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Leonhard Euler and the Theory of Ships

A condensed version of this article will be presented in the Captain Ralph R. and Florence Peachman Lecture at The University of Michigan in Ann Arbor on April 16, 2007. This occasion provided an additional incentive for preparing these notes in commemoration of Leonhard Euler's tricentennial and of his contributions to ship theory.

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Leonhard Euler and the Theory of Ships

Horst Nowacki, Berlin

Abstract

On April 15, 2007 the scientific world will commemorate *Leonhard Euler's* 300th birthday. *Euler's* eminent work has become famous in many fields: Mathematics, mechanics, optics, acoustics, astronomy and geodesy, even in the theory of music. This article will recall his no less distinguished contributions to the founding of the modern theory of ships. These are not so widely known to the general professional public. In laying these foundations in ship theory like in other fields *Euler* was seeking "first principles, generality, order and above all clarity". This article will highlight those achievements for which we owe him our gratitude.

There is no doubt that *Leonhard Euler* was one of the founders of the modern theory of ships. He raised many fundamental questions for the first time and through all phases of his professional lifetime devoted himself to subjects of ship theory. Thereby he gave a unique profile to this young, still nascent scientific discipline. Many of his approaches have been of lasting, incisive influence on the structure of this field. Some of his ideas have been become so much a matter of routine today that we have forgotten their descent from *Euler*. This article will synoptically review *Euler's* contributions to the foundation of this discipline, will correlate them with the stages of *Euler's* own scientific development, embedded in the rich environment of scientific enlightenment in the 18th c., and will appreciate the value of his lasting aftereffects until today. The same example will serve to recognize the fertile field of tension always existing between *Euler's* fundamental orientation and his desire to make contributions to practical applications, which has remained characteristic of ship theory to the present day. Without claiming completeness in detail this article aims at giving a coherent overview of *Euler's* approaches and objectives in this discipline. This synopsis will be presented primarily from the viewpoint of engineering science in its current stage of development.

1. Introduction

Leonhard Euler (1707-1783) is famous for many brilliant scientific achievements in mathematics, in solid and fluid mechanics, as a physicist in optics and acoustics, in astronomy and geodesy, and even in the theory of music. *Euler* excelled also in the application of science to practical problems, which in our current awareness has been relegated to the background. One of his favorite application themes was also the *theory of ships*, i.e., the application of the scientific principles of solid and fluid mechanics to practical and technical problems in the design, performance evaluation and in the operation of ships. *Euler* throughout his scientific career devoted no small share of his investigations and publications to such topics in the theory of ships.

It is all the more surprising that *Euler's* role in the foundation of modern ship theory and the substance of his contributions are not widely known today, even among specialists who are working on matters of ship theory today. In this regard the statement made by the French geometer *Taquet* in the 17th c., referring to *Archimedes*, the classical ancestor of ship theory, may also be applied to *Leonhard Euler*:

> „All praise him, few read him, all admire him, few understand him".

Many causes may have contributed to creating this gap in our historical memory. First, *Euler* wrote the majority of his publications in Latin, the classical language of

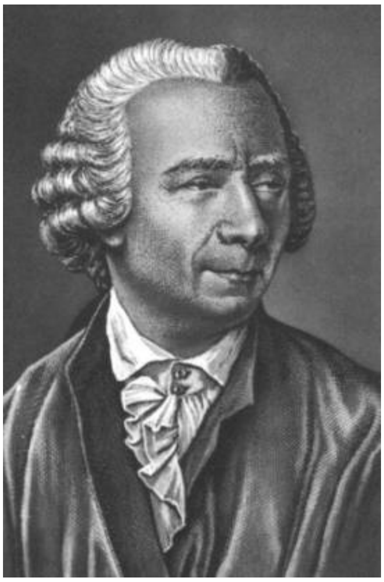

Fig. 1: Leonhard Euler (1707-1783)

Fig. 2: Pierre Bouguer (1698-1758)

science at the Academies in his era, in particular also his principal opus on Ship Theory, the famous „*Scientia Navalis*"[1] (1749). Thus the access thereto by practitioners of shipbuilding must have been difficult, even during *Euler's* lifetime. Until today there exists no complete translation of "*Scientia Navalis*", a two volume work of ca. 900 pages, into a modern language. Second, it is true that he gave an analytical exposition of the subject in brilliant clarity of style, but he rarely ever concerned himself with numerical examples for specific, built or designed ships. Third, he presumed from his reader an understanding of the mathematical notation of analysis and of the methods of infinitesimal calculus, which was not widely spread in his days. Thus his publications on the theory of the ship became accessible and comprehensible to the general public only after a long delay. Nevertheless his influence in science was always well perceived and even without many readers did reach those who later continued his work in developing ship theory, certainly no later than in the 19^{th} c.

Euler is not the only founder of the modern theory of ships. Among his contemporaries the French scientist *Pierre Bouguer* (1698-1758), hydrographer, mathematician, geodesist, physicist and in particular also ship theoretician must be mentioned who had played a similar eminent role in the 18th c. *Bouguer* has by his principal opus "*Traité du Navire*" [2] (1746), written in French, during his lifetime no doubt gained faster and more direct influence on developments in practice, in particular since he was willing also to furnish practical examples and numerical calculation methods. *Bouguer* and *Euler* both were first to use infinitesimal calculus as an approach to subjects of ship theory. This is why I count both of them as founders of "*modern*" ship theory. Their contemporaries, especially during the second half of the 18^{th} c., such as *D'Alembert, Jorge Juan de Santacilia* or *Chapman,* also made other, influential contributions to the fundamentals of the theory of ships. In this illustrious field *Euler* is eminent by his long range impact on the whole field of ship theory which he founded on the first principles of mechanics and to which he gave a systematic structure. This is why *Euler's* role as a founder of ship theory will be particularly stressed in this article.

In the following sections a chronological survey will be given first illustrating how topics of ship theory accompanied *Euler's* whole scientific career. Then *Euler's* individual contributions are sorted by thematic aspects and are assigned to the most important subtopics of ship theory: Hydrostatics and ship stability, resistance, propulsion, maneuvering and ship motions in rigid body degrees of freedom. His impact on the development of general fluid mechanics will be briefly recorded, too. This overview will demonstrate the close interrelation between fundamentals and applications in his work, and will illustrate on the other hand how even a brilliant scientist like *Euler* cannot offer all definitive solutions in one strike, but must continuously learn by experience to achieve a mature form of his insights. *Euler* accepted and succeeded in this lifelong learning process.

In later literature *Euler* by some commentators, and not infrequently, was characterized as a pure mathematician without motivation for practical applications. I must dismiss such views (cf. also *Truesdell* [3], *Eckert* [4]). Rather it is essentially demonstrated by his contributions to the foundation of the theory of ships that his goal in this context was to achieve practical success in applications by a well constructed physical foundation. This message will also be presented in this article.

2. The Theory of Ships in Euler's Biography

Euler's biography is well known from multitudinous presentations (e.g. Fellmann [5]). His oeuvre has been very systematically documented and reprinted in his Collected Works (Opera Omnia Euleri [6]), although these comprehensive series have not yet been fully completed. In this collection his treatises dealing with the theory of ships are contained, too (Series II, vols. 18-21). *Table I* gives an overview of *Euler's* contributions to this subject area with reference to the Enestroem Index Nos. of his publications. In these volumes generally very elaborate appreciations and interpretations of his works are given. The following must be mentioned in particular in the context of fluid mechanics and ship theory: *Truesdell* [3], *Habicht* [*7*, *8*, 9]. Further in the same vein important summaries are given in: *Burckhardt et al.* [10], *Szabó*[11], *Mikhajlov* [12], *Calinger* [13]. In addition several general textbooks on the history of fluid mechanics contain sections appreciating *Euler's* contributions (e.g. in *Calero* [14], *Darrigol* [15]). The monograph by *Ferreiro* [16] on the history of ship science in the 17^{th} and 18^{th} c. also touches on many details of *Euler's* life and work. The contents of such available sources shall not be repeated here. Suffice it to acknowledge in gratitude that I owe them much inspiration.

Rather I just want to retrace here how it happened that *Euler* developed a predilection for subjects of the theory of ships, a preference that is not obvious for the son of an Alpine country, and how he consistently returned to his early maritime application interests during many stations of his vita. *Table I* shows evidence that his related publications appeared between 1727 and 1782. How did this lifelong affinity arise?

Leonhard Euler was born in Basel, Switzerland on April 15, 1707 as the son of the Swiss reformed priest *Paulus Euler* and his wife *Margarethe née Brucker.* He attended a Latin Grammar School in Basel at the age of 8 to 13, as it was customary, and then enrolled at Basel University in preparatory courses. Here before he was age 15 he met his later mentor and patron *Johann Bernoulli* (1667-1748), one of the leading scientists of this era in mathematics and mechanics , from whom he took courses in these two subjects. *Bernoulli* very early recognized *Euler's* eminent talent and invited him to take part in the series of Saturday private meetings in his house which he held with his sons and selected guests and where mathematical subjects were discussed, literature was reviewed and problems were solved. This is where *Euler* also got acquainted with *Johann Bernoulli's* sons, in particular with *Daniel Bernoulli* (1700-1782), with whom he developed a long lasting friendship. After completing his first degree in 1723 by the Magister examination, although *Euler* at his father's urging then enrolled in the divinity faculty, he continued intensively to concentrate on mathematical studies under *Johann Bernoulli*, who much promoted him.

