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["Apuntes de Mecánica Social", by Antonio Portuondo]



# Does more abstraction imply better understanding? ("Apuntes de Mecánica Social", by Antonio Portuondo)

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## Abstract

This paper studies the contribution of the Spanish engineer and mathematician Antonio Portuondo to the mathematisation of Sociology in his 1912 book *Apuntes de Mecánica Social*<sup>1</sup> –An Essay on Social Mechanics. Portuondo describes his approach to Sociology in the language of Rational Mechanics, translating sociological and psychological concepts, both for individuals and social groups, into mathematically tractable entities, and then proceeding to the derivation of a series of theorems and conclusions.

The mathematical and physical content of *Apuntes de Mecánica Social* is analysed from a critical viewpoint, showing that the influence of Ernst Mach was a decisive one in the construction of the ideas expressed in the book. Curiously enough for an engineer, the main drawback of his approach is that the obtained results are not reinterpreted into actual sociological instances, for Antonio Portuondo sticks to abstract ideas with a minimum contact, if any, with real-word circumstances.

The paper also contains some considerations on the role of Mathematics in the sociological research, and a description of a possible priority conflict with the Romanian scientist and politician Spiru Haret.

## Introduction

Leviathan (1651), the gloomy utopia by Thomas Hobbes (1588-1679), is written in a deductive style that allows including it in the realm of mathematical literature. On the other hand, the bitter social critique expounded in Gulliver's Travels (first edition, 1726) by Jonathan Swift (1667-1745) could be considered as the other end of the spectrum: Emphasis is made on social structures as a result of man-made assumptions on how to understand natural law, *i.e.*, on cultural aspects. Since its very beginning as a science, Sociology has always oscillated between both extreme visions. Today this methodological seesaw seems to have finally found a scientific equilibrium in the field of nonlinear processes where deterministic, statistical and hermeneutical approaches peacefully coexist (See *e.g.*, Ball 2002, Ball 2004).

In the cover page of the foundational book (Quételet 1835), his author, the Belgian Adolphe Quételet (1796-1874), attributes to the biologist and philosopher Jean Baptiste de Lamarck (1744-1829) an *Essai philosophique sur les probabilités*, from which the following words are extracted:

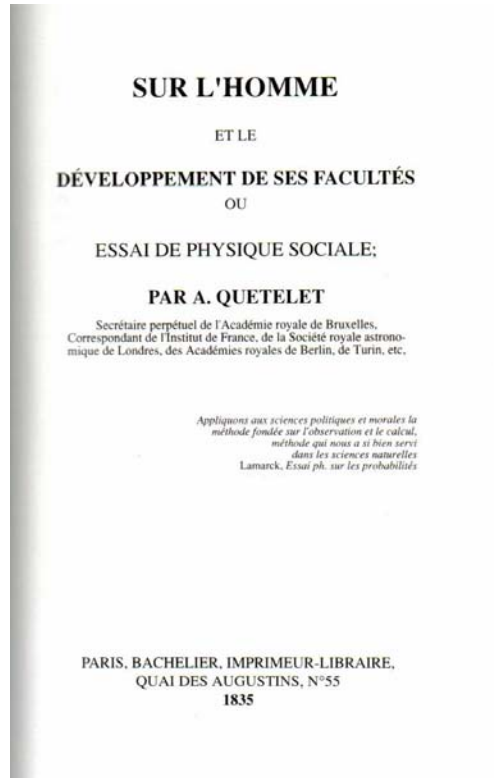
*“Appliquons aux sciences politiques et morales la méthode fondée sur l’observation et le calcul, qui nous a si bien servi dans les sciences naturelles”*

Although Quételet’s aim is clear, his quotation is most possibly a mistake, or simply a by-product of wishful thinking: Lamarck never wrote an *Essai philosophique sur les*

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<sup>1</sup> All expressions in languages other than English are italicised. Spanish is (nearly always) translated into English, but French and German are not.

*probabilités*. Pierre-Simon de Laplace (1749-1827) did write such a book (Laplace 1814), but these exact words have not been found in it. Laplace's *Essai* has a long section on applications to demography whose organisation and insight prefigure the work of Quételet.



Cover page of Quételet's *Sur l'homme...*

In Quételet's book detailed analyses of masses of data on the most various social and moral facts are performed, and the main objectives are expressed in the form of two questions before dealing in Book I with the determination of the 'average man' (*l'homme moyen*) (*Introduction*, II, p. 31):

- *Les actions de l'homme sont elles soumises à des lois?*, and
- *Comment il convient d'étudier et d'interpréter les lois relatives à l'homme?* (*ibid.* III p. 38).

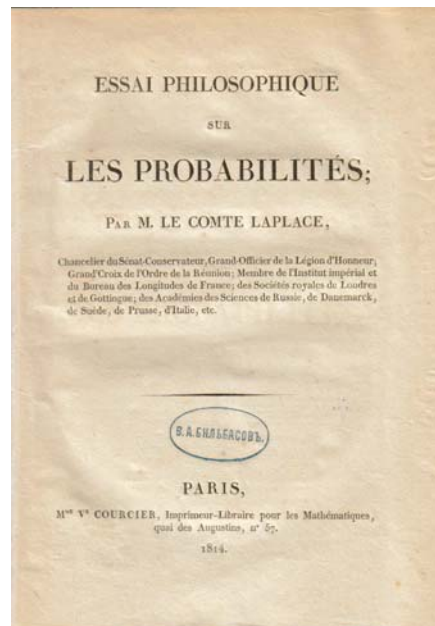
The *homme moyen* is the basic study object, and through careful observation it can be characterised by several statistical properties. Book II is devoted to the physical properties



The Belgian theoretician of Social Science Adolphe Quételet (1796-1874)

(height, weight, strength, agility...), while Book III considers the development of moral and intellectual properties in the average man. Finally, Book IV displays the most significant title: *Des propriétés de l'homme moyen et du système social, et des progrès ultérieurs de leur étude*.

Quételet is indeed well aware –and his extremely detailed work proclaims it- of the power of Statistics as the ideal tool for unveiling the hidden laws of social behaviour. But one thing is establishing the laws, and a very different one is observing them in an evolutionary manner, trying to use them as models to formulate predictions. Therefore, it is no wonder that the search of new ideas turned to the already deductively organised science of Physics, and it was the French Auguste Comte (1798-1857) who made this step forward.



Cover page of Laplace's *Essai philosophique sur les Probabilités*

The expressions ‘Social Physics’ and ‘Sociology’ were coined by the political philosopher Comte in his monumental treatise *Système de Philosophie Positive*, whose six volumes were published in Paris in a twelve-year period (Comte 1830-1842). His positivism as ‘metaphysics free’ thinking can be considered in a first instance as a reaction triggered by the acceptance of both power and accuracy of predictions formulated by deterministic

methods, especially in the field of celestial Mechanics, through the work of Isaac Newton (1643-1727) and the developers of his methods. Among these followers the Austrian philosopher and physicist Ernst Mach (1838-1916) –although also a serious criticist- is a most interesting one for the aims of this paper.

In the central years of the 19<sup>th</sup> Century, the application of strict positivism to social studies was greatly hindered by two main circumstances:

- The current status of Psychology and Physiology -and of Statistics as well- as Sciences on their own right was not strong and clear enough to scientifically support a purely deductive development of Sociology.
- The resistance of several schools to accept the application of these ‘scientific’ methods to human social questions.

The above argument shows that Hobbesian and Swifteans were always there: Some other scientists argued quite early, among them the distinguished Wilhelm Dilthey (1833-1911), laws ruling the natural world should differ from those of the social one, because some aspects like symbolic thinking and the existence and acceptance of certain value systems are only cultural, *i.e.* man-made, artefacts.

A ‘third way’ to overcome these difficulties was represented by some attempts to free sociological ideas from any non-objective consideration and to identify the resulting processes with some abstract construct having a deductive status similar to Mechanics. This idea is presented by Guillaume de Greef (1842-1924) in (Greef 1893), and its spirit is based on Machian ideas contained in the beautiful book by Ernst Mach *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt* (Mach 1883).

In this paper a Spanish contribution belonging to this third group will be studied: The 1912 book *Apuntes de Mecánica Social* –Notes on Social Mechanics, *Apuntes* from now on- and its sequels, written by the engineer and mathematician Antonio Portuondo Barceló (1845-1927, Portuondo in what follows).

