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Zhang Baichun and Jürgen Renn (eds.)

**Transformation and Transmission:
Chinese Mechanical Knowledge
and the Jesuit Intervention**

Joint Research Group of the MPIWG
and its Partner Group at the IHNS of the CAS

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The work related to this publication was undertaken by a research group dedicated to the *Development of Mechanical Knowledge in China* and coordinated by Matthias Schemmel. The group integrates research of the project on *Mental Models in the History of Mechanics*, headed by Jürgen Renn, at the *Max Planck Institute for the History of Science* in Berlin, of its *Partner Group*, headed by Zhang Baichun, at the *Institute for the History of Natural Sciences* in Beijing (IHNS) of the *Chinese Academy of Sciences* (CAS), as well as work of an international group of experts on sinology and Jesuit science.

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Preface

ZHANG Baichun and Jürgen RENN

The present volume explores the development and interaction of two cultures of science, Chinese and Western, focusing on mechanical knowledge with its long-standing tradition going back to the very origins of science. The parallelisms between both traditions are sometimes striking, their differences offer deep insights into the role of cultural contexts for science, and their encounter in the course of the Jesuit mission to China opens up a unique occasion for understanding transformation and transmission processes of scientific knowledge.

The work related to this publication was undertaken by a research group dedicated to the development of mechanical knowledge in China and coordinated by Matthias Schemmel. The group integrates research of the project on *Mental Models in the History of Mechanics*, headed by Jürgen Renn at the *Max Planck Institute for the History of Science* (MPIWG) in Berlin, of its *Partner Group*, headed by Zhang Baichun, at the *Institute for the History of Natural Sciences* in Beijing (IHNS) of the *Chinese Academy of Sciences* (CAS), as well as work of an international group of experts on sinology and Jesuit science.

The Partner Group of the MPIWG at the CAS Institute for the History of Natural Sciences was established in July 2001 on the basis of a general agreement that was signed by the President of the *Chinese Academy of Sciences* and by the President of the *Max Planck Society* (MPG) on 23 February, 1999. The group consists of the leader, Prof. Zhang Baichun, and two senior scholars, Associate Prof. Tian Miao and Associate Prof. Zou Dahai, as well as the Ph.D. candidates Yin Xiaodong, Chen Yue, and Xiao Yunhong (since 2004 Associate Professor at *Ganna Normal University*). Work on the development of mechanical knowledge in China at the Max Planck Institute for the History of Science has been pursued by Prof. Peter Damerow, Prof. Jürgen Renn (director), Simone Rieger (research coordinator), Dr. Matthias Schemmel (project coordinator), and Urs Schoepflin (head of MPIWG library). It is now being complemented by the work of a newly founded Junior Research Group *From Invention to Innovation: Cultural Traditions of Technical Development in China*, headed by Prof. Dagmar Schäfer.

The work of the Partner Group has been supervised and accompanied by an international scientific advisory board. Its members are: Prof. William G. Boltz, chairman of the advisory board (*University of Washington*, U.S.A.); Prof. Hans Ulrich Vogel, vice-chairman of the advisory board (*University of Tübingen*, Germany); Prof. Fung Kam-Wing (*University of Hong Kong*, China); Prof. Liu Dun (*Institute for the History of Natural Sciences*, China); and Prof. Jürgen Renn (*Max Planck Institute for the History of Science*, Germany). Members of the advisory board have also directly participated in the work of the research group. The work has furthermore benefited from the collaboration with other scholars, in particular with Prof. Rivka Feldhay (*Tel-Aviv University*, Israel), a leading expert on Jesuit science. We would like to take this occasion to thank all of them for their engagement, their encouragement and critical support.

The success of the *Partner Group* was by no means obvious from the outset. The concept of a close interaction of junior scholars with Max-Planck scientists on a specific research theme originated in the natural sciences, and the Partner Group at the *Institute for the History of Natural Sciences* was the first such endeavor in the humanities. In contrast with the natural sciences, research themes in the humanities are still often shaped by national intellectual traditions, methods, and standards. The scope of research that the Partner Group intended to pursue in collaboration with scholars at the *Max Planck Institute for the History of Science*, as well as with a wider network of scholars, was ambitious. It went beyond a traditional history of ideas and rather focused on the long-term development of science in its contexts, comprising different historical periods and different forms of knowledge that are often neglected in the history of science or studied in isolation from each other. The joint research team thus investigated the question of the interaction between practical and theoretical knowledge, and that of the interaction between Chinese and Western knowledge traditions over a period of more than a millennium. In the course of our work, the interaction of Western mechanical knowledge with Chinese cultural and scientific traditions at the time when Jesuits introduced Western knowledge into China in

the 17th and the 18th centuries became a central focus of our research and also of this presentation of our results.