Now it was essentially a coincidence that *Johann Bernoulli* had already for some time concerned himself with special questions of ship theory because he regarded this field as a promising area for applying the still new methods of infinitesimal calculus

Table I: Euler's Publications on Subjects of Ship Theory

Enestroem Index No.	Title	Place in Opera Omnia, Series vol., p.	Manu-script Date	Publi-cation Date	Publi-cation Place
E.4	Meditationes super problemate nautico, quod illustrissima regia Parisiensis Academia scientiarum proposuit (Thoughts on the Nautical Problem Posed by the Most Illustrious Royal Parisian Academy of Sciences)	II 20, 1-35	1727	1728	Paris
E.78	Dissertation sur la meilleure construction du cabestan (Treatise on the Best Construction of the Capstan)	II 20, 36-82	1741	1745	Paris
E.94	De motu cymbarum remis propulsarum in fluviis (On the Motion of Barges Propelled by Oars on Rivers)	II 20, 83-100	1738	1747	St. Peters-burg
E.110/ E.111	Scientia Navalis, seu tractatus de construendis ac dirigendis navibus (Ship Theory or Treatise on the Construction and Steering of Ships)	II18/19, 2 vols.	1741	1749	St. Peters-burg
E.116	Mémoire sur la force des rames (Memorandum on the Force of Rudders)	II 20, 101-129	1747	1749	Berlin
E.137	Examen artificii navis a principio motus interno propellendi (Examination of the Thought to Propel A Ship by the Principle of Internal Motion)	II 20, 130-145	1747/48	1750	St. Peters-burg
E.150	Meditationes in quaestionem observationibus temporis momentum determinandi (Thoughts on the Question of Determining the Momentum by Observation of Time)	II 20, 130-145	1747	1750	Paris
E.413	De promotione navium sine vi venti (On the Propulsion of Ships without the Force of Wind)	II 20, 146-189	1753	1771	Paris
E.415	Sur le roulis et le tangage (On Rolling and Pitching)	II 21, 1-30	1759	1771	Paris
E.426	Théorie complette de la construction et de la manœuvre des vaisseaux (Complete Theory of the Construction and Maneuvering of Ships)	II 21, 80-222	1778	1781	St. Peters-burg

Table I: Euler's Publications on Subjects of Ship Theory (cont.)

Enestroem Index No.	Title	Place in Opera Omnia, Ser., vol.,p.	Manu-script Date	Publi-cation Date	Publi-cation-Place
E.520	Essai d'une théorie de la résistance qu'éprouve la proue d'un vaisseau dans son mouvement (Attempt at a Theory of Resistance Referring to the Forebody of a Ship in its Motion)	II 21, 223-229	1778	1781	Paris
E.545	De vi fluminis ad naves sursum trahendas applicanda (On the Force to Apply to Tow Ships upriver)	II 21, 230-242	1780	1783	St. Peters-burg

developed by *Newton* and *Leibniz. Johann Bernoulli* in a treatise as early as 1714 [17] had been first to attempt to apply *Newton's* impact theory of resistance (cf. below) to determine the forces acting on a ship sailing before the wind, viz., both the hull resistance due to underwater hydrodynamic forces and the thrust acting on the sails due to the aerodynamic effects of wind. He achieved this in a mathematically valid way by integration of the fluid dynamic impact forces acting on their respective areas of application. However the agreement of these predictions with empirical observation was disappointing, which was to be attributed to the inadequate premises stipulated in *Newton's* impact theory for the determination of the resistance in water. The subject remained of intriguing interest to *Bernoulli* and met a fashion of this time which favored subjects aiming at improvements in the performance of technical systems such as ships by means of scientific methods.

When the Parisian Royal Academy of Sciences launched a prize contest in 1726 in order to determine the optimal configuration, number and height of masts for sailing vessel propulsion, *Johann Bernoulli* encouraged his student *Leonhard Euler* to submit a contribution. Why *Johann Bernoulli* did not participate in the contest himself, cannot be clearly stated. Perhaps he wanted to promote *Euler* and at the same time avoid the risk of laying open his cards to the competition? In any case it was in this way that *Euler's* first treatise [18] on a subject of ship theory was initiated, which he submitted in 1727 at the age of. 20.

Euler earned the recognition of a runner-up award (Grade: "Accessit") The first prize went to *Pierre Bouguer*, the already better known French scientist, who had been working on the issue of ship masting for some time. It is futile to compare the merits of the two treatises today. For on the one hand the two authors departed from very similar fundamental premises, in particular from *Newton's* impact theory of resistance as well as a precise knowledge of contemporary sail force theory, which had been intensively and controversially debated during the decades before the contest (*Renau* [19], *Huygens* [20]). On the other hand neither author yet achieved a satisfactory solution to the problem. Both suffered from deficits in the fundamentals of hydrostatics which were required for determining the equilibrium floating condition by trim and heel angle when the ship was displaced by wind loads. This issue was circumnavigated. *Euler* e.g. assumed the vessel to be fixed (or frozen in ice) temporarily in a heeled and trimmed condition. It appears however that with respect to such weak assumptions the results of the prize contest and the knowledge achieved were not to the satisfaction of *Euler* at least, perhaps not of *Bouguer* either. For both continued to work on the subject and certainly had recognized the deficits in the fundamentals of ship hydrostatics.

In any event *Euler* by this experience had established a first closer acquaintance with subjects of the mechanics of ships, has familiarized himself with the concepts of ship geometry, sailing equipment and nautical topics and from now on maintained a lasting interest in problems of ship theory as issues of applied mechanics and mathematical analysis.

Still in 1727 *Euler* received an attractive offer from the Russian Imperial Academy of Sciences in St. Petersburg, founded by Tsar *Peter I* in 1725. He was pleased to accept this offer, initially with a modest salary as a research fellow. He traveled by ship on the Rhine from Basel to Mainz, from there by horse drawn coach to Lubeck where he embarked on a ship to cross the Baltic Sea to reach the Neva in St. Petersburg in June 1727. He was scientifically quite successful right away and soon advanced to be appointed a full member of the Academy (1731) with responsibilities in physics, and especially in mathematics and mechanics. His first St. Petersburg period extended from 1727 to 1741. During this time he also intensively dealt with topics in ship theory.

This is attested first of all by the minutes and reports of the Academy, which in 1735 assigned to *Euler* a review of a treatise submitted by *La Croix* on the transverse stability of the ship and, as his reply makes evident, found him well prepared to point out errors in *La Croix'* derivation and to provide the correct answer, at least for the simple shape of a floating parallelepiped. *Euler's* stability criterion from the beginning was *a positive restoring moment when the ship was inclined by a very small (infinitesimal) angle.* A general solution for ships of arbitrary hull form was already in preparation in 1735.

His reputation for outstanding knowledge in the theory of ships became well known at the Academy, so in 1737 – probably at his prior request – he was commissioned by the Academy to write a comprehensive treatise on the whole range of subjects in ship theory. Therefrom resulted *Euler's* most famous opus on this subject, the two volume treatise *Scientia Navalis. Euler's* manuscripts for this work were completed by 1741, but he failed to find a publisher for this voluminous, very specialized treatise in Latin so that after very long delays this monumental work of early ship theory did not appear until 1749 in St. Petersburg. (*Pierre Bouguer's Traité du Navire* [2], a work of comparable significance, had been developed simultaneously and independently (see [21]), but was published in 1746 in Paris soon after *Bouguer's* return from an Andean scientific expedition conducted from 1735 to 1744 by the Parisian Academy [16]).

After many scientifically fertile years in St. Petersburg *Euler* in view of a certain political turmoil in Russia accepted an invitation by *King Frederick II* of Prussia to move to Berlin in 1741, where he was involved in the foundation of the Royal Academy of Sciences (1745) and were he worked through 1766. During his Berlin years *Euler*, officially responsible for the class of mathematics, was active in many fields of science and remained very creative. Fluid mechanics from these years owes him many important, even today still fundamental results:

- *Euler's* equation of the motion of a fluid volume element (1752),
- The rejection of *Newton's* impact theory of resistance, practised as „common rule“ until that time (1753),
- The establishment of a new field theory of fluids, i.e., a continuum theory of fluid mechanics, based on the modern concepts of pressure and velocity, initially applied to ideal fluids (1755),
- Beginnings of potential flow theory (1755),
- Considerations on fluid friction (1751).

These fundamental results led to a new level of understanding of flow phenomena, furnished the exposition for a catalogue of questions for a large class of applications and had lasting effects on the further development of fluid mechanics. The discipline of ship theory was included in this spectrum.

Although the most important branches of ship theory had been addressed already in his *Scientia Navalis*, *Euler* found the opportunity also in his Berlin years to go deeper into certain questions and to arrive at new results. The most important subjects during this period include ship propulsion [E.116, E.413/[22]] and ship motions in oscillatory degrees of freedom [E.415]. Several of these new studies were motivated by prize contests of the Parisian Academy in which Euler participated with success.

After some irritating quarrels with *Friedrich II Euler* in 1766 returned to St. Petersburg, where he was welcomed with open arms. During his second St. Petersburg period (1766-1783) several further studies in ship theory were performed. The most important lasting effects probably stem from his *„Théorie Complète...“* [E.426/[24]], a French abridged and extended translation of *Scientia Navalis* in popularized form. This work soon was used as a textbook, too, in the education of French naval constructors.

Euler by his lifetime oeuvre has lastingly formed and enriched ship theory. He has based it on the first principles of mechanics and thus placed it on a stable foundation for its future development. He was not able to provide definitive answers to all questions raised. Much remained open. But he had cast a basic structure of the field in which scientific work could steadily progress.

3. Individual Contributions to Ship Theory

3.1 Hydrostatics und Ship Stability

The foundations of hydrostatics were laid by *Archimedes* (ca. 287-212 B.C.) in antiquity. In his famous treatise “On Floating Bodies” [25] he derived what later became known as *the Principle of Archimedes,* i.e., the theorem of equilibrium between the forces of weight and buoyancy for a floating object of arbitrary shape. In the same treatise, Part II, he also established a criterion of stability of this equilibrium for a body floating at rest, though only for the special case of a body of simple shape, the axisymmetric paraboloid. His justification was that in an inclined position of the body the couple formed by the forces of buoyancy and weight (=displacement) must provide a positive restoring moment for the floating position to be stable. Else the body heels over further and may capsize. For further details cf. *Nowacki* [26]. *Archimedes* as far as we know did not yet apply this criterion to actual ships.