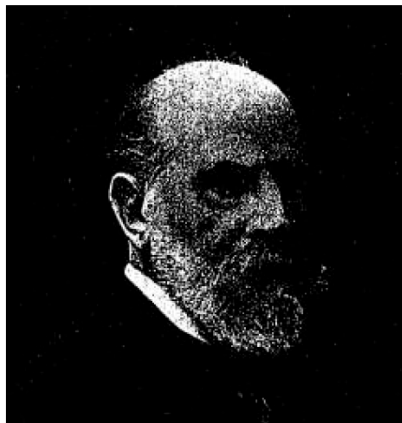
Today, Mechanics is a part of Physics, but in 19<sup>th</sup> Century they were considered as independent Sciences. This is plain in the already cited *Mechanik* by Mach, as well as in Portuondo’s adoption of this viewpoint in his exposition, where Mach is often cited and quoted. Nevertheless, at first sight it is not clear in *Apuntes* whether his attitude is a mere pretext in order not to enter into deep social matters –whose clues would be either unfamiliar or unclear to the author- or a thoroughly made decision. Some comments will be offered at the end of this work.

### **Portuondo, and how he came into Sociology**

Antonio Portuondo was a civil engineer of Cuban-Spanish origin who spent most of his life in Madrid. He was rather involved in the cultural and educational life of the Spanish capital city, where he became a close friend and supporter of Francisco Giner de los Ríos (1839-1915), one of the founders of the school system *Institución Libre de Enseñanza*, grounded on pedagogical and cultural ideas of the German philosopher Christian Friedrich Krause

(1781-1832). Other prominent Spaniards in Portuondo's liberal circle were the sociologist Gumersindo Azcárate (1840-1917), the biologist Juan Rodríguez Carracido (1856-1928), and the historian and sociologist Eduardo Saavedra (1829-1912). Portuondo was a member of the learned society *Ateneo de Madrid*, where the mathematician and future Nobel Prizewinner for Literature José Echegaray (1832-1916) delivered a series of lectures between 1897 and 1903 on the subjects of Galois Theory and Elliptic Functions. Most biographical data have been obtained from the obituary published in the *Revista de Obras Públicas* the year 1927 (Machimbarrena, 1927). The life and some of the mathematical work of Portuondo are studied in a forthcoming paper (Pacheco *et al.* 2007).

He was a Professor at the *Escuela de Ingenieros de Caminos, Canales y Puertos* (School of Civil Engineering, the Spanish version of the French *Ingénieur des Ponts et Chaussées*) from 1883 until his retirement in 1913, where he served as Director for many years. On the mathematical side, he became well known for his 1879 translation into Spanish - with his brother Joaquín- of the classical two-volume *Traité de Géométrie* by Rouché and Comberousse. They authored as well a companion volume to this translation, *Notas al Tratado de Geometría de E. Rouché y C. Comberousse*, which had several editions in Spanish and later in French. He also wrote some more books, or rather lecture notes, for the Engineering School students, among them *Apuntes para la clase de Mecánica* (1893-1894) and *Apuntes sobre Probabilidades, teoría de los errores y método de los mínimos cuadrados* (Lecture notes on Probability, Error Theory, and the Least-squares Method). Though he was never elected to the Academy, he acted as an editor of the *Revista de la Real Academia de Ciencias* for some time.



The Spanish engineer and mathematician Antonio Portuondo (1845-1927).

As regards the aim of this paper, for several years Portuondo was in charge of teaching *Economía Política*, then a broader topic than its name now suggests. This task led him to the field of 'Social Mechanics', where he published *Apuntes*, and he finally became a respectable member of the *Institut International de Sociologie* during the last part of his life. In order to understand this interest in Sociology, emphasis must be made on two points, at least:

1. First, as a civil engineer he thought that engineering practitioners should be well informed on many topics, even on those not directly related with their work. This

conception of engineering as the paradigm of positive social utility and success is very well described by the French writer Jules Verne (1828-1905) in several of his novels, among them *L'Île mystérieuse* (1874) or *Les cinq-cent millions de la Béguem* (1879), as shown in (Fernández & Pacheco 2005).

2. Second, Portuondo tried hard, as a member of the *Ateneo de Madrid* and editor of the *Revista de la Academia* to popularise viewpoints emphasising the role of scientific, positive methods in everyday life. His connections with the *Institución Libre de Enseñanza* and his acceptance of its Krausean ideas support this portrait of the author.

Two books written by Portuondo are representative outputs of these two aspects: *Ensayo sobre el Infinito* (Portuondo 1880, extracted in Portuondo 1812c) is the milestone of the first one, and *Apuntes de Mecánica Social*, the aim of this paper, of the second. Their common appearance in French translations at the end of Portuondo's life –many years after the original publication dates- gives a hint on how his author valued them.

On reading *Apuntes* it is clear that Portuondo was well aware of the current sociological literature; the large number of authors and books cited, and the quotations interspersed in the main text and footnotes show familiarity with the leading ideas of various sociological schools. Whether he made crucial contributions to the mathematisation of Sociology will be studied in the rest of this paper.

### **A survey of *Apuntes***

*Apuntes* was first published during the year 1912 as a series of ten papers in the *Revista de Obras Públicas*, from whose on-line edition<sup>2</sup> can be retrieved (Portuondo 1912a-1/10). This *feuilleton* method was also used by Lester Ward in his twelve-article suite published in volumes 1 and 2 of the *American Journal of Sociology* the years 1895-1897, whose joint edition appeared in book form shortly after the publication of the articles (Ward 1897). Especially the seventh and eighth papers (Ward 1896a, Ward 1896b) must be clearly considered inspirations for the author of *Apuntes*.

An edition in book format of *Apuntes* was printed the same year (Portuondo 1912b) with a small change in the title –*Apuntes sobre Mecánica Social* was changed into *Apuntes de Mecánica Social*- and a later French version appeared in 1925 under the title *Essais de Mécanique Sociale* (Portuondo 1925). The 1912 book edition was reviewed by Lester Ward shortly before his death for the *American Journal of Sociology* (Ward 1913) in a joint review of *Apuntes* and the *Mécanique Sociale* written by the Romanian scientist and politician Spiru Haret (1851-1912), that had been published in 1910 (Haret 1910). The French edition was also reviewed for the same journal by Floyd House the year 1926.

In this study the original papers in the *Revista de Obras Públicas* have been used. The headings of chapters and sections are exactly the same in all three editions, and the only

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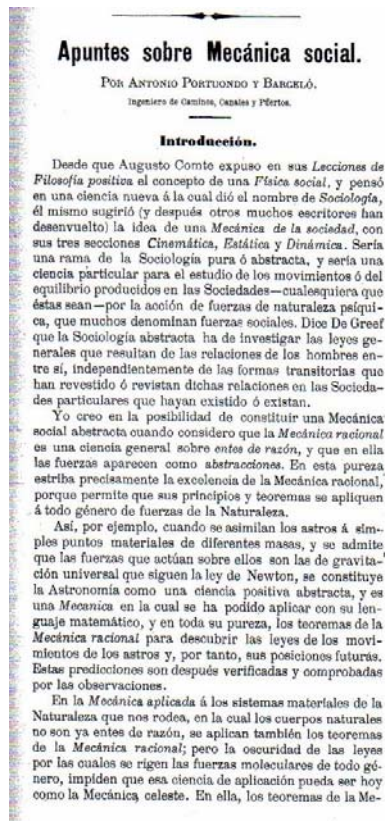
<sup>2</sup> [http://ropdigital.ciccp.es/public/detalle\\_articulo.php?registro=11940](http://ropdigital.ciccp.es/public/detalle_articulo.php?registro=11940) . The online copy of (Portuondo 1912a-5) is short of the last page.



difference between the Spanish versions and the French one is that this one includes as a foreword the 1913 review by Ward in pp. IX-XVIII. There is still another difference: a reference to Haret's book appears as a footnote both in the original Spanish article suite and the 1912 book edition, but is incorporated into the text in the French book edition. Later on some considerations will be made on this fact. Here is the index of *Apuntes*:

1. Introduction
2. Preliminary ideas. Definitions. Hypotheses
3. Kinematics
4. Statics and Dynamics
  - a. First Part: Equilibrium and movement for individuals
    - i. Theorems on the movement of individuals
  - b. Second Part: Equilibrium and movement of social groups
    - i. Social Statics
    - ii. Social Dynamics
    - iii. Theorems on the movement of social groups.
5. The Universal Energy

Moreover, there are more detailed divisions at each level, offering a very accurate account of the topics within each subdivision.



