over reinterpretations and even over fundamental conceptual breaks. A conspicuous illustration of this fact is the similarity of the triangular figures of Oresme's scheme and those occurring in the modern derivation of the law of fall.

5 The epistemic role of symbolic means in early modern derivations of the law of fall

To conclude, let us summarize what role the symbolic means of knowledge representation may play in answering the three historiographic questions raised in the beginning.

In early modern times, the means available to describe relations between physical quantities, which today we would describe by means of functions and their derivations, were limited. One major means were proportions. Another means were the medieval diagrams of change. The shared knowledge embodied by these symbolic means conditioned the possibilities and the limitations of the early modern scientists' work on motion, thus explaining many of the similarities between the independent early modern derivations of the law of fall. As an example, all early modern scientists we considered used, in one way or another, the Aristotelian proportions on motion for interpreting the diagrams in terms of space, time, and velocity. This fact further illustrates the role of external representations of knowledge for the stability and longevity of knowledge structures.

This circumstance does not imply, however, that the knowledge structures represented by the ancient and medieval symbolic means remained unchanged in early modern times. The symbolic means functioned as tools for thinking and, like all tools, could be applied to purposes other than the ones for which they were originally developed. So, what was new about the early modern derivations of the law of fall as compared to the medieval discussions of change was primarily the new purposes for which the symbolic means were used. The question concerning the functional relationship between space and time in the motion of fall was closely related to the practical concerns of the time. This new context of argumentation brought about changes in the meaning attached to the symbolic means, which resulted in ambiguities and contradictions. These ambiguities and contradictions stimulated the further exploration of the implications of the symbolic means, an exploration that was pursued to a different degree by the different mathematicians.

Since the symbolic means were not rigidly related to concepts, but rather represented more complex mental structures involving different concepts, working with them could lead to changes of the relations between concepts and, as a consequence, to changes of the concepts themselves. In this way the modern concepts of motion emerged from work that still employed medieval concepts. The most obvious example for conceptual change in the case considered here was the change of velocity from a concept globally characterizing an overall motion to a concept characterizing motion in an infinitely small moment in time.

In the process of knowledge transformation, the medieval diagrams thus served as catalysts of conceptual development. In combination with the Aristotelian proportions on motion, they propelled conceptual change through their iconic quality, since they geometrically represented relations between quantities which had otherwise remained obscure. At the same time, they constituted a stable iconic representation of uniformly increasing motion and thereby served as a bridge from one conceptual scheme to a new, incompatible one.

Acknowledgments

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