This knowledge possessed by *Archimedes* was almost completely forgotten for many centuries, although fortunately a few handwritten copies of his treatise [25] in Latin and in Greek were preserved [26]. But it took many centuries, almost two millennia, before the essential insights in this treatise were rediscovered and applied. It was first *Stevin* [27], then *Pascal* [28] and also *Huygens* [29] who resurrected hydrostatics (and aerostatics) and applied them to modern systems. *Stevin*, the Flemish/Dutch scientist, also first introduced the concept of *hydrostatic pressure*, based on the weight of the fluid column. *Huygens* first investigated the hydrostatic stability of simple, homogeneous bodies as they were rotated about their longitudinal axis by 360 degrees, however did not publish his results so that these appeared only posthumously in the beginning of the 20th c. in his Collected Works [29].

A few further cuts were taken at the stability problem by *Hoste* [30] (1698) and by *La Croix* [31] (1735), but they remained still without success. *Bouguer* und *Euler*, too, in their prize contest treatises of 1727 did not yet offer a practically promising approach. It was only by means of integral calculus that they later succeeded, independently of each other, to develop criteria for the hydrostatic stability of ships of arbitrary shape, which were published in their treatises, *Traité du Navire* {2] and *Scientia Navalis* [1] in 1746 and 1749, respectively. *Euler's* approach will be discussed here in more detail.

To understand *Euler's* objectives in his *Scientia Navalis*, it is useful to take a look at the title of this work and the subtitles of the two volumes, which in free English translation are:

"Ship Science or Treatise on the Construction and Operation of Ships:

Vol. I: General Theory of the Position and Motion of Bodies Floating in Water,
Vol. II: Reasons and Rules for the Construction and Steering of Ships."

Euler never chose the wording "*Theory of Ships*", as later authors did. But since his opus concerns the "*Science of Ships*" and he aspired to furnish a *general theory* of the equilibrium at rest and the motion dynamics of floating bodies, applicable in shipbuilding and ship operations, as his title and subtitles claim, it is fair to state that his objective was the development of a *Theory of the Ship*, which comprises the fundamentals of mechanics for the design, construction and operation of ships. This claim is justified by the systematic structure of his work, even if he was not able, being limited to the methods of his time, to do justice to all related questions of ship theory. He did already provide very significant contributions to the hydrostatics and stability of ships. These subjects were treated in Chapters I-IV of Vol. I and Chapters I-III and V of Vol. II of his *Scientia Navalis*, both in their fundamentals (Vol. I) and their applications to ships (Vol. II).

Euler opened Chapter I of Vol. I (Equilibrium of floating bodies) with the sentence:

"*The pressure which the water exerts on an immersed body at a specific point is normal to the body surface, and the force which an individual surface element experiences is equal to the weight of a cylindrical water column whose basis is equal to the surface element and whose height is equal to the submergence (z) of the element under the water surface*."

This implies the defition of *hydrostatic pressure* $p = \gamma z$ and also of the resulting *buoyancy force F* by integration over the body surface S

$$F = \int p \, dS = \gamma V \quad \text{with } V = \text{displacement volume}$$

As already underscored by *Truesdell* [3], *Euler* formulated here in a single sentence the necessary and sufficient axiomatic premises on which hydrostatics are entirely based. These premises can hardly be stated more concisely and clearly. In contrast to *Archimedes*, who knew only the resultant hydrostatic buoyancy force, but not the pressure, *Euler* (and similarly *Bouguer*) derived the buoyancy force as the integral of the hydrostatic pressure distribution. This cleared the way to calculating the buoyancy force for arbitrary body shapes.

By moment equilibrium it also holds that the buoyancy and weight resultants must act in the same vertical plane.

In the course of Chapter I *Euler* further calculated the equilibrium floating positions of simple prismatic shapes (triangular, trapezoidal and rectangular cross section) through a rotation of 360 degrees from the condition that the volume and weight centroids must lie in the same vertical plane. Each of these shapes has several equilibrium positions which may be stable, unstable or indifferent. The stability of each position requires its own investigation.

In Chapter III (Stability of the equilibrium of floating bodies) *Euler* dealt with the derivation of a stability criterion. He proceeded in the following steps:

<u>STEP 1: Premises and Axioms</u>

In his stability considerations like on the issue of equilibrium *Euler* proceeded from the same premises as cited regarding the hydrostatic pressure in a fluid at rest and its action normal to the surface.

<u>STEP 2: Resultant Buoyancy and Weight Forces</u>

The resultant buoyancy force in the upright floating condition of the ship by Euler was determined as the resultant of the hydrostatic pressures acting on the submerged part of the body surface. This force acts through the volume centroid F of the underwater hull form (in English today: Center of Buoyancy CB), which *Euler* called "centrum magnitudinis".

Analogously *Euler* combined all weight components of the ship into the resulting weight force acting through the center of gravity G.

<u>STEP 3: Volumes and Centroids</u>

Euler defined the geometrical relationships for volumes and centroids in analytic form by integral expressions for arbitrary hull forms and floating conditions, but left the evaluation of these expressions to numerical methods. The closed form evaluation of the integrals succeeded only for simple, regular shapes, which he used as examples.

<u>STEP 4: Stability Criterion</u>

Euler just tersely stated:

„The stability by which a floating body is retained in equilibrium will be determined from the restoring moment arising when the body is displaced from equilibrium by an infinitesimally small angle".

This basic idea stemmed from *Archimedes*, too, but was here applied to ships. *Euler* pointed out that the ship must have a sufficient reserve of stability to be able to resist the external effects of heeling moments.

<u>STEP 5: Evaluation of the Criterion</u>

Euler began his derivation by considering a planar cross section of the ship (Fig. 3). The cross section was intentionally assumed to be non-symmetrical to keep the derivation general. The ship was displaced by a very small angle from the floating position AB to the new floating position ab. Thereby the triangle bCB was immersed, the triangle aCA emerged, so that area centroid of the triangles moved from the emerging to the immersed side. The area centroid of the whole cross section thereby moved parallel to this shift to the immersed side (shift theorem of *Archimedes*). From this action a restoring couple resulted, formed by the weight force (acting through G) and the buoyancy of the cross section (acting through the shifted area centroid of the cross section). By integration of these effects over the entire ship length *Euler* obtained the restoring moment for the whole ship. Therefrom resulted after a few intermediate steps the expression for the *restoring moment* (for a symmetrical ship), which is very well known today:

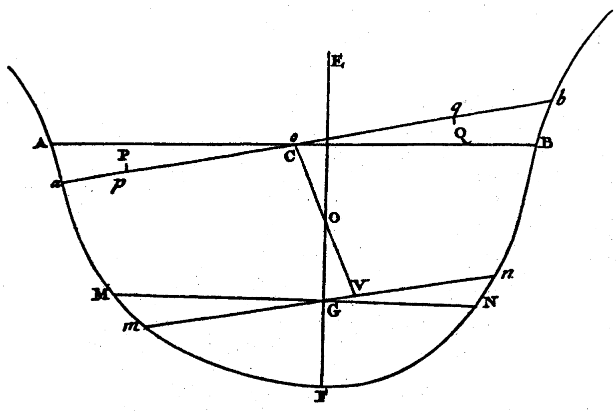

Fig. 3: Derivation for a cross section (according to *Euler* [1]): Displacement of the volume centroid O to the immersed side,
G = center of gravity

$$M_{REST} = \Delta\,(GB + I_T/V) = \Delta\,(GB + BM) = \Delta\,\overline{GM}\,,$$

where $\Delta = \gamma V$ = displacement
V = volume under the water surface
GB = distance from the center of gravity to the center of buoyancy, positive for B above G
I_T = area moment of inertia
$BM = I_T/V$
GM = GB +BM = „metacentric radius“

Thus the floating position is stable at small angles of inclination provided that the restoring moment $\Delta\,\overline{GM}$ is positive. This is called *positive initial stability* of the ship.

This result is entirely equivalent to *Bouguer's* in [2], who for a stable ship postulates a positive metacentric radius GM. The name "metacenter" stems from *Bouguer* and was never used by *Euler*, who was not familiar with this terminology. But in fact both derive the magnitude of GM from the ship form by the same expression in order to judge the initial stability. *Euler* uses a physical quantity, *Bouguer* a geometric one in formulating the stability criterion.

Further details on the derivation of the stability criterion by *Euler* und *Bouguer*, also in comparison, are given by *Nowacki* and *Ferreiro* [21].

<u>Extensions:</u>

The knowledge of the metacentric radius GM for the transverse stability and by analogy of GM_L for the longitudinal stability of the ship now also permitted deriving the equilibrium floating condition of the ship provided that the internal weight distribution in the ship and the external loads, e.g. wind loads in the sails, was known by magnitude and direction. Thereby one could predict the angles of heel and trim for any desired internal weight distribution and external load case. Thus both *Bouguer* and *Euler* closed a knowledge gap still existing in their 1727 prize contest treatises.

Further *Euler* demonstrated in his *Scientia Navalis*, Vol. I, Ch. IV, how the stability of a ship can be improved, e.g., by lowering the CG, by raising the CB or by broadening the design waterplane (raising of M). The effects of weight displacements aboard the vessel or changes of the cargo distribution during loading and unloading as well as the effects of ballast placement by quantity and position were analyzed as practical questions. These results have not lost any practical relevance and accuracy until today.

3.2 **Ship Resistance**

In order to better appreciate *Euler's* contributions to the theory of ship resistance, a few remarks on the earlier developments on this subject will be useful. The interest in predicting the resistance of a hull form and in improving the design of the hull shape in order to reduce the resistance and to increase the achievable ship speed in practical applications is probably as ancient as seafaring. Theoretical methods for resistance prediction, however, based on scientific grounds were not developed before the stage of the „Scientific Revolution" in the 17^{th} and 18^{th} c.