The beginning of *Apuntes* (Portuondo 1912a-1, p. 63)

The review by Lester Ward contains a most vivid description of the book (Ward 1913, p. 817, our italics):

“The fundamental *principle that underlies the entire work is that of the existence of true social forces, identical with the forces that control the physical world.* Social Mechanics is for him simply the laws of mechanics operating in society. But the social forces are psychic, and become social in their collective action. The subdivision of Social Mechanics is therefore necessarily into social statics and social dynamics, and these are systematically treated in that order. He uses the regular postulates, elements, or units of the science of mechanics -mass, space, time, and their combinations- only that he employs different symbols for some of them from those used by Clerk Maxwell and other English writers and the several combinations of these, velocity, momentum, force, energy, power, etc. are formulated as in other works. He makes much use of the principle of acceleration, which is the velocity divided by the time. *He thus shows the fallacy of regarding either force or energy as an ultimate, irreducible postulate, since they both involve mass, space, and time in different relations.* It is remarkable how closely he clings to these postulates and formulas throughout, and it would seem that almost the entire science of mechanics may be worked out in all fields by the aid of these alone”.



The American sociologist Lester F. Ward (1841-1913)

On the other hand, the 1925 French translation only deserved a short, slightly melancholic comment (House, 1926, p. 129). It reads:

“Portuondo’s *Essays in Social Mechanics* is a reissue of a book first published in 1912. The first edition was reviewed by Lester F. Ward, and the review was published in the May, 1913, number of the *American Journal of Sociology*, the same number in which the notice of Professor Ward's death appears. Ward’s review is printed as a preface in this edition of the book. It is surprising that the book has not made more impression upon American sociologists than it appears to have made, in view of the favorable report which Professor Ward gave. Perhaps the reason is that Ward emphasized in his review the assumption made by the author that “the social forces are the desires of men”, in confirmation of Ward’s own doctrine -a doctrine which on the whole has not been favorably received by sociologists in this country since that time. In fact, the book is of interest for the stimulating methodological suggestions which it presents, and which are of value quite apart from the author’s attempt to develop them in a series of mathematically stated theorems of ‘Social Mechanics’”.

The programmatic sections in *Apuntes* are the Introduction and parts 2 and 5. Somehow, this last one apparently falls apart from the original idea of the author, but he thought it to be his most personal contribution to the progress of Sociology. The main principles of these sociological conceptions, extracted from the book, appeared in the *Revue philosophique* (Portuondo 1917). In turn, parts 3 and 4 contain a non-technical exposition of Rational Mechanics where theorems are offered either without proof or with brief heuristical

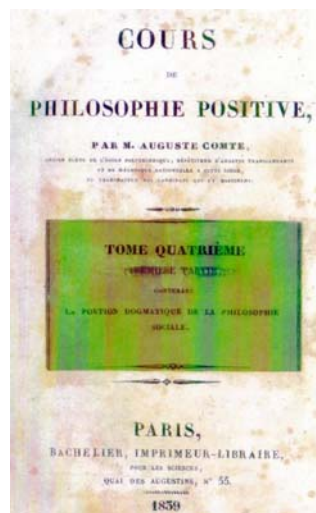
comments. Each result is then translated into the jargon of Social Mechanics, although very cautiously: Throughout the exposition the reader will often find ‘maybe’, ‘hopefully’ and similar expressions showing that the author did not feel sure enough to clearly state the social counterparts of the introduced mechanistic concepts.



The French sociologist Auguste Comte (1798-1857)

In the Introduction, written in a rather complicated Spanish, the author explains his viewpoint and tries to point out his aims (Portuondo 1912a-1, pages 63-66). As usual, he starts by attributing to Comte the origin of the expressions ‘Social Physics’, ‘Social Mechanics’, and ‘Sociology’, and then he heads toward the realm of Abstract Sociology, defined by de Greef as the study of the general laws governing the relationships between individuals, social groups and so on, abstracted from any particular transient or local aspects (de Greef 1893, pp. 24-25), see also (Durkheim 1886):

*“La sociologie abstraite complète la série logique et historique des autres sciences abstraites. Elle a pour objet la recherche et la connaissance des lois générales qui résultent des rapports des hommes les uns avec les autres, abstraction faite des formes originales, variables et transitoires dans lesquelles ces rapports se manifestent dans les sociétés particulières; celles-ci sont le domaine réservé de la sociologie concrète”.*



Cover page of the volume of Comte's *Cours* dealing with Sociology

Enter Portuondo, with his initial statement (see Note 1)<sup>3</sup>:

“When I say that *Rational Mechanics* is a general science dealing with *rational entities*, I express my belief in the possibility of constructing an abstract Social Mechanics in whose discourse forces would appear as *abstractions*. The excellence of Rational Mechanics is the expression of this purity that allows the application of its principles and theorems to all sorts of natural forces”.

Therefore he declares himself a follower of de Greef, and according to this programme, the principles of Rational Mechanics should also be applicable to forces of a *psychical* nature, the so called ‘social forces’. In order to work with them, he deems that under special conventions it would be necessary (Note 2):

1. “To define the points of application [of the forces], by accurately establishing their positions.
2. To define direction and sense for the intervening forces.
3. To define the masses of individuals and of other social elements.
4. To conceive that the intensities of psychical forces be measurable, even though their intimate nature might be unknown to us, as is the essence of all classes of forces in Nature”.

Here the author recalls the authoritative opinions of Henri Poincaré (1854-1912) and the biologist Félix Le Dantec (1869-1917). He borrows from the French mathematician the concept that Science is only a family of observed relationships between things whose true nature (of things) remains undisclosed: as a rule, just a few relationships of a permanent nature can be determined, an idea already found in the *Antimetaphysische Vorbemerkungen* of Mach’s *Analyse der Empfindungen* (Mach 1885), where a programme already found in (Mach 1883) is developed. When this conception is applied to human social behaviour, Le Dantec’s view adds even more complexity, for he maintains that what we know as ‘a thing’ does not only depend on the world’s nature, but also on the observer.



The Austrian physicist and philosopher Ernst Mach (1838-1916).

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<sup>3</sup> In quotations from the Spanish original, italics are always Portuondo’s. Translations are ours.

According to the above remarks, Portuondo also thinks that things under study are mere abstractions. Compare with the words of Mach:

“*Das Ding ist eine Abstraktion, der Name ein Symbol für einen Komplex von Elementen, von deren Veränderung wir absehen. Dass wir den ganzen Komplex durch ein Wort, durch ein Symbol bezeichnen, geschieht, weil wir ein Bedürfnis haben, alle zusammengehörigen Eindrücke auf einmal wachzurufen*” (Mach 1883, p. 454).

Therefore, Portuondo feels comfortable because his scientific discourse is developed and evolves within the safe world of Mathematics –or rather, Mechanics. He now explains his Programme (Note 3):

“In these *Notes* we shall try to assimilate social movements –considered from some particular viewpoint- to those of material systems studied in Mechanics, and we shall consider social facts as natural ones. We shall admit the possibility that all our assumptions could be assessed some future day through *Experimental Psychology*...”

A long paragraph after these words indicates that, nevertheless, nothing would please more the author than to follow the usual road from experimentation and data collection to the formulation of laws and to checking their validity. He invokes Henry Maudsley (1835-1918), Albion Small, and Lord Kelvin (1824-1907) in his support, for he thinks that the impossibility of this task lies on the by then scarce reliability of Psychology and Sociology as Sciences. In his own words (Note 4):

“As a rule, sociologists develop their views of Social Mechanics by considering the *quality* of the forces acting on individuals while in society. Moreover, they take care of economical, moral, etc. trends or *goals*. I try to state a completely different way of considering Social Mechanics. I shall think only on *how* psychical forces act, independently of their specific nature. For me, these forces are *pure abstractions*, as are forces in Rational Mechanics. On the other hand, consideration of possible practical goals is alien to our study”.

Thus the evolution of social structures, *i.e.* what sociologists know as Social Dynamics, is beyond a purely mechanical study. Social Mechanics must restrict its scope to forces acting on individuals and social groups and dealing only with a single topic: Can the reader recognise here Mach’s *Denkenökonomie*?

### ***Apuntes, and Mathematics***

Some hints and valuable informations on Portuondo’s knowledge of Mathematics and his mathematical background can be obtained from *Apuntes*. Parts 3 and 4 of the book contain a small course on the mathematical foundations of Mechanics, whose aim is to prepare the reader to the adaptation of physico-mathematical arguments to interpretation in terms of sociological ideas. As has been noted a few paragraphs above, he feels at ease within the abstract domains of Mathematics, and proclaims (Note 5) that:

“I shall think only on *how* psychic forces act with independence of their specific nature; for me these forces are *pure abstractions*, like those in Rational Mechanics”.



The famous *Mechanik* by Ernst Mach, 1<sup>st</sup> Edition, 1883.

A few lines below, (Note 6) the following explanation is offered, where he adheres to the economy of thought:

“...today [Mechanics] is a predominantly *deductive* Science, built upon the abstract entities of Rational Mechanics, where Mathematics play the main role through Infinitesimal Analysis. This allows what Mach has called –in a most celebrated and repeated statement- *economy of thought*, *i.e.* the least intellectual effort”.