Research referring to different historical periods, cultures, and forms of knowledge has to cover a broader basis of sources than traditional studies in the history of science. All papers presented here were based on a close analysis of the original sources, often presenting considerable challenges in view of the difficulty of expressing scientific concepts from one culture in the language of another. Work on the sources was accompanied by transcriptions, translations, and the use of new media for a digital presentation of working materials on the Internet as part of the ECHO framework and in accordance with the open-access policy adopted by both the Chinese Academy of Sciences and the Max Planck Society (<http://www.mpiwg-berlin.mpg.de/CHINA/> and <http://mech-history.ihns.ac.cn>). A key source for our work is the *Yuanxi qiqi tushuo luzui* (*The Record of the Best Illustrations and Descriptions of the Extraordinary Devices of the Far West*, for short *Qiqi tushuo*), composed by the Jesuit Johannes Schreck and the Chinese scholar Wang Zhang in 1627. A copy of this book is held by the library of Institute for the History of Natural Science; its digital facsimile and a transcription have been made freely available on the Internet. In addition to the studies of this work presented in this volume, an English translation produced with computer assistance for ensuring consistency and for extracting technical terminology is in preparation. Also the work on other sources relevant to the research of the group is being continued and will complement the existing digital archive of the project.

More generally, it is planned to continue the cooperation between the CAS Institute for the History of Natural Sciences and the Max-Planck-Institute for the History of Science, also beyond the present period of funding. On the basis of the work so far accomplished, it makes sense to reexamine the history of later contacts between Chinese and Western science, in particular in the 19th and 20th centuries, but also to extend the perspective from mechanics to an equally ancient field of both practical and theoretical knowledge, optics. Apart from these more or less direct lines of continuing the present work, the research of the group has also opened up new vistas towards a history of global processes of the transformation and transfer of knowledge, including also other scientific cultures as well as the present challenges facing a world in which the encounter between local and globalized forms of knowledge has become a potential both for conflicts and innovation.

The idea to make this collection of essays available goes back to the 22nd Congress of History of Science, held in 2005 in Beijing. On behalf of the Partner Group, Zhang Baichun and Hans Ulrich Vogel organized a special panel entitled “Mechanics in Chinese and Western Traditions,” in which preliminary results of the international research team were presented. Meanwhile most of the contributions could be completed, while others are still work in progress, in particular with regard to overcoming difficulties connected to the language barrier, resulting in some passages that are still merely raw versions, for instance in the paper by Xiao Yunhong. Due to the considerable interest with which our work has met among colleagues interested in comparative perspectives in the history of science, we have nevertheless decided to publish all works in their present form.

Most papers underwent thorough language revisions, editing, and “homogenization” in view of a joint publication. The lion’s share of the considerable effort involved in this reworking was accomplished by Peter Damerow and Matthias Schemmel, who both spent many weeks carefully reading all papers, improving the English, making suggestions to the authors, engaging in discussions with the authors, and implementing changes. It is due to their stamina and to the assistance by Carmen Hammer (also from the Max Planck Institute for the History of Science) that this volume could finally appear. We would furthermore like to acknowledge Joseph Dauben, professor of history of mathematics at City College, New York; Philip Cho, postdoctoral fellow at the CAS Institute for the History of Natural Science; and John Moffet, curator of the Needham Institute’s library in Cambridge for their efforts in editing the language of contributions to this volume. Finally, we express our most sincere gratitude to Barbara Spielmann of the Headquarters of the Max Planck Society for her unflinching and competent support of our collaboration.

The Transformation of Mechanical Knowledge – an Introduction

Peter DAMEROW, Jürgen RENN, and Matthias SCHEMMEL

1. Prescientific Knowledge as a Basis of Science

Science is not only based on the specific kind of knowledge usually characterized as “scientific” and expressed in theoretical texts, but essentially involves a broader knowledge base comprising all we have to know in order to master our environment, our technology, as well as the specific equipments necessary to gain and validate scientific knowledge. Nowhere does the role of this prescientific knowledge as the basis of science become more evident than in the case of mechanics, one of the earliest sciences, which was developed on the basis of the accumulation of practical experiences over centuries and established long before the invention of the experimental method.