In experimental work *Christiaan Huygens* (1629-1695) and *Edmé Mariotte* (1620-1684) must be named as important precursors. *Huygens* [32] already in 1669 in a small towing tank performed resistance tests with simply shaped ship models, which were towed by a falling weight apparatus (Fig. 4), in order to determine the dependence of the resistance R on the speed V. He found a quadratic resistance law: $R \sim V^2$. *Mariotte* [33] investigated other simple shapes, which he exposed to a current, e.g., in a current in a river or in air, in order to measure their resistance. He found the relationship $R \sim \rho V^2$, hence also a proportionality to the density of the fluid medium ρ. Both sets of experimental results were published only posthumously (1698 for *Huygens*, 1686 for *Mariotte*).

Newton who first published his *Principia* in 1687 certainly was not familiar with these results at that time. But in his own way he quite independently arrived at corresponding conclusions. Thus at the beginning of the 18^{th} c. from various sources there was agreement that the resistance law for objects in parallel inflow had the following structure:

$$R \sim \rho V^2 S \qquad \text{or} \qquad R = C_D \rho V^2 S \,,$$

where S = reference area, e.g., the projection of the maximum cross section (midship section)
ρ = density of the fluid
C_D = resistance coefficient

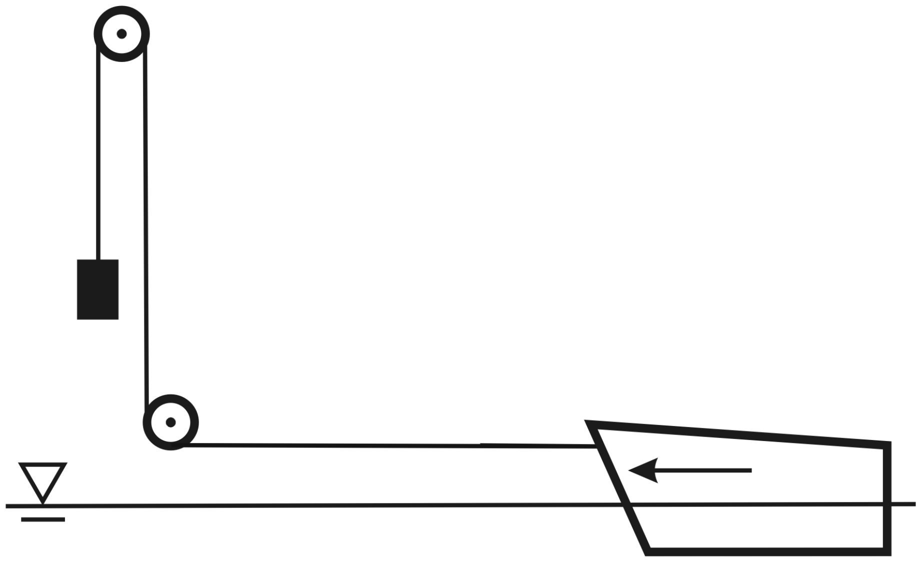

Fig. 4: Towing test apparatus with falling weight according to *Huygens* [32]

If this assumption was accepted, the most important open question was the dependence of the resistance coefficient on the body shape, the direction of inflow and other potential influences. Resistance research, at least in the 18^{th} c., concentrated on this key question, strongly motivated by the goal of finding favorable shapes in fluid flow.

Newton devoted the second volume of his *Principia* completely to fluid mechanics whose theory he newly conceived from fundamentals. *Newton* took an experimental and theoretical approach, but in his theory had to confine himself to simple cases. In his models of thought he introduced many distinct cases and for each set of assumptions argued very cautiously and with incisive simplifications. Although he clearly recognized that in fluid mechanics the influences of inertia forces, gravity forces and viscous effects all play a certain role, in studying resistance he much favored the situation with pure inertia effects and for this case developed a corpuscular resistance theory with the following further assumptions:

- Let the onflow be composed of mass particles (corpuscles) that move on parallel paths with uniform velocity V toward the body (or obstacle).

- Let the fluid medium be so „*thin*“, i.e., of such low density, that the particles maintain a small, but finite distance from each other without colliding with or influencing each other.

- The fluid be either *elastic* so that the particles bouncing on the obstacle are repelled as in an elastic impact without loss of kinetic energy (Fig. 5A); or the fluid be inelastic so that the particle motion upon impact is completely stopped. In the event of oblique inflow the particle paths are deflected and mirrored relative to the body normal at the point of contact (Fig. 5B).

- According to the laws of impact in the elastic or inelastic case the resistance of the object can be determined by the momentum balance of the fluid mass stream. The resistance in the elastic case is twice as high as in an inelastic fluid. *Newton* found the resistance coefficients for several simple body shapes on the basis of these laws and assumptions.

Newton himself was very cautious when justifying this experiment of thought for a body in a „*thin*“, corpuscular medium subject only to inertia forces. He never claimed the existence of thin media in nature. He explicitly pronounced that water was not a thin medium. Rather he considered this case as a hypothetical scenario and perhaps as a limiting case that could never be reached. In a different place he directly mentions viscous and gravity effects. But unfortunately his disciples and adherents were not so cautious. They quickly and uncritically proceeded to apply *Newton's* theory to thin media, which they called "*impact theory*", to real fluids and e.g. to bodies in water and air. The results were entirely disappointing, but due to *Newton's* authority such misleading concepts were widespread for a considerable time. The main deficits of impact theory became clearly evident in such applications which were well outside the range of validity which *Newton* had claimed:

- The corpuscular theory does not permit any particles to reach the rear side of the body in its inflow. Rather they are all reflected from the front side. Thus only the front side will incur any resistance. The orbits of all particles near the body are unrealistic.

- The neglect of viscous and gravity effects gravely impairs the prediction of resistance. If inertia effects are assumed to be the only existing forces, then each body shape will have its own, speed independent C_D value. In a real fluid, however, as we know today, the resistance coefficients depend on several categories of forces and hence on several parameters of similitude.

These critical insights were not pronounced in the beginning of the 18th c. Rather for at least half a century impact theory remained the only available, even though increasingly distrusted method for predicting the ship resistance.

The beginning of a critical reanalysis and reformulation of fluid mechanics was made in 1727 by *Daniel Bernoulli* [35], when in experiments he measured the force exerted by a jet impinging on a flat plate and detected fundamental contradictions between his results and *Newton's* impact theory. He then developed a new theory for pipe flows, based on the energy conservation principle, which for this case yielded a new relationship between pressure and velocity in a „*stream* tube", the predecessor of *Bernoulli's* equation. In its further development this led to a new paradigm for fluid mechanics for the parallel onset flow of a flat plate [36], where in contrast to impact theory the streamlines (and thus the particle orbits) are no longer reflected upon impact, but are deflected laterally before impact (Figs. 5C and 5D). This concept was also elaborately presented in *Daniel Bernoulli's* famous book "*Hydrodynamica*" [37].

His father *Johann Bernoulli* in his no less famous book „*Hydraulica*" [38], which appeared in 1742, carried these ideas a little further in a more general vein and based his derivations on *Newton's* Lex Secunda (in place of the energy principle), which he applied to a free volume element in the flow. He also introduced the concept of "*internal fluid pressure*" in a moving fluid, whereby the *Bernoulli* equation for a "*stream tube*" obtained the form well known today.

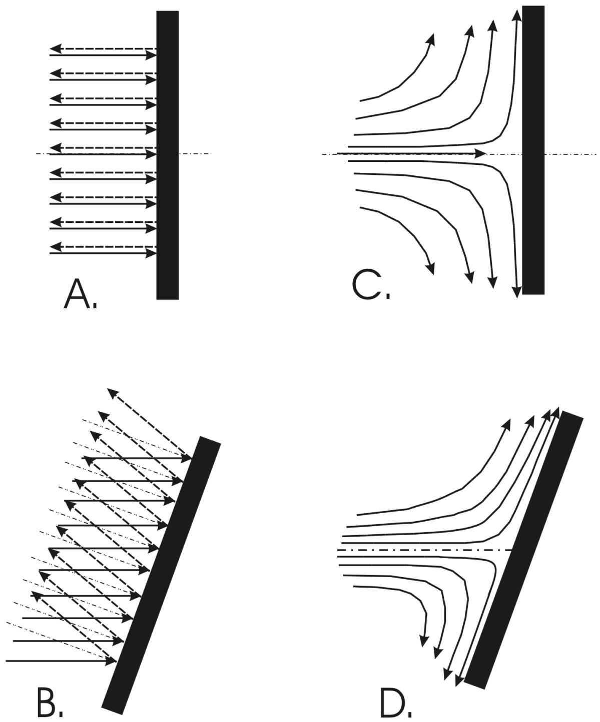

Fig. 5: Flow characteristics in parallel onset flow of a flat plate according to *Newton* and *Daniel Bernoulli*:

A. *Newtons* impact theory in an elastic, thin medium (reflected particles), inflow normal to the plate.
B. The same theory, oblique inflow.
C. *Daniel Bernoulli's* theory of deflected streamlines, normal inflow.
D. The same for oblique inflow.

This was the state of knowledge that *Euler*, who corresponded frequently with both *Bernoullis*, found by around 1742, before he began to base his own ideas on it and to arrive at an original new view of fluid mechanics. At the time when his *Scientia Navalis* was written, viz., between 1737 and 1741, *Euler* was still a user of *Newton's* impact theory of the resistance. Therefore in this principal work of his [1] and also in the later adapted French translation, the *Théorie Complète* [21], we find only deductions based on impact theory. *Euler* expressed his later contrasting views in other places. Nonetheless it is worthwhile to briefly reexamine *Euler's* thoughts in *Scientia Navalis*, also with respect to resistance, at least because of his systematic approach.