In order to present , attention is restricted to only one social topic. Portuondo’s Spanish word for topic is *asunto*, that may refer to any particular field, activity, institution or whatever else the particular individual or group is involved or interested in. On the group idea, the generic Spanish word used in *Apuntes* is *agrupación* (grouping, but ‘group’ appears to be the best choice in English), and in fact it describes the idea of a group structured by some type of links<sup>4</sup> between its individuals. Higher degree groups are those whose members are other [structured] groups.

In order to locate individuals or groups some reference system must be defined. Read Portuondo’s definition (Note 7):

“The *position about some topic* of an individual or group in a given instant will be *the set of everything psychical -in any form whatsoever- found in the individual or group which has some relationship with the given topic*”.

Thus social individuals or groups are identified through their sets of values –something like coordinates, and any social topic defines a ‘sociological space’, where individuals or groups are located according to these ‘coordinates’. Therefore social changes on the topic under study may be represented by properties of ‘curves’ describing the social trajectories of individuals or groups in this space.

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<sup>4</sup> The idea of a link (Spanish: *enlace*), otherwise undefined, is crucial in the group concept. Portuondo considers Family, Town, Province, State as higher order groups, although he also asks himself whether Race or Religion could belong to the category of groups.

Indeed the idea of a social space is not explicitly formulated by Portuondo. Only in a footnote towards the end of the book does (Portuondo 1912a-8, page 546), the author refer to the Theory of Sets developed by Georg Cantor (1845-1918) in the following terms (Note 8), where the word ‘complex’ is used in the sense of ‘*n*-tuple’<sup>5</sup>:

“...it comes to my mind that maybe this [the simultaneous consideration of the individuals in a group] could be considered as an example of the abstract conception of some German mathematicians –especially *Georg Cantor*- on what they call a complex number of *n* real numbers. [...] Let us remember that these mathematicians think that any complex number defines the position of a point –so they say- in an *arithmetical space of n dimensions...*”

In the footnote Portuondo also makes some considerations on whether what has been called above ‘the social space’ could be ‘an arithmetical space of *n* dimensions’. The moral is that he takes the notion of ‘arithmetical space of *n* dimensions’ as synonymous to our ‘vector space’, but does not dare to go any further and to operate with these new entities. This is the only time Cantor is cited, but with no allusion to set-theoretical notions, and the closer the author is to introduce this sort of abstraction in his analysis of *Social Mechanics*.

Portuondo writes everything in plain Spanish, but he could have written (of course, he knows the notation, but his expository mood prevents him to use it):

$$X(t) \rightarrow X(t + \Delta t), \text{ where } X(t) = (X_1(t), X_2(t), \dots, X_n(t)),$$

where the  $X_i(t)$  are the ‘measures’ –hopefully obtained by means of Statistics, but otherwise abstract quantities- of the various psychological features considered as coordinates of the topic. Of course, if  $X(t) = X(t + \Delta t)$  for every time increment, then  $X(t)$  is termed a stationary position or rest point. Portuondo illustrates it with the example of religious beliefs and how they evolve (Portuondo 1912-1a, p. 68). Religion as an example will appear again a couple of times in *Apuntes*.

The foundations of Mechanics are described in (Portuondo 1912a-2) where he insists in following the Newtonian paradigm. The concept of inertial mass is described with some detail, because its interpretation in social terms is to endow every element of the social space with a constant –not time vaying- quantity describing obstruction to social change. Here one more allusion to Mach is found (Note 9):

“In his [book on] Mechanics, Professor Ernst Mach emphasises that the notion of mass –as a determining feature of movement- must be experimentally obtained...”

The exact phrasing of Portuondo in the definition of mass is given in (Note 10), and it reads:

“We can define the mass of an individual –for some given topic- as a coefficient measuring the capacity of the individual to modify his position...”

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<sup>5</sup> In Spain, ordinary complex numbers  $a+ib$  were called *cantidades imaginarias* for many years, while *complejo* was used for ‘composed of several elements’.

Many practical difficulties appear when it comes to actually measuring the quantities needed to define the location of an individual in the social space. Portuondo is aware of them, and in order to be able to make some progress in his study, he proposes in a very obscure way the existence of a continuously varying ‘symbolic parameter’ or time-dependent quantity  $\lambda(t) = F(X_1(t), X_2(t), \dots, X_n(t))$  for some adequate though unknown real valued function  $F$ . The additional requirement on this abstract parameter is that for a fixed individual the evolution  $\lambda(t) \rightarrow \lambda(t + \Delta t)$  symbolises some psychical change whose sense and direction are quantitatively expressed by the vector difference  $X(t + \Delta t) - X(t)$ .

A few comments are found on the dimensionality of social space and on how possible future psychological developments could yield a reduction in the number of dimensions. It is remarkable that this idea, expressed by Portuondo in terms of the implicit function theorem –without referring to it– is nowadays a most fruitful idea exploited in multivariate Statistics as a routine tool. When dealing with the continuity of the parameter Portuondo cites his own *Ensayo sobre el infinito* of 1880 to explain the idea of an infinitely decreasing magnitude.

A short comment can be found in page 197 on the alternative formulation of Physics derived from the consideration of energy and its conservation as first principles. And it is interesting to note this slightly disdainful comment on an alternative which might have been a more adequate framework for the exposition of a ‘universal energy’ system. (Note 11):

“In their system for the exposition of Mechanics –the so-called energetic system– they must start by defining the kinetic and potential energies, they do not want to use the notion of force [...] We think that this expository system has not attained a degree of maturity and popularity...”

The next papers of the suite deal with Kinematics, Statics, and Dynamics, indeed all of them applied to social entities. In the first one (Portuondo 1912a-3) Kinematics is considered with all generality, and it contains a remarkably long digression of four pages on Probabilities and Statistics falling quite out of the mainstream of Portuondo’s ideas. Most possibly its inclusion was just a matter of demonstrating his knowledge and mastery of these mathematical techniques, of a more applied nature than the abstractions which are the core of *Apuntes*. More interesting are the footnotes, especially the long one in page 238 where the paradox of Achilles and the turtle is solved through summation of a geometric series. The paper ends with a very long exposition of the subtleties of relative movements and of the composition of movements. Few or no Mathematics can be found in this last section: Examples on religious behaviour and family relationships replace considerations on vector addition. This paper is followed by the study of Statics and Dynamics of individuals in (Portuondo 1912a-4), where the rhetorical style is the same, and the three Newtonian laws are presented one more time with side comments of a philosophical nature.

The usual theorems of Rational Mechanics, *i.e.* the theorem of live forces, theorems on momenta, and the theorem of the least action, are presented and proved in a standard though schematic way for the case of a single individual in (Portuondo 1912a-5), while in



(Portuondo 1912a-6, 1912a-7) the same questions are addressed for the case of groups. As expected, the theorem of D’Alembert –on the reduction of a problem in Dynamics to a problem in Statics via the cancellation of all external and inertial forces through the internal links between the group components- is studied in greater detail in (Portuondo 1912a-6). A most interesting point is footnote 1 on page 397 (Note 12):

“We cannot either consider here the second order differential equations appearing on the general problem of Dynamics or refer to the formulation given to them by Lagrange by limiting the number of variables to the strictly necessary. I cannot see how Social Mechanics could be led into such depths”.

The first consideration reminds the reader of *Apuntes* of the ‘divulgative’ character of the text, but the second one contains a warning on how formidable the task of mathematising Social Mechanics can be. The candid affirmation contained in this second sentence is the only instance where Portuondo seems overwhelmed by his effort.

Next comes (Portuondo 1912a-7) with the theorems on the movement of social groups. The theorems are the same as in the case of individuals, with the addition of the theorem on the movement of the centroid and the principle of least effort. On the centroid, the social version imagined by Portuondo would be a Chief of State, or Premier, of a hobbesian nature (Note 13):

“Obviously such a Chief of State does not depend on written Constitutions, or on electoral procedures. At most it would depend on the intimate Constitution of the nation, i.e. on the political nature of all its individuals and social elements”.

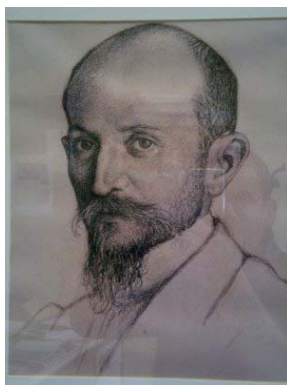
A standard proof of the least action principle is offered in page 509, but from the mathematical viewpoint the considerations in (Portuondo 1912a-10, p. 568) are more interesting, because the principle is presented in the Hamiltonian variational formulation: If the social group moves from position *A*, occupied at time  $t_0$ , to position *B* (at time  $t_1$ ), then the variation<sup>6</sup> of the integral

$$H(\Gamma) = \int_{t_0}^{t_1} \left[ \frac{1}{2} \sum mv^2 - W \right] dt$$

representing the difference between the joint kinetic energies and the work exerted by the group along any curve  $\Gamma$  joining *A* and *B* in social space must vanish: in other words, the movement must be performed along the  $\Gamma$  for which  $H(\Gamma)$  attains its minimum value. This presentation is in sharp contrast with the general spirit of *Apuntes*, and it can be considered as an opportunity for Portuondo to show his familiarity with the Calculus of Variations.