Taking this prescientific knowledge into account one can distinguish between three sources of knowledge. In the case of mechanics, knowledge acquired through universal human experiences accounts for the understanding of basic properties of bodies and motion such as the property of heavy bodies to fall down if they are not supported. Knowledge acquired in connection with the invention, the production, and the use of technology accounts for the understanding of non-obvious properties of natural phenomena such as the possibility to compensate weight with distance using the movable counterpoise of a steelyard. Knowledge acquired with the help of experiments accounts for the possibility to explore relations between theoretical questions and natural phenomena as may be illustrated by the exploration in early modern Europe of the motion of fall in the light of medieval conceptions of acceleration.

In view of the different functions of the representation of knowledge it makes sense to distinguish two different forms: *internal representation* by means of mental structures and *external representation* by means of actions, instruments, images, speech, symbol systems, and writing. These representations may take on three basic forms: (*mental*) *models*, *rules*, and *theories*.

Furthermore, three forms of communication and transmission of knowledge can be distinguished accordingly: *sharing practices*, oral and written *representation of rules*, and the *transmission of texts* with an explicit argumentative structure. In our context, a typical example for the transmission of mental models by means of sets of rules is the literary tradition of compilations of problems and problem solutions, widespread in all cultures here under consideration without implying any mutual influence, such as the Aristotelian *Mechanical Problems* in the West and the *Jiuzhang suanshu* 九章算術 [*Nine Chapters of Mathematical Procedures*] in China.

For short, three forms of knowledge determine the role of prescientific knowledge as a basis of science, which may be called (*purely*) *intuitive*, *practical*, and *theoretical* knowledge.

Intuitive knowledge is the implicit knowledge of the coordination of sensori-motor activities. In its developed form this coordination is based on mental models which allow to draw conclusions about processes or situations even when only incomplete knowledge about them is available, intuitively filling gaps by default-assumptions suggested by prior experiences. The stability of such mental models results from the fact that their implicit default assumptions may be changed in the light of new experiences without abandoning the model. An example is the intuitive knowledge about the causation of motion structured by the “motion-implies-force model” inducing inferences from an observed or conceived process of motion to the assumption that it is generated by the application of a force. While intuitive knowledge is first built up in ontogenesis in interaction with our natural environment and is hence, at its core, of a universal nature, it may be extended by culturally specific experiences.

Practical knowledge is typically represented by explicit rules prescribing certain actions which may induce an intuitive understanding of their object. Since these actions involve material tools, as for instance those necessary to produce or to use a balance, practical knowledge is, from the outset,

generated, acquired and transmitted within a culturally and historically specific technological environment. It is typically transmitted by sharing practices or by the oral communication of rules.

Theoretical knowledge is the result of reflection on human action in the broadest sense. The actions which are the subject of reflection may be material activities that are the basis of intuitive and practical knowledge. They may be internal operations of the human mind determined by mental models, operations such as the application of rules, transformations in space and time, or mental derivations within systems of propositional knowledge. They may also be manipulations with material symbols that externally represent mental models in written descriptions, symbol systems, semantically ordered propositions, or formal theories. In order to be transmittable between individuals and thus capable of being interculturally and historically transferred, theoretical knowledge has itself to be externally represented in such written forms, thus giving rise to higher order reflection processes.

In the case of mechanical knowledge, the outcome of the processes of theoretical reflection is knowledge coded in theoretical categories such as the concept of the *center of gravity*. Its meaning can neither be grasped by visual perception, nor can it be immediately derived from experiences obtained from using mechanical devices such as the balance or the lever. To build up knowledge about the *center of gravity* requires written language as a medium of transmission since it involves a controlled use of language and the determination and consistent use of argumentative structures resulting in the formulation of “theories” such as the Archimedean theory of the equilibrium of planes.

The developmental pathway leading from universal intuition to theories results in what one may call a “dynamical hierarchy of knowledge,” in which the different levels persist as lower levels necessary for the application and further development of higher levels. Thus, new knowledge may be created by the feedback of higher to lower levels, giving rise, for instance, to new intuitive forms of thinking. An example is the intuitive perception of mechanical motion as seemingly taking place “against nature,” a perception that is shaped by the emergence, both in Europe and in China, of theories of motions taking place in accordance with nature.

2. Intuitive Origins of Mechanical Knowledge in China

Universal human experiences that account for the understanding of basic properties of bodies and their motions leave traces in the language. Certain terms with equal or similar meanings related to mechanical phenomena such as terms that designate *forces* occur independently in various cultures even if any direct influence is historically impossible. The review of meanings of the term *li* 力 in early China by ZOU Dahai in this volume (chapter 2) shows that the Chinese culture