In *Scientia Navalis*, vol. I, Chapter V, *Euler* first considered the resistance of planar figures or planar cross sections of cylindrical bodies (Example: A rectangular rudder with constant profiled cross sections). He determined the resulting resistance in normal or oblique inflow in three different ways: By *Newton's* impact theory in an elastic or an inelastic medium or thirdly by the energy principle. All three results have the same structure (in modern notation):

$$R = C_D \rho V^2 S ,$$

as stated by *Newton* above. *Euler* did not commit himself to any of the C_D values, but argued that it was only the structure of the expression which mattered, while the coefficient had to be brought into agreement with experiments anyway. Besides the theoretical C_D value represented an upper bound because the theory neglected that the fluid particles were not strictly repelled, but could reach the rear side of the body. This was an elegant way out of the dilemma, but not a quantitative solution.

Euler then proceeded to determine the resistance coefficient for several simple shapes of cross sections (triangle, circular and elliptical segment etc.). It is of interest that he already searched for an optimal profile form with minimal resistance with certain profile dimensions (length, thickness) as given constraints. *Newton* had already posed the problem of an axisymmetric body shape of least resistance and had formulated a variational problem for this special case. *Euler* went one step further and specified here the general problem statement for a variational problem with equality constraints whose general theory he published later in 1744 [39], hence after completing the manuscript for the *Scientia Navalis*. He solved it here for special cases. Further examples by *Euler* were related to the effects of oblique inflow on foils (under an angle of incidence), hence on their resistance, lift and moment. His systematic approach is admirable, though unfortunately the quantitative results based on impact theory are useless.

In a corresponding way in *Scientia Navalis*, vol. I, Chapter VI, *Euler* approached the resistance problem for spatial solid shapes. He started out with the basic expression for the resistance and moment of a solid in parallel inflow according to impact theory, but then restricted himself to the special case of inflow parallel to the axis of symmetrical shapes. His procedure in generating families (or systematic series) of shapes is remarkably modern. He created mathematical hull form representations that can be systematically varied so that a whole range of typical shiplike form parameters is covered. The ship form or hull surface was represented in parameter form by $\boldsymbol{r}(u,v) = \{x(u,v),\ y(u,v),\ z(u,v)\}$ and was applied to the special case where separation of variables is feasible, e.g., $x(u,v) = f(u)\, g(v)$. Then it is easy to generate families or types of shapes in which all stations or all buttocks or all waterlines are affine curves. (A similar approach was frequently taken much later in ship theory for systematic investigations of hull form variation, e.g., by *Weinblum* [40] in wave resistance studies). *Euler* then examined the resistance of various types and treated some of them by optimization based on variational calculus. Further he discussed special shapes in which the forebody, the only part relevant to resistance in impact theory, was conical, pyramidal or cono-cuneiform (cono-cuneus by *John Wallis* [41]) or was an axisymmetric shape (with circular section shapes). It is clearly evident that by these variations in ship form he was aiming at a systematic overview, almost a compendium, on the dependence of resistance on shape design, a very practical purpose. His plan was again brilliantly logical, we may call it "systematic engineering". His results regrettably shared the fate of impact theory which soon became obsolete. *Euler* recognized the weak points early, but during his lifetime there did not yet exist a viable alternative for resistance prediction. A fundamental new start was required.

The first steps in this new direction were still taken by *Euler* himself, inspired by a few discoveries by his contemporaries. An important impetus came from the ballistic experiments performed by the Englishman *Benjamin Robins*, whose results were described in his book *"New Principles of Gunnery"* [42] in 1742. *Robins* had shot with spherical projectiles against a target disk suspended as a pendulum and had found out that *Newton's* impact theory was completely untenable for the resistance of his projectiles. *Euler* [43] had the opportunity in 1745 to translate *Robins'* book into German and provided its text with many elaborate comments and footnotes. He fully agreed with *Robins* in his criticisms, fundamentally rejected the idea of a "*thin medium*", because such a fluid was not found in nature, and he developed his own first ideas, inspired by the *Bernoullis*, on how the flow about a body could be described more realistically by "*stream tubes*" that enclose the body shape and remain attached to its afterbody. (On this episode, cf. also *Truesdell* [3], *Szabó* [11] and *Calero* [14]).

3.3 **Field Theory of Fluids**

In the following years *Euler* turned above all to general fundamentals of fluid mechanics, which were not yet applicable to ship theory, but were necessary to create a new basis on which later also the theory of ships could be safely founded. *Euler* was no doubt strongly influenced by the results obtained by *Daniel Bernoulli* (stream tube concept, energy conservation theorem, deflection of the flow before impact), by *Johann Bernoulli* (Second law of dynamics applied to a volume element, internal pressure, Bernoulli equation for a fluid filament or in a stream tube) and by *Robins* (refutation of impact theory). *D'Alembert* [44] somewhat earlier had already formulated a first "*field theory*" for the flow about *bodies* in a fluid. *Euler* likewise wanted to establish the laws of continuum mechanics in fluids, i.e., the laws for the state variables pressure and velocity in any desired point of the fluid flow, e.g. in the flow about a body. *D'Alembert's* field theory had consistently avoided the use of concepts like *pressure* as a state variable and *force* according to *Newton's* dynamics. *Euler* based his approach on *Newton's* laws and chose *pressure* and *velocity* as state variables. He thus created the foundation of the modern field theory of fluids, as we know it and use it today

Euler pursued the goal of replacing *Newton's* impact theory by a new theory which dealt with the physical state variables in the whole domain of the fluid treated as a deformable medium. His axiomatic premises were:

- *Newton's* laws as principles of dynamics.
- Constitutive equations, i.e., information about the material properties of the fluid, initially regarded as an ideal fluid, and on the boundaries of the fluid domain..
- Conservation laws, especially on the conservation of mass (hence the continuity equation).

From this information alone pressure and velocity can be determined. These variables of state are multivariate functions of the location in the field.. *Euler's* approach, the application of *Newton's Lex Secunda* to a fluid volume element, yielded the equations of motion of the element, the famous *Euler equations* in the form of a set of partial differential equations. The *continuity equation* is of a comparable form.

The *Euler equations* and the *continuity equation* constitute the foundation of the field theory of fluid dynamics. *Euler* developed this theory in general form in a few classical papers [45], [46], [47], [48], [49] between 1752 and 1756. His theory holds for planar and spatial flows in incompressible and compressible fluids.

Thereby the equations of state of the flow problem are established. Therefrom a form of the *Bernoulli* equation can also be derived that holds for any arbitrary streamline in the entire fluid domain. If the boundary conditions are added, a complete problem formulation results in a form that is called *boundary value problem* today. In *Euler's* time the mathematical theory for solving partial differential equations and boundary value problems was developed concurrently with the treatment of this flow problem of field theory. *Euler* himself created important foundations although initially solution methods for arbitrary given body shapes were still missing. *Euler* showed that the field under certain conditions may possess a velocity potential for which solutions can more readily be built up. This was the starting point for the later development of singularity methods to solve this type of boundary value problem of potential theory.

If in some specific case the solution for the state variables is known, a streamline field map can be developed and therefrom, as *Euler* suggested, especially for the streamlines on the body surface velocity and pressure distributions can be derived. Pressure integration then yields the resulting force on the body. In an ideal fluid, as *D'Alembert's* paradox anticipated and *Euler* was able to confirm by pressure integration, the result which is obvious to us today is achieved, viz., that the resistance of a deeply submerged body in a steady flow vanishes. Thus field theory proved the following facts to start with:

- The assumptions of *Newtons* impact theory for the resistance are not tenable.

- A flow that remains attached to the entire body surface and whose afterbody therefore contributes to the resistance is feasible.

- In an ideal, lossfree fluid the resistance vanishes.

By these results fluid mechanics had overcome a difficult stalemate and was able to develop further without impairing contradictions. For application in ship theory the new insights gained were apt to produce a rich harvest later in the 19^{th} and 20^{th} c. Today the practically successful analytical and numerical methods for calculating the flow about an arbitrary ship form in an ideal fluid all are based on *Euler's equations* of fluid mechanics on the one hand, and on *Euler's field theory* on the other hand.

The other major obstacle, which prevented the development of a realistic theory of resistance and which unfortunately could not be removed in the 18th c. any more, was the lack of understanding the causes of wavemaking and viscous resistance. For the resistance cannot be realistically estimated without taking into account the energy losses caused by the effects of gravity and hence wavemaking on the one hand, and those caused by the viscosity of the fluid on the other hand. In the case of a real fluid the resistance law must account for several parameters of similitude (later called Froude number, Reynolds number etc.) and therefore must provide more than one freely allocable resistance coefficient. In the 18^{th} c. the influence of both resistance components was underestimated. The effects of wavemaking were first more clearly recognized in model experiments (*Juan* [50], *Chapman* [51]) and there followed first, immature theoretical hypotheses [50]. The friction on the body surface was judged as low [cf. also *Euler* [52]) and was regarded as neglible in the interior of the fluid. It was only by the thorough experiments performed by *Beaufoy* [53] beginning in 1793 on the frictional drag on plates that the relatively considerable significance of frictional resistance, also on ships, was realized. It still took a long time before *William Froude* [54] presented a method that took into account several parameters of similitude in ship resistance and thereby permitted faithful prediction of full scale ship resistance from model tests. The theories of wave resistance and viscous resistance could not reach maturity before that insight.

3.4 <u>Ship propulsion</u>

In the 18th c. ship propulsion by wind energy in sailing and by human energy in rowing were the predominant methods and energy sources. *Euler* investigated the mechanical principles for both methods of propulsion. Besides he analyzed certain innovative, not yet practically applicable propulsion systems like the paddle wheel, the screw propeller and jet propulsion.