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<sup>6</sup> In the technical sense of the Calculus of Variations, where ‘variation’ is the analogue of ‘the derivative of ordinary Differential Calculus.



The French biologist Félix Le Dantec (1869-1917)

To end this section on Mathematics in *Apuntes*, a comment on the interesting footnote 2 in (Portuondo 1912a-8, page 554) will be made. The biologist and philosopher Le Dantec appears several times in *Apuntes* and his opinions are respected and shared by Portuondo. In this footnote, in a notation invented by Le Dantec, we learn that in every vital activity there are two intervening factors:  $A$ , the inner contents in the body of the living being, and  $B$ , the environment surrounding it. Therefore, vital activity can be expressed formally as a product  $A \times B$ . When  $A$  is interpreted as ‘the form’ of the living being<sup>7</sup>, its temporal evolution is described as (page 554)  $A_n = A_{n-1} + (A_{n-1} \times B_{n-1})$ , where the subscript  $n-1$  means ‘at time  $t$ ’; and  $n$ , ‘at time  $t + dt$ ’. It would have been better as  $A_{t+dt} = A_t + (A_t \times B_t)$ , but the absence of the differential  $dt$  in the second term of the right hand side is a notable formal feature that does not hinder Portuondo to immediately write the ‘equivalent’ integral formulation for any two time instants:

$$A_n = A_0 + \int_{t_0}^n (A \times B) dt .$$

This is a typical instance of dubious formalism. In fact, the first equation is a simple balance law, and if carefully written as  $A(t + \Delta t) \approx A(t) + (A(t) \times B(t))\Delta t$ , under the hypotheses of a small time increment and continuity of the product term, after operating and taking limits, an integration between the given time instants correctly yields the above integral formula. It is rather shocking that the exquisite care taken by the author in definitions and interpretations be missing in this last mathematical commentary.

### ‘The Universal Energy’ (*Apuntes*, part 5)

The Universal Energy is an ideological construct where Portuondo states that ‘psychic energies’ must be considered on equal foot with classical forms of energy, on the basis and hope that sometime in the future the progresses of Psychological Physics, the name given to Psychology in de Greef’s work, could explain how to do it in some way. Ward’s 1913 review highlights this opinion (our italics):

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<sup>7</sup> Form is the inner structure on which the external shape is modelled.

“The author's views on the universal energy are highly enlightening, and *he is not afraid to declare that vital and psychic energy exist as simply higher forms* into which physical and chemical energy have been transformed under the great law of the conservation and transmutation of energy, and he says: “All physical, chemical, physiological, and psychic energies are different manifestations of one sole universal energy”. He insists on the importance of man's utilizing all forms of energy. He does not speak of “wasting energy”, which, in view of its indestructibility, is a meaningless phrase, but of economizing it, and turning it to man's use, the extent to which this is done being the true measure of civilization” (Ward 1913, p. 818)

The meaning of the expression ‘higher forms’, as has been remarked before, remains unknown to both the author and the reviewer. Portuondo writes (Note 14):

“If we think that transformations observed in the organic world are different from those taking place in the inorganic one, it is because the first ones are so peculiar for different individuals [...] that we do not dare to apply the laws we know to rule energy transformations in the inorganic world *in an identical* way. This is why many writers have called them *vital energies*.”

And a few paragraphs below, he considers that (Note 15):

“In general terms, and omitting the details, one could say that physiological energy manifestations are always some chemical potential energy transformations triggered by the destruction of some organic material and the consequent loss of potential energy. Inversely, physiological energies are transformed into physical ones and yield some thermal (that is dissipated) or mechanical energy manifestations”.

Of course he is aware of the debate on the true nature of nervous and psychical activities, where the relationship between them and their physiological counterparts was trying to be established. Among others, he cites the chemist Wilhelm Ostwald (1853-1932), the philosopher William Pepperell Montague (1873-1953), the psychologist and sociologist Henry Maudsley (1835-1918), the psychologist Hermann Ebbinghaus (1860-1909), and his old friend Juan Rodríguez Carracido. Portuondo explains that according to Ostwald psychological phenomena can be conceived as energetic ones, and that it has been observed that all processes of the central nervous system entrain some energy consumption in their psychical activity. In any case, physical and physiological energies are connected, and there exists a correspondence between them and the psychical energies. Nevertheless, Portuondo cautiously observes, according to his inspirer, Mach (Note 16):

“We do not investigate the causes of these connections and correspondences. Mach remarks that the notion of *cause* is a very metaphysical one –as we have remarked on some other occasion- and this is not the place for it”.

But there must be some environmental limits or bounds to the psychical energies, and therefore (Note 17):

“...we can affirm –from the mechanical viewpoint- that the psychical energies of an individual are limited by the energies of the physical world to which it belongs. Its body, with his organs and material elements, constitutes a system subjected to the laws of physical Mechanics”.

His solution to this problem is to introduce some sort of force field under whose influence individuals live –indeed he is thinking about the gravity field as a model, but his description is somehow different. Let any individual and his position or opinion about some social topic be given, then (Note 18, our underline):

“We see that this force field under which *the individual* lives is defined at a given time instant by:

1. Anything of a physical or psychological nature and lying outside the body occupied by the individual and having a sensible influence on him.
2. Anything physical or physiological inside his own organism or body.
3. Any general psychic features *of his own* that could influence his position on the particular topic under consideration”.

In the footnote at (Portuondo 1912a-9, p. 552) these conditions are identified with the Machian idea of the *U-limit* (*U* from *Umgebung*), *i.e.* what we would now call the ‘boundary of the system’. Items satisfying condition 1 are ‘exterior to the *U-limit*’, while those falling under 2 and 3 are ‘interior’ to it. The underlined words are explained at length a few lines below, and it seems that the existence of a time-dependent field is a great novelty to Portuondo. After these considerations the standard ideas of kinetic and potential energies and the theorem of energy conservation are formulated for the (social) movement of a single individual under the restrictions imposed by the force field: The final aim should be an estimation of how much ‘positive social work’ can be obtained from the single individual.

Once the Social Mechanics of individuals have been studied, Portuondo ends this chapter – and the book itself as well - by analysing the Mechanics of social groups or structures. He starts with a wishful thought (Note 19):

“It is most remarkable that, with the help of all knowledge provided by Physics, Chemistry and other Natural Sciences, unfolding and transformation of the total energies of individuals and elements of some group, these energies markedly influence the external physical environment of the group. With the help of the Psychic and Social sciences –should they progress enough- it would be possible to markedly influence the internal and external psychic environments as well”.

It has been already observed that the distinction between ‘individuals’ and ‘elements’ of a social group is that an individual is just a single person, while an element is some sort of organised group within a larger group, an idea which amounts to a translation of the difference between ‘physical’ and ‘juridical’ persons. Portuondo employs this distinction in order to eliminate the external environmental force field when dealing with the Mechanics of social groups. In this case, the field is defined by the relationships or links between the individuals and elements of the social group, and nothing else<sup>8</sup> (Note 20):

“When a system of points is moving through space, and their masses are considered to exert dynamical influences *on each other* (and nothing else), the field force acting on one element is defined by the family of all other forces”.

The chapter on the universal energy includes the concept of entropy in its final considerations, where the author recalls that in any physical process a fraction of the input energy is dissipated, *i.e.* it is transformed in such a way that it is impossible to retrieve or store it for any future use, and he adds (Note 21):

“... and this “*bend towards dissipation of mechanical energy in Nature*” is quite contrary to human interest, as Lord Kelvin has stated”.

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<sup>8</sup> This reminds of Jean-Paul Sartre's '*Huis clos*': *L'enfer, c'est les autres*.

He thinks this idea could be translated to the world of psychic energies, and proposes that Psychology and the Social Sciences should try to tell apart useful social work from useless and therefore dissipative activities. The final recommendation is (Note 22):

“Should the Sciences (all Sciences) consecrate themselves to the discovery of laws and the investigation of the most adequate processes in order to achieve the maximum possible utility [by making the transformation coefficients as large as possible], then could they contribute, as far as possible, to the economy of the *Universal Energy*”.