Already in his prize contest treatise [18] of 1727 *Euler* –not unlike *Bouguer*- devoted his attention to sail propulsion. In the spirit of *Newton* and other precursors he analyzed the forces acting on a sail and the resistance of the hull by means of impact theory. Later in *Scientia Navalis* and even still in his *Théorie Complète* –like many of his contemporaries- he remained committed to this approach. This unfortunately led to rather misleading results regarding the forces acting on the hull and the sails. Fig. 6 shows the component decomposition in the forebody for resistance and thrust. According to impact theory the hull resistance acts on the forebody normal to its surface, and hence obliquely upward. In the example of the figure *Euler* assumed a spherical segment bow shape so that all impact force contributions acted through the center of the spherical shell W. The resultant WR acted in a direction normal to the spherical segment in its area centroid. Now it was the dominant opinion in the 18th c. that masts and their sails in design should be arranged in such a way that the resultant sail force would have a horizontal forward thrust component, intended to act also through the point W, called "*point vélique*", and then in steady motion should be in equilibrium with the horizontal component of the resistance. Thereby it was intended to prevent a trimming moment formed by the couple of resistance and thrust. However these considerations on the *point vélique* were grossly misleading and also superfluous. For firstly the resistance resultant in actual fact by no means acts obliquely upward, secondly it is possible to compensate the inevitable trimming moment by the nose of the sailing vessel by means of ballast or other design measures.

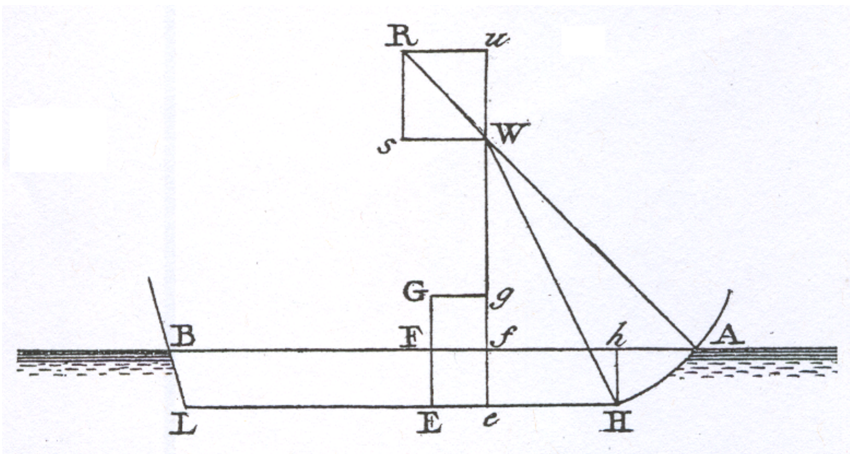

Fig. 6: Hull resistance and sail forces with point of intersection in the *point vélique* W (*Euler* [1])

In addition it caused difficulties that the sail forces could not be accurately estimated according to impact theory. It was assumed that the sail area of several masts could be lumped in the centroid of all sail areas and that the resultant sail force would act through this point, as impact theory without accounting for any sail interactions would suggest. Aerodynamic effects as the cause of lift and drag of the sails were still unknown. Therefore the predictions of sail forces were unrealistic by magnitude and direction. The further assumption of impact theory that the sail force resultant in oblique inflow would vary with the square of the angle of incidence ($\sin^2 \alpha$ law) was false, which was not recognized before some experiments in the second half of the 18th c. In conclusion in *Euler's* time the propulsive forces acting on hull and sail and hence the resulting ship speed could not be realistically predicted.

It must be acknowledged as a positive element of understanding that the force components acting on ship and sail and their interactions were correctly identified, and the equilibrium position of the sailing vessel by drift angle, trim and heel angle were qualitatively properly understood. Regarding trim and heel the progress made in ship hydrostatics prior to 1750 had furnished important prerequisites (cf Section 3.1). On further details in the development of sailing theory during this period, see also *Ludwig Rank* [55].

Although *Euler's* results on rowing, paddle wheel and screw propulsion still suffered from their reliance on impact theory for force predictions and hence failed to be of quantitative value, they still made a lasting contribution to propulsion theory which lies primarily in the analysis of the acting physical principles and mechanisms, which are fundamentally based on the momentum balance. *Euler's* scientific courage must be admired to study the mechanical principles of propulsion systems which in his day were not yet ready to be technically realized.

In 1753 on the occasion of another prize contest by the Parisian Académie Royale des Sciences *Euler* had submitted a treatise [22] that dealt with the propulsion of ships without windpower, i.e., without sails („De promotione navium sine vi venti", E.413). This award winning treatise written at a time when sail propulsion was still by far the dominant propulsion method for all major ships gave a basic overview of alternative propulsion methods, whose power was to be provided essentially by the humans on board, be it by known means like rowing, be it by mechanisms to be newly developed similar to the paddle wheel or the screw propeller, yes, even by jet propulsion as propagated earlier by *Daniel Bernoulli* . Though *Euler* did not develop such new propulsion systems to technical maturity as patents, he still qualitatively described correctly their physical principles of operation, also for propulsion systems which could only be realized in the 19^{th} c. by means of steam power. Thus he cannot be regarded as the inventor of such later solutions, but he anticipated by more than half a century before their realization the physical explanation of the performance of paddle wheel, screw propeller and jet propulsion.

Human Performance Limits

Euler began his considerations with the question of how much mechanical power a human according to his physiological capacity can provide continuously. He estimated that a man under favorable load conditions may be able to move a load of about 15 kp at a velocity of about 0.65 m/sec for an extended period of time. This in today's units corresponds to providing a power of about 100 Watts continuously. This power capacity agrees surprisingly well with current data on the continuous power performance of humans, if we disregard sportive peak performances in shorter time intervals. *Euler* adapted the power absorption of his ship propulsion systems to this human performance potential.

Oar Propulsion

The principle of ship propulsion by rowing was schematically regarded by *Euler* as if a submerged planar plate (FF, Fig. 7) was attached to a linkage system gliding on a roller C and was arranged forward of the bow (or abaft the stern) so that it could be horizontally shifted in a direction opposite to the motion of the ship. Thereby water mass is accelerated backward and a reaction force is generated on the plate which drives the ship forward as a thrust. After this work cycle the plate must be raised from the water, transferred back through the air and lowered into the water again (as the oar in rowing).

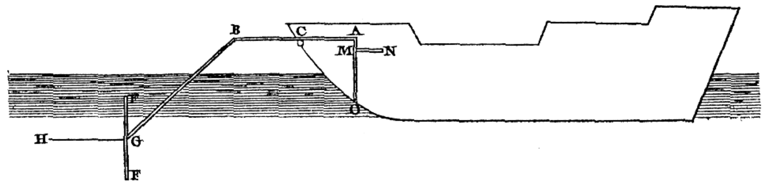

Fig. 7: Ship propulsion by backward shifting of a plate in water (*Euler* [22])

Let the ship velocity be C, the horizontal velocity of the plate relative to the ship be V, hence the horizontal velocity of the plate relative to water V – C. Then if V is greater than C, a positive thrust is acting on the plate which drives the ship and in steady motion exactly overcomes the resistance. *Euler* thus equated thrust and resistance and determined the achievable velocity C of the ship from the power input by the crew at an assumed velocity V of the plate or "oar blade". Unfortunately *Euler*, as was still common practice at this time, used *Newton's* impact theory for estimating the influence of hull form and velocity upon the resistance coefficient, which was unrealistic and resulted in quantitatively misleading conclusions, which were later also rejected by *Euler* himself. Qualitatively it was correct that an increase in the area of the plate and a reduction of the hull resistance would improve the propulsion of the ship and increase the achievable ship speed.

Euler expected an improvement in the efficiency of this propulsion method if the blade was fitted with rotatable laminae like Venetian blinds (Fig. 8) which during the backward stroke of the blade would be turned into a horizontal position and hence would have a low resistance. Thus this kind of unidirectionally permeable blade could be arranged on both sides of the hull and connected to a system of levers OA-NN so that the blades could be continuously moved back and forth.

<u>Propulsion System with Cranked Shaft</u>

The previous idea was further simplified if the blades on both board sides were driven by a horizontal shaft DD (Fig. 9) whose center part CC is cranked so that the propelling forces can act on the crank. The blades by their laminae are again unidirectionally permeable and are intended to move continuously back and forth in their submerged condition.

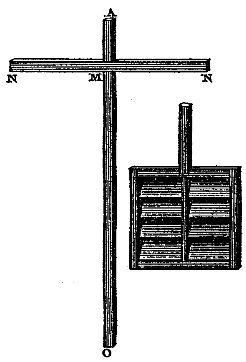

Fig. 8: Venetian blind blade with rotatable laminae and lever system for attachment and position control of the blades [22].

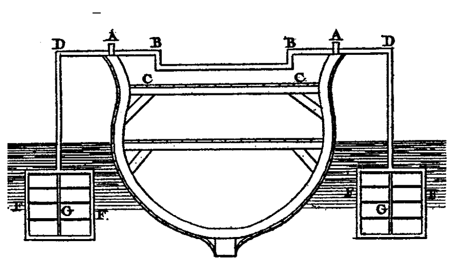

Fig. 9: Drive of Venetian blind blades by means of rotatable, cranked shaft [22].

The paddle wheel principle

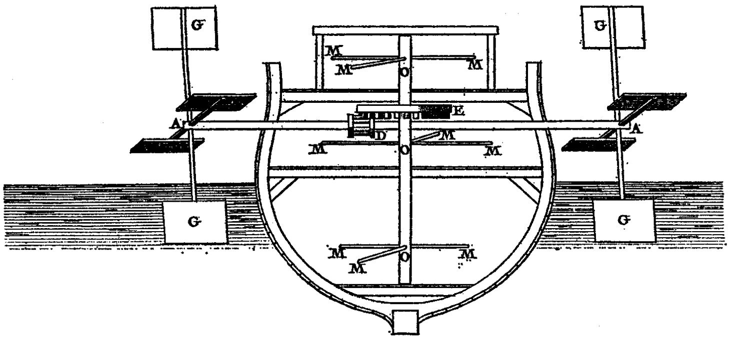

Fig. 10: Ship propulsion by two „paddle wheels“ with plates as blades [22]

In order avoid to this pendulum like motion, which was not very practical as *Euler* probably realized, too, to make permeable blades dispensable and to operate in a continuous motion, it was almost cogently necessary to arrive at the principle of the paddle wheel (Fig. 10). Here several plates are attached to the spokes of the driving shaft on both sides of the hull which are driven by the shaft AA, which in turn is kept in steady rotation via a whim gear (E, D) by the crew rotating the arms M about the vertical axis OO. The blades are not profiled like paddles, but they do provide a steady thrust while they are immersed in the water. The "*paddle wheel*" as a steadily rotating engine according to *Euler* is a logical further advance of the idea of the "*oar blade*", which works only intermittently in a horizontal translation. Thus *Euler* succeeded immediately to generalize the balance of input power vs. usefully delivered thrust power from the oar blade to the paddle wheel. "*Paddle wheel propulsion*" is thus regarded as a generalized, continuously operating form of "*oar blade propulsion*" with improved efficiency.