### ‘Social Mechanics’, not ‘the Mechanics of the Social’

Portuondo’s conception of Social Mechanics as an abstract construction deeply contrasts with other popular schools where Energy was also the fundamental concept. Social energy expenses had been more or less effectively computed and explained in terms of energy units ( $Joule = N \times m$ ) and translated into equivalent land areas by social scientists and theoreticians, following a line whose ancestry can be tracked as back as Antoine de Lavoisier (1743-1794), the founder of modern Chemistry. This early notion of an ‘Ecological Footprint’ has led to discover in a nearly forgotten book by Eduard Sacher<sup>9</sup> the original spirit of the ‘Ecological Economics’ movement (Blanco 2006 and references therein) in the last years of the 20<sup>th</sup> Century (Sacher 1881, especially pp.19-31 and 32-40).

In the foreword the reader will find the standard positivist statement (p. 2):

“Der Verfasser glaubt in die Naturwissenschaften die Basis für eine rationelle Volkswirtschaft gefunden zu haben”.



The ‘lost’ book by Sacher, *Grundzüge einer Mechanik der Gesellschaft*

<sup>9</sup> No birth and death dates available for Sacher. Unfortunately, this Eduard Sacher is not the same Eduard Sacher (1843-1892) associated with the famous Viennese *Sacher Torte*.



concept expressed for the first time by the American Lyda Hanifan (1879-?) in 1916 on his studies on the American school system, when he wrote about fostering solidarity networks within the citizenship (Putnam 2002). Here is Portuondo, on how to define education (Note 23, our underline).

“Taking into account the potential energy available for every topic at a definite moment, one could think that e.g. for a national group, the people’s education –in the broadest sense of the word- should try to *effectively* obtain *positive work* for every social topic...”

### **A supplementary story: Haret vs. Portuondo, a would-be priority dispute?**

The already cited *Mécanique Sociale* (Haret 1910) by the Romanian scientist and politician Spiru Haret was another contemporary attempt to introduce scientific method, or rather some mathematical formalism, in the study of sociological questions. The book is very similar to *Apuntes*, but with a clearer feeling of actual sociological practice: Although analytical Mechanics is the basic tool, the final Chapters include political, geopolitical and geographic as well as historical considerations which make it more readable than *Apuntes*. Portuondo was quite concerned about this book and acknowledged its existence in an *Advertencia* –remark- as a footnote on p. 246 of (Portuondo 12a-3) (Note 24):

“**Advertencia.**- I have now known about a book by Professor *Sp. C. Haret* from Bucarest with the title Social Mechanics, published by the end of 1910. I can see that work to be completely different from mine: though in both the Theorems of Rational Mechanics are applied to individuals and social groups, they are considered from different viewpoints and under different criteria. He who read both books could appreciate these divergences. My study is predominantly a psychologic one (as a ground for the social), as well as abstract and theoretical, while Mr Haret aspires with his to develop (at least in a first approximation) an applied Mechanics, the impulse to his work are the political applications.

Thus, the second half of the book [Haret’s] deals with things without any analogy with the topics dealt by me in these *Apuntes*, and he ends with some reflections on the future of civilisation.

As refers to Social Mechanics itself, I can see that Mr Haret had already studied some questions treated by me in this work [*Apuntes*], but in a completely different way. It is remarkable, though, that I have been able to develop –through a different way- an extension of the inertia principle which in some way is coincident with his. I have as well formulated ideas analogous to Haret’s on the concept of mass, in order to consider it as a constant”.



The Romanian mathematician, sociologist and politician Spiru Haret (1851-1912).

In the book edition of 1912, this remark appears as a footnote on p. 87, while in the French edition of 1925 it is included in the main text (Portuondo 1925, p. 97) and appears independently in the index.

Haret died the same year 1912, so a priority dispute could not take place. The *Advertencia* emphasises on the differences, for coincidences between both texts are clear because they use the same formalism. No more information on this topic has been found, and the independence of both attempts must be accepted. Nevertheless, the remarkable statement “*My study is predominantly a psychologic one (as a ground for the social), as well as abstract and theoretical...*” undoubtedly tries to establish some distance between Portuondo and Haret, while the phrasing “*...Mr Haret aspires with his to develop (at least in a first approximation) an applied Mechanics, the impulse to his work are the political applications*” points to an unsurmountable difference: The intention of the Romanian to formulate and explain political questions through a scientific discourse.

Here are the contents of Haret’s book, to be compared *Apuntes*:

1. *Rappel de quelques notions mathématiques*
2. *Emploi de la méthode mathématique dans l’étude de quelques phénomènes et problèmes sociaux.*
3. *Mode de représentation des phénomènes sociaux.*
4. *Statique sociale.*
5. *Dynamique sociale.*
6. *Les Forces Sociales.*
7. *Aperçu générale sur la civilisation.*

The review by Ward reads (Ward 1913, p. 815):

“In formulating the fundamental principles to be used in his calculations Dr. Haret assumes that ‘society, or the social body, is a union of individuals subjected on the one hand to reciprocal influences among themselves, and on the other to external influences’. In other words, he accepts without modification the theory set forth by De Greef that the basis of all society is ‘territory and population’”.

Further on, in p. 816, this statement is found (our italics):

“The task of demonstrating mathematically that democracy is an essential condition of civilization, has been well met by Dr. Haret. *This work shows that a great field of research is open in the line of quantitative social dynamics, that may compare favorably with the achievement in quantitative social statics, or social statistics*”.

To conclude this section, let Haret express himself. He declares, in the same mood as Sacher thirty years before, (p. 1) that

“*Ce travail est un essai d’application de la méthode scientifique á l’étude des questions sociales*”.

He also states the main obstacles found when trying to translate sociological jargon into a more formal language (p. 2):



*“Les causes que déterminent l’équilibre et le mouvement sont plus nombreuses en Sociologie qu’en Mécanique; et puis l’étude de la manière d’agir de ces causes est loin d’être aussi avancée en Sociologie qu’en Mécanique...”*

And he hopes, in an interrogative phrasing, in some future Sociology will be considered as a science (p. 6):

*“Le jour viendra-t-il jamais où cette science sera constituée? Nous l’espérons, et le bout de ce travail est de montrer la possibilité d’y parvenir...”*

And finally, a quotation of incredibly current interest one Century after the book was written (p. 194):

*“On peut se poser la question, que deviendrait la vie des peuples de l’Europe centrale, si tous les glaciers des Alpes venaient à disparaître. Les conséquences s’en feraient ressentir jusque dans les régions du bas-Rhin, de la Provence et du bas-Danube. Le régime des fleuves serait bouleversé, les saisons aussi, un climat extrême et sec prendrait la place de celui qui régnait actuellement, par suite toute l’Agriculture devrait changer ses pratiques ce qui, sans parler d’autres causes, forcerait les populations à modifier profondément leur genre de vie”.*

After these short excursions into Haret’s thinking, it is clear that the intellectual differences between Haret and Portuondo are really deep ones. The sharpest contrast lies in the rigid intellectual position of the Spaniard, sticking to abstraction and theoretical speculation, whereas the Romanian writes with much broader insight on the problems of Sociology and their application to real-life situations. As has been indicated above, it is shocking for an engineer to try to understand the most abstract concepts underlying the social Sciences without simultaneously considering their applicability to real-world problems.

## **Conclusions and views**

The attempt to mathematisation of Sociology by Antonio Portuondo is an interesting piece throwing some light on the scientific and cultural status of the Spanish learned class in the years before World War I. When the turbulent 19<sup>th</sup> Century ended with the loss of the last remains of the Spanish overseas colonies in Cuba and the Philippines, many educated Spaniards became culturally inclined towards Europe, in a sort of intellectual fleeing from the colonial times. In the work of Portuondo a remarkable knowledge of European Science can be observed, especially of foundational studies in Mechanics and Sociology. His familiarity with Mathematics is of a different nature: A skillfull practitioner of a large number of classical topics, he only made a single contribution to pure Mathematics with his *Ensayo sobre el infinito* in 1880.

On the Mechanics and Social Science side, Portuondo fervently adherent to positivism, so he can be considered a follower of Ernst Mach in Mechanics and of Auguste Comte and Guillaume de Greef in the Social Sciences. These influences, mixed with his engineering background and social interests, are the substratum underlying *Apuntes*, where an attempt to establish that social and psychic energies can be considered as a new form of energy that can be transformed into the more familiar chemical, thermic, mechanical energies.

His approach to Sociology through Social Mechanics is purely abstract; an election that favours a mathematical treatment of the subject where non-objective thoughts have no place and the rational *Denkenökonomie* is the rule.