The "screw propeller principle"

Encouraged by the proven idea of the windmill *Euler* in his next step arrived at a propulsion system whose configuration resembled a modern screw propeller (Fig. 11). A system rotatable about the longitudinal axis AB is arranged in front of the bow or abaft the stern of a ship to whose spokes planar blades FF are attached with some angle of inclination relative to the longitudinal direction. Thereby in their rotation they experience a longitudianl force (thrust) and a circumferential force, similar to a modern screw propeller, though without the helical curvature of the blade surface and without modern profiled blade sections.

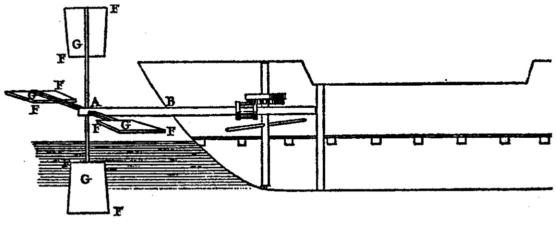

Fig. 11: Propulsion by propeller with planar blades [22]

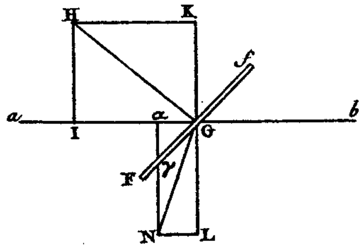

Fig. 12: Velocities and forces acting on the blade [22]

Euler in his analysis of propeller operation took into consideration the mean effects acting on the blade, lumped into its area centroid G. He combined the components of inflow in the direction of advance (αG) and in the circumferential direction (GL) into the resultant GN (Fig. 12) acting with an angle of incidence γ to the blade. In this context, too, *Euler* remained prepossessed by *Newton's* impact theory of resistance and therefore in his analysis of the force acting on the blade section (through G) accounted only for the normal force GH, perpendicular to the blade, which has components in the circumferential and advance directions. Thus he neglected the tangential forces of the blade section and all effects of foil theory acting on the blade, which are known today. Thereby his analysis remained crudely approximative. Qualitatively his theory did correctly explain the chain of phenomena by which a planar blade propeller or later a screw propeller in its rotation in order to overcome blade resistance in the circumferential direction absorbs propulsive power and at the time generates a thrust in the direction of advance.

<u>Jet propulsion</u>

The efflux from a containment vessel or the flux through a vessel or pipe causes a reaction force acting on the boundaries of the vessel as is known from the garden hose. The flow vessel or pipe can be arranged in a ship in such a way that the resulting reaction force provides a thrust for ship propulsion.

Euler picked up this idea following suggestions by *Daniel Bernoulli* („Hydrodynamica", 1738), analyzed the acting forces and conceived a jet propulsion system for a ship (Fig. 13). The flow through the system of pipes is induced either by the pressure head of a tank arranged in a high position or by a reciprocating piston pump (EE) in the pipe. This pump in principle can be driven by human operator power.

Euler first calculated the resulting reaction force on the boundaries of the vessel for a system with arbitrary cross section distribution and in some arbitrary spatial position. He demonstrated that this force depended only on the flow rate and on the area and orientation of the inlet and outlet cross sections, but not on the cross section variation of the vessel between the end sections.

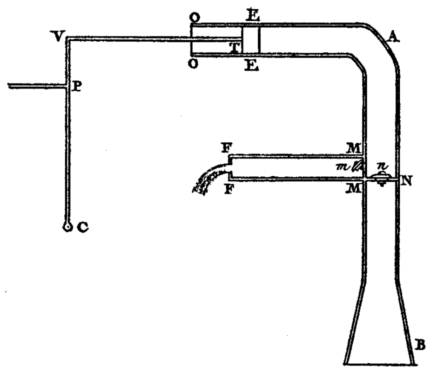

Fig. 13: Jet propulsion system according to *Euler* [22] with piston pump (EE)

He then addressed the most favorable case for ship propulsion (Fig. 13) where the fluid enters the system through a horizontal pipe of cross section EE from abaft and after a deflection of 180 degrees leaves the system rearward via a horizontal pipe nozzle with the orifice FF. The cross section of the orifice FF can be made very small relative to the intake cross section at EE, thus very high jet velocities can be achieved.

Mechanically the process was subdivided by *Euler* into two cycles (Fig. 13), the intake cycle and the ejection cycle: During the *intake cycle* fluid is sucked into the pipe system from below (at the suction funnel at B), the valve at n is open, the valve at m closed. Because of the deflection of the fluid by 90 degrees at A a certain horizontal force already arises which acts as thrust. During the *ejection cycle* the piston is advanced forward, the valve at n closes, the one at m opens, the jet can now exit at FF. This cycle generates a very high thrust due to the great jet velocity and the complete deflection by 180 degrees. In order to alleviate the thrust fluctuations between cycles, *Euler* advocated two parallel jet systems in counter rhythm.

In his example powering calculations *Euler* then recognized the limitation that jet propulsion can profitably operate probably only at great propulsive power which exceeded the powering potential from human energy, i.e., from the energy sources available in his day.

Aftereffects

Although first experiments with and patents for paddle wheels, screw propellers and jet propulsion had existed for some time before *Euler's* publications, the realization of those ideas still failed in *Euler's* time, essentially due to the lack of a power source of sufficient capacity aboard ships. It was only after the introduction of steam propulsion that more advanced inventions were able to achieve technical and commercial success. *Euler's* physical explanations and calculations for such propulsion systems most likely were not known to those inventors. It was only much later that his thoughts were reassessed and were apt to be taken into consideration in modern theories of ship propulsion. As successful inventors of some of the earliest, patented and more mature solutions deserve to be named:

- Steam ship with paddle wheel: Symington (1802), Robert Fulton (1807)
- Screw propeller: Ressel (1826), Ericsson (1834-36), Smith (1838)
- Jet propulsion: Ruthven (1851), Seydell (1856) and the ship.*WATERWITCH* (1867)

3.5 **Maneuvering**

In ship theory the term „maneuvering“ generally comprises all motions of the ship taking place in the horizontal plane as translations (parallel and normal to the ship's longitudinal axis) and as rotation about the vertical axis (yawing). Let the coupling with other degrees of freedom of the ship be neglected here as a first approximation. Thus maneuvering does include the dynamic behavior of the ship on a straight course, also in oblique inflow, and in turning maneuvers. Great technical and practical significance was attributed above all to the optimization of sailing performance of the big sailing vessels of the 18th c. in different wind conditions and directions. This is why studies of ship theory already in the early 18th c. had been concerned with maneuvering theories (*Renau* [19], *Huygens* [20], *Johann Bernoulli* [17]). *Euler* continued the work of these precursors by his own contributions, above all on the dynamics and aero- and hydrodynamics of sailing ships, and gave essential new momentum to this topic area. The most essential results can again be found in the *Scientia Navalis* [1] and in the *Théorie Complète* [24]. The synopses by *Habicht* in [7], [8], [9] are very helpful for the understanding, too.

The analysis of ship maneuvers requires physical and analytical knowledge in the following areas:

- The magnitude and direction of forces and moments acting on hull, sail and rudder, also for wind directions obliquely to the course.
- The dynamics of the system, especially under the influence of inertia, sailing rig and resistance forces.
- The solution of the equations of motion.

Euler concerned himself thoroughly with all these aspects. His determination of forces and moments here again suffered from the weaknesses of *Newton's* impact theory, but his contributions to the system dynamics and the integration of the equations of motion remained unaffected by this. They were in part breakthroughs and are valid until today. These applications of mechanics and infinitesimal calculus to ship motions in maneuvers belong to the first practically and technically successful contributions by modern dynamics and fluid mechanics.

In *Scientia Navalis* [1], vol. I, Chapter II *Euler* developed an approach for the rotational motion of an extended body system of arbitrary mass distribution and about a given, fixed axis of rotation through the body center of gravity. Here he still presumed "*free rotation*" about the axis through the CG, i.e., he disregarded any "bearing reactions" that may result from the coupling with other inertia effects stemming from simultaneous rotation about other axes (by deviational moments). These assumptions would hold strictly only if the principal axes of inertia were chosen as coordinate system. For ships he chose an orthogonal coordinate system through the center of gravity with the longitudinal axis being horizontal in the ship center plane. Thereby the simplification in neglecting the couplings holds in good approximation. Then by integration over all mass elements of the ship he obtained the rotational momentum M in a turning motion (where $\mathbf{r}$ = distance of the mass element dm from the center of rotation):

$$M = \Theta\,(d\omega/dt), \qquad \text{where } \Theta = \int \mathbf{r}^2\,dm = \text{the axial mass moment of inertia, and } \omega = \text{angular velocity.}$$

The mass moment of inertia was first introduced by *Euler* in this place and for the present purpose (he named it „*momentum inertiae*"). With these preparations the motion of the maneuvering ship could now be derived by integration of the translational and rotational laws of dynamics, if the external forces, i.e., thrust and resistance, and their moments, were known. However *Euler* still neglected the influence of the hydrodynamic mass moment of inertia which in a turning maneuver may be of comparable magnitude as the mass moment of inertia of the body.