On the mathematical aspect of *Apuntes*, Portuondo was very close to the formulation of his theories in a really modern style, but he did not dare to do it. His manipulation of Le Dantec's symbolic modelling language is far from rigorous, and some hints indicate that he was not familiar enough with the growingly abstract trends in Mathematics during the first years of the 20<sup>th</sup> Century. Although his main idea was 'the universal energy', he also thought that Mechanics could not be derived by starting from energy as a first principle; and the second order differential problems of Lagrangian Mechanics seem to him very distant from application to Social Mechanics, even though he is well aware of the role of supplementary conditions.

Most intriguing is his choice of such an abstract viewpoint and the rigid manner in which he sticks to it. The final result is a dry treatment where any view of applicability has been eliminated. One would expect from an engineer more sensitivity towards the practical side of his model of Sociology, but he even thinks that the political side of Haret's book is rather anomalous for a treatise on Social Mechanics.

Ward cites him in the 1913 review as a prominent antimetaphysicis (Ward 1913, p. 818):

"Portuondo has no patience with any metaphysical attempts to explain the phenomena of mind. The 'empirical' character of Kant is the only one he recognizes, and of his 'intelligible' character he significantly says: 'to me unintelligible' (*para mí ininteligible*<sup>10</sup>)".

To end this study, note that some authors have written on supposedly metaphysical bases for any scientific enterprise (Espinoza 1994, Gissis 2002). So a natural question can be asked after reading *Apuntes*: Was Portuondo's adhesion to purely deductive methods just another type of metaphysical belief? Or, in other words, *does more abstraction imply better understanding?*

## Notes

Note 1. (Portuondo 1912a-1), p. 63.

"*Yo creo en la posibilidad de constituir una Mecánica social abstracta cuando considero que la Mecánica racional es una ciencia general sobre entes de razón, y que en ella las fuerzas aparecen como abstracciones. En esta pureza estriba precisamente la excelencia de la Mecánica racional, porque permite que sus principios y teoremas se apliquen a todo género de fuerzas de la Naturaleza*".

Note 2. (Portuondo 1912a-1), p. 63.

1. *Definir bien los puntos de aplicación, determinando de un modo preciso sus posiciones.*
2. *Determinar las direcciones y los sentidos en que actúan las fuerzas.*
3. *Definir las masas de los individuos y de los elementos sociales.*

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<sup>10</sup> 1912a-4, p. 303, footnote.

4. *Concebir como medibles las intensidades de las fuerzas psíquicas, aunque nos sea desconocida su esencia íntima, como lo es la esencia de todo género de fuerzas de la Naturaleza*”.

Note 3. (Portuondo 1912a-1), pp. 63-64.

*“En estos Apuntes vamos a intentar un ensayo de asimilación de los movimientos sociales –vistos de un modo peculiar- a los movimientos de los sistemas que estudia la Mecánica de los sistemas materiales, mirando a los hechos sociales como fenómenos naturales, y admitiendo que por la Psicología experimental se pudiera llegar algún día a precisar y determinar todo lo que dijimos antes”.*

Note 4. (Portuondo 1912a-1), p. 64.

*“Los sociólogos que han escrito sobre Mecánica social han desenvuelto generalmente sus ideas, preocupándose de las cualidades de las fuerzas que actúan sobre los individuos en sociedad, y también de los fines o tendencias económicas, morales, etc.*

*Este modo de considerar la Mecánica social es totalmente distinto del que yo intento seguir. Habré de pensar sólo en el cómo de la acción de las fuerzas psíquicas independientemente de su naturaleza específica; puesto que para mí esas fuerzas serán puras abstracciones, como lo son las de la Mecánica racional. La consideración de los fines es, por otra parte, enteramente ajena a nuestro estudio”.*

Note 5. (Portuondo 1912a-1), p. 64.

*“Habré de pensar sólo en el cómo de la acción de las fuerzas psíquicas, independientemente de su naturaleza específica; puesto que para mí esas fuerzas serán puras abstracciones, como lo son las de la Mecánica racional”.*

Note 6. (Portuondo 1912a-1), p. 65.

*“...hoy la encontramos [Mechanics] ya como ciencia predominantemente deductiva, construida sobre las entidades abstractas de la Mecánica racional, en la cual las Matemáticas con su análisis infinitesimal desempeñan el principal papel. Esto permite ya lo que Mach llamó, con frase tan celebrada y repetida, la economía del pensamiento, es decir, el menor gasto intelectual”.*

Note 7. (Portuondo 1912a-1), p. 67.

*“Llamaremos posición en un asunto de un individuo o agrupación en un instante dado: El conjunto de todo lo psíquico que haya que haya, de cualquier modo que sea, en ese instante en el individuo o en la agrupación y que se refiera al asunto”.*

Note 8. (Portuondo 1912a-7), p. 546, footnote 2.

*“...me ocurre que esto quizá pueda ser considerado como un ejemplo de la concepción abstracta de algunos matemáticos alemanes –especialmente Georg Cantor- acerca de lo que llaman número complejo de  $n$  números reales [...] Recordemos que esos matemáticos piensan que cada valor del número complejo define la posición de un punto –así lo llaman- en un espacio aritmético de  $n$  dimensiones...”.*

Note 9. (Portuondo 1912a-2), p. 192.

*“El profesor Ernst Mach insiste mucho en su Mecánica en que la noción de masa –como característica determinante para el movimiento- debe de arrancar del hecho de experiencia...”.*

Note 10. (Portuondo 1912a-2), p. 193.

*“Podremos definir la masa de cada individuo para un determinado asunto como un coeficiente de capacidad de ese individuo para modificarse en el asunto que se considere...”.*

Note 11. (Portuondo 1912a-2), p. 197.

*“En ese sistema de exposición de la Mecánica –que denominan energético- tienen que empezar por definir las energías cinéticas y potencial; no quieren usar la noción de fuerza [...] Creemos que este sistema de exposición no ha alcanzado un grado suficiente de madurez y de vulgarización...”*

Note 12. (Portuondo 1912a-6), p. 397.

*“No podemos entrar aquí en la consideración de las ecuaciones diferenciales de segundo orden a que conduce el problema general de la Dinámica, ni mucho menos referirnos a la forma dada por Lagrange a las ecuaciones diferenciales, limitando el número de variables a las estrictamente necesarias. No alcanzo yo a ver cómo se pueda ahondar para la Mecánica social hasta esas profundidades”.*

Note 13. (Portuondo 1912a-7), p. 507, footnote 1.

*“Claro es que un Jefe del Estado, así concebido, no depende de Constituciones escritas ni de procedimientos electorales. En todo caso dependería de la Constitución íntima de la nación, o sea del modo de ser político de todos los individuos y de todos los elementos sociales”.*

Note 14. (Portuondo 1912a-8), p. 547.

*“Si nos parece que las transformaciones en el mundo orgánico son algo distinto de las que se realizan en el inorgánico, es porque aquéllas se concretan y realizan de un modo tal en los individuos [...] que no acertamos a aplicar a ellas de idéntico modo leyes como las que conocemos para las transformaciones de las energías en el mundo inorgánico, por lo que muchos las han llamado energías vitales.”*

Note 15. (Portuondo 1912a-8), p. 548.

*“Prescindiendo de los detalles, se puede decir, en términos generales, que es siempre alguna manifestación de energía química en el organismo la que provoca inmediatamente cada energía fisiológica, en la cual se transforma aquélla mediante alguna destrucción del material orgánico y la consiguiente pérdida de energía potencial. Inversamente, las energías fisiológicas del animal, al transformarse en físicas, conducen a alguna manifestación de energía térmica (para disiparse), o a energía mecánica”.*

Note 16. (Portuondo 1912a-9), p. 549, footnote 2.

*“No entramos a investigar las causas de estas conexiones y de esa correspondencia. Según advierte Mach, la noción de causa es muy metafísica –como hemos dicho en otra ocasión- y no encuentra lugar aquí”.*

Note 17. (Portuondo 1912a-9), p. 550.

*“... lo que sí puede afirmarse –desde el punto de vista mecánico- es que las energías psíquicas del individuo encuentran limitaciones en las energías del mundo físico a que el individuo natural pertenece. El cuerpo de este, con todos sus órganos y elementos materiales, constituye un sistema sometido a las leyes de la Mecánica Física”.*

Note 18. (Portuondo 1912a-9), p. 552.

*“Este campo o región en que él se halla, lo vemos constituido, en un instante dado, como sigue:*

- 1. Por todo lo físico y psíquico que –fuera del espacio que ocupa el cuerpo del individuo natural- rodea, por decirlo así, a éste; y alcanza hasta donde sea sensible la acción que pueda ejercer sobre el individuo que consideramos.*
- 2. Por todo lo físico y fisiológico de su propio organismo corporal.*
- 3. Por todo lo psíquico general suyo, que pueda influir sobre él para modificar su posición en el asunto de que se trate”.*

Note 19. (Portuondo 1912a-9), p. 555.

*“Y es muy de notar que mediante el despliegue y las transformaciones de las energías totales de los individuos y elementos de una agrupación, éstos influyen a su vez, muy marcadamente, sobre el medio ambiente físico externo, y lo modifican, ayudándose de todos los conocimientos adquiridos por las ciencias físicas, químicas y naturales. Con la ayuda de las ciencias psíquicas y sociales –si éstas progresaran bastante- se influiría muy marcadamente también sobre el ambiente psíquico interno y externo”.*

Note 20. (Portuondo 1912a-10), p. 568.

*“Cuando un sistema de puntos [...] se halla en movimiento en el espacio, y se miran estas masas como como ejerciendo acciones dinámicas unas sobre otras (y nada más), el campo de fuerzas, para cada una de ellas, en un instante dado, está constituido por todas las demás”*

Note 21. (Portuondo 1912a-10), p. 568.

*“... y esta inclinación hacia la disipación de la energía mecánica en la Naturaleza es contraria a los intereses de la Humanidad, como ha observado Lord Kelvin”.*

Note 22. (Portuondo 1912a-10), p. 571.

*“Consagrándose las ciencias (todas las ciencias) al descubrimiento de las leyes y a la investigación de los procedimientos adecuados para alcanzar la mayor utilización posible [disminuyendo para ello más y más la energía disipada, es decir haciendo que los coeficientes de transformación sean los mayores posibles], contribuirán hasta donde ellas alcancen a la economía de la Energía Universal”.*

Note 23. (Portuondo 1912a-10), p. 568.

*“Concebida la energía potencial de que se disponga, para cada asunto, en un instante, y haciendo aplicación a una agrupación nacional, por ejemplo, se debe de pensar que el fin de la educación de un pueblo –tomando la palabra educación en su más amplio sentido- debe de consistir en que las fuerzas interiores en conjunto hagan efectivamente trabajos positivos en todos y cada uno de los asuntos de carácter social...”*

Note 24. (Portuondo 1912a-3), p. 246.

**“Advertencia.-** Tengo ahora conocimiento de un libro del Profesor Sp. C. Haret, de Bucarest, titulado *Mecánica Social* y publicado a fines de 1910. Veo que ese trabajo es enteramente distinto del mío, aunque en ambos se aplique a los individuos y agrupaciones los Teoremas de la Mecánica racional; porque se hace la aplicación desde puntos de vista diferentes y con muy diverso criterio, como podrá apreciar quien leyere uno y otro trabajo. Mi estudio es predominantemente psicológico (como base de lo social) y además puramente abstracto y teórico. Mientras que Mr. Haret aspira en el suyo a hacer una Mecánica aplicada (al menos en primera aproximación), habiendo sido la aplicación a la política el móvil que le ha impulsado a hacer su trabajo.

*Así, en toda la segunda mitad del libro, se trata de cosas que no tienen analogía alguna con con lo tratado por mí en en estos Apuntes; y termina con reflexiones sobre la marcha de la civilización.*

*En la Mecánica social propiamente dicha, veo que Mr. Haret había estudiado ya alguna de las cuestiones que yo he abordado en este trabajo, pero lo había hecho de manera muy diferente. Es de notar, sin embargo, que yo haya llegado –aunque sea por distinto camino- a un modo de extender el principio de inercia que de algún modo coincide con el de Mr Haret, y que yo haya llegado a concepciones algo análogas a las suyas sobre la noción de masa, para poder mira a ésta como constante”.*

## References

Ball P (2002) The physical modelling of society, a historical perspective, *Physica A* 314, pp. 1-14.

Ball P (2004) “Critical Mass: How One Thing Leads to Another”, Heinemann/Farrar, Straus & Giroux, London.

Blanco E (2006) “Influencia de la legislación en la información medioambiental...” Ph. D. Thesis, online at <http://www.eumed.net/tesis/2006/erbr/>

- Clausius R (1885) “Über die Energievorräthe der Natur und ihre Verwerthung zum Nutzen des Menschen”, Verlag von Max Cohen & Sohn, Bonn.
- Comte A (1830-1842) “Système de Philosophie Positive”. Bachelier, Paris. 6 Vols. The *Physique Sociale* begins with Volume 4, published in 1839.
- Durkheim E (1886), La science sociale selon de Greef, *Revue philosophique (de la France et de l'étranger)*, 22, pp. 658-663.
- Espinoza E (1994) Le réalisme scientifique: une métaphysique tronquée, *Archives de Philosophie*, 57, pp. 325-340.
- Fernández I, Pacheco J (2005) On the role of Engineering in Mathematical Education, *Eur. J. Engin. Educ.*, 30(1), pp. 81-90.
- Gissis S (2002), Late nineteenth-century Lamarckism and French Sociology, *Perspectives on Science*, 10 (1), pp. 69-122.
- Greef G de (1893) “Les lois sociologiques, esquisse d’une Sociologie”, Félix Alcan, Paris.
- Haret S (1910) “Mécanique Sociale”. Gauthier-Villars (Paris) and C. Göbl (Bucarest).
- House F (1926) Review: Essais de Mécanique Sociale, *American Journal of Sociology*, 32(1), pp. 128-129.
- Lagrange J B (1814) “Essai philosophique sur les probabilités”, Courcier, Paris.
- Mach E (1883) “Die Mechanik in ihrer Entwicklung historisch und kritisch dargestellt”, (first edition), F. A. Brockhaus, Leipzig.
- Mach E (1918) “Analyse der Empfindungen und das Verhältnis der Physischen zum Psychischen” (seventh edition, first one 1885), Gustav Fischer Verlag, Jena.
- Machimbarrena V (1927) Portuondo. *Revista de Obras Públicas*, 75 (Nº 2472), pp.85-86.
- Pacheco J, Pérez-Fernández, F, Suárez C (2007) Infinitesimals in Spain: Portuondo’s *Ensayo*. Submitted to *Archive for History of Exact Sciences*. A first draft is available from the web page [www.dma.ulpgc.es/profesores/personal/jmpc](http://www.dma.ulpgc.es/profesores/personal/jmpc)
- Portuondo A (1880) “Ensayo sobre el infinito”, Aribau y Cia, Madrid.
- Portuondo A (1912a-1) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1898), pp. 63-71.
- Portuondo A (1912a-2) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1908), pp. 191-198.
- Portuondo A (1912a-3) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1912), pp. 237-246.
- Portuondo A (1912a-4) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1917), pp. 297-303.
- Portuondo A (1912a-5) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1919), pp. 321-327.
- Portuondo A (1912a-6) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1925), pp. 393-399.
- Portuondo A (1912a-7) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1934), pp. 506-510.
- Portuondo A (1912a-8) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1937), pp. 546-548.
- Portuondo A (1912a-9) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1938), pp. 549-555.
- Portuondo A (1912a-10) Apuntes sobre mecánica social. *Revista de Obras Públicas*, 60, (1939), pp. 567-571.

- Portuondo A (1912b) *Apuntes de Mecánica Social*. Imprenta de Prudencio Pérez de Velasco, Madrid.
- Portuondo A (1912c) Les lois infinitésimales dans l'Analyse mathématique, *Revue générale des Sciences*, 23, pp. 741-746.
- Portuondo A (1917) Principes fondamentaux pour la Sociologie Mécanique, *Revue philosophique (de la France et de l'étranger)*, 83, pp. 219-245.
- Portuondo A (1925) Essai de Mécanique Sociale, *Bibliothèque Sociologique Internationale*, Vol. LVII, Marcel Giard, Paris.
- Putnam R (ed) (2002) "Democracies in Flux", Oxford University Press, New York..
- Quételet A (1835) "Sur l'homme et le développement de ses facultés, ou Essai de Physique Sociale", Bachelier, Paris.
- Sacher E (1881) "Grundzüge einer Mechanik der Gesellschaft, Theil I", Gustav Fischer, Jena (part II was never published).
- Sacher E (1899) "Die Gesellschaftskunde als Naturwissenschaft", Pierson's Verlag, Dresden and Leipzig.
- Sacher E (1901) "Die Massenarmut: ihre Ursache und Beseitigung", Akademische Verlag für soziale Wissenschaften, Berlin.
- Ward L (1896a) The Social Forces, *American Journal of Sociology*, 2(1), pp. 82-95.
- Ward L (1896b) The Mechanics of Society, *American Journal of Sociology*, 2(2), pp. 234-254.
- Ward L (1897) "Outlines of Sociology", McMillan, New York.
- Ward L (1913) Review: Two books on Social Mechanics, *American Journal of Sociology*, 18(6), pp. 814-819.