Euler now first calculated a few examples for the axial mass moments of inertia of simple, homogeneous bodies. Then he investigated the motions of the ship on a straight course before the wind, e.g., in a stopping maneuver with the ship slowed down by its resistance with all sails reefed, or in accelerating the ship from rest by a given sail force. Finally he considered also the case of oblique wind, i.e., with the wind acting obliquely to the course and the rudder laid for coursekeeping at steady speed. In this application the drift angle was estimated empirically from plausible assumptions and was assumed to be speed independent. The heel angle under wind load was estimated hydrostatically and the sail force was adjusted thereto. In systematic series investigations the sail forces were then investigated for wind directions before the wind, with quartering winds and pointing high. Even the deflection of the sail cloth was taken into account, though everything only according to impact theory. With these "*polar curves*" of the rig (in today's terminology) *Euler* was also able to deal with nautical problems, e.g., finding favorable strategies for cruising against the wind.

Regarding the placement of masts and the arrangement of sail area, a subject to which *Euler* returned several times, he applied the dynamics of maneuvering to arrive at very practical recommendations. The masts and their sails should be placed in such a way that the sail force resultant through the sail area centroid would act slightly abaft the center of action of the transverse hull resistance ("lateral plan centroid") so that the couple of these two forces would turn the forebody into the wind. This tendency could be compensated by minor rudder action to keep the ship on course. By contrast ships tendings to drift leeward suffered from increasing drift angles, resistance increases and difficulties in coursekeeping. *Euler* was able to explain such observed phenomena plausibly by his mechanics of maneuvering. The prediction of forces by magnitude and direction was more difficult. The centers of action of the resulting forces on the sails in air and on the hull under water were estimated in accordance with impact theory to lie in the centroids of the respective areas, thus in the sail area centroid and the lateral plan area centroid, respectively.

The simplifications made in the choice of axes of inertia continued to concern *Euler* for some more time. It was only later in the context of his analysis of the arbitrary rotational motion of a body and in connection with the *equations motion of the gyroscope* [56], [57] that he arrived at a general solution for the arbitrary rotation of a body about its centroid, formulated in terms of the "*principal axes of inertia*" through the center of gravity. If the rotational motion of the body was represented with reference to these three orthogonal axes, then the deviational moments of inertia would vanish and the equations of motion with uncoupled inertia terms would hold exactly in this reference frame. The principal axes of inertia were defined and determined by *Euler* according to an idea by *Segner* (1707-1777), who had published this in 1758.

The theory of ships in this field owes much gratitude to *Euler* efforts and insights that have remained of classical, lasting value throughout the field of mechanics, i.e., well beyond the initially motivating field of ship motions.

3.6 Ship motions

The ship, considered as a rigid body, has three oscillatory degrees of freedom in which inertial and restoring forces or moments exist so that a periodic oscillation may arise: *Heaving* (translation parallel to the vertical axis), *rolling* (rotation about the longitudinal axis) and *pitching* (rotation about the transverse axis of the ship). These motions are designated in this subsection as "*oscillatory ship motions*", or briefly as "*ship motions*". Since in all of these degrees of freedom hydrostatic forces or moments are involved, it is rather easy to date the time since when the treatment of these oscillations became feasible, viz., only after *Bouguer* and *Euler* had created a foundation for the calculation of such restoring forces or moments acting on the ship in its position displaced from equilibrium. Thus it is no coincidence that both, again independently and almost simultaneously, published first theories on oscillatory ship motions, viz., in their monumental principal works *Théorie du Navire* [2] und *Scientia Navalis* [1] (appeared in 1746 and 1749). The solutions and even calculation methods proposed by both are not equal, but in practice equivalent. Thus we may limit ourselves here to the narration of *Euler's* contributions.

Fortunately in addition a very substantial part of the correspondence between *Euler* and *Johann* as well as *Daniel Bernoulli* has been conserved ("*Commercium Epistolicum*" [58], contained in Series IVa of [6]), which contains several letters dating from 1738 to 1740 on the subject of ship oscillations. *Euler*, who at this time was writing a chapter on ship motions in *Scientia Navalis*, had succeeded in convincing his teacher *Johann* and his friend *Daniel Bernoulli* to work on similar tasks. Therefrom resulted, in addition to *Euler's* own results, also some publications by the *Bernoullis* during the same period, e.g., by *Daniel Bernoulli* [59] in the years 1738/39.

The studies initially concentrated on the determination of *natural frequencies* and *periods* of ship oscillations in order to predict or avoid resonances. For this purpose it was required to know inertia and restoring forces or moments, which could now be predicted quite realistically by available methods. (However the influence of hydrodynamic masses, which may be of considerable magnitude in many degrees of freedom, was not yet taken into account).

All three scientist – and *Bouguer* likewise- had noticed the analogy between a physical pendulum, which had been investigated earlier by *Galilei* and *Huygens,* as it occurs in a pendulum clock, and the ship moving in an oscillatory degree of freedom. This analogy is founded on the fact that both system types to the first approximation (small amplitudes) constitute linear oscillators with isochronous periodic oscillations, as the equations of motion will already demonstrate. If we follow *Euler's* derivation for the pendulum and for the rolling ship (small roll angle φ), the following comparison can be drawn:

Pendulum (Mass m, center of gravity distance from center of rotation s):
Equation of motion: $\Theta\,(d^2\varphi/dt^2) + m\,g\,s\,\varphi = 0$
Equivalent pendulum length: $l_{EQU} = \Theta/(m\,s)$

Natural period: $T = 2\pi\sqrt{l_{EQU}/g}$

Rolling Ship (Mass m, metacentric radius GM):
Equation of motion: $\Theta\,(d^2\varphi/dt^2) + m\,g\,GM\,\varphi = 0$
Equivalent pendulum length: $l_{EQU} = \Theta/(m\,GM)$

Natural period: $T = 2\pi\sqrt{l_{EQU}/g} = 2\pi\sqrt{\Theta/(\Delta\,GM)}$
where Θ = mass moment of inertia in rolling (φ)

The analogies are clearly visible, especially between s and GM.

Corresponding results were obtained by *Johann Bernoulli* for *heaving* and by *Euler* and *Daniel Bernoulli* for *pitching*, by *DanielBernoulli* also already for *coupled rolling and heaving* [59].

In design it was the purpose to avoid great accelerations at resonance, thus to reduce the natural frequencies and to increase the natural periods. In rolling, e.g., where GM must not be chosen too small, *Euler* (like others) recommended to increase the mass moment of inertia Θ by shifting any movable masses inside the ship as far away from the center of gravity as possible. A plausible idea, but only practicable within narrow limits.

It should be noted with interest, too, that *Euler* in a later treatise [23] on rolling and pitching almost in passing ((§16-§19) also mentions how to determine the interior loads of the ship, e.g., at midship section. In his ingenious way *Euler* takes a planar vertical section through the hull girder and determines the *longitudinal bending moment* as an internal load in this cross section. By this influence he also explains the deflection of the hull in longitudinal bending, the much feared "*hogging*". The article further elaborates on how dynamic loads must be added to to the static loading. This to my knowledge is the first historical entry point into those methods which later developed into longitudinal strength calculations for ships and became indispensable in dimensioning the structure of the hull.

The treatment of ship oscillations by *Euler* and his contemporaries however still had significant gaps and limitations:

- Neglect of hydrodynamic mass and damping forces,
- Lack of load assumptions for excitation forces and moments, especially by the seaway, hence lack of data for forced oscillations,
- Simplification in the choice of oscillation axes through the center of gravity as being parallel and orthogonal to the waterplane in place of the principal axes of inertia,
- Limitation to small amplitudes, linearization,
- Absence of statistical methods for frequency and extreme value analysis.

Euler was aware of the majority of these limitations, as his cautious premises and disclaimers usually indicate. Nevertheless we must recognize and pay our tribute to the useful knowledge already achieved in *Eulers* era, both in scientific substance and in qualitative practical insights.

4. <u>Conclusions</u>

By consistent application of the first principles of mechanics and fluid mechanics, which *Euler* in part had created or extended himself, he was able to base the new, application oriented scientific discipline of ship theory on a firm foundation and thereby to help lay the ground for modern ship theory. He left his mark on the structure of this field. Many of his results are still valid and of lasting value. The following achievements deserve to be singled out as especially noteworthy:

o The foundation of criteria and calculation methods for the hydrostatic stability of ships, derived by integration of the pressure distribution in the fluid at rest, acting on the ship slightly displaced from equlibrium. The application of stability criteria already at the design stage as a starting point for more systematic analysis of the safety of ships.

o Initially application of *Newton's* impact theory of resistance, which yielded false predictions, yet combining correct system dynamics with false force coefficients. Later after intensive efforts discarding of the misleading impact theory and creation of the promising field theory.

o Contributions to the laws of ship propulsion by sailing, rowing, paddle wheel, screw propeller and jet propulsion. Correct application of the momentum theorem to explaining the principles of propulsion, unfortunately with wrong force coefficients.

o Fundamental studies on system dynamics of the maneuvering vessel with flow effects on hull, sail and rudder, also solution of the equations of motion.

o Contributions to ship oscillations by deriving the natural periods for rolling, heaving and pitching based on inertia and hydrostatic restoring forces.

In all these activities we owe *Euler* our gratitude not only for his important insights in the theory of ships, but also for what *Truesdell* [3] already praised as the special characteristic of style in all of *Euler's* work:

„First principles, generality, order and above all clarity“.

It took a major span of time before *Eulers* insights were understood by the engineers and found acceptance in practice. But even in his lifetime there were a few enlightened contemporaries and practitioners who recognized and appreciated the value of theoretically well founded knowledge. The eminent Swedish naval constructor *F.H. af Chapman* [56] be quoted here as a key witness who had comprehensive practical experience in shipbuilding, but was also familiar with the literature of his era, hence also with *Bouguer's* and *Euler's* treatises, which he held in high esteem. He is quoted with the sentence:

„Without a good theory design is only a game of hazard!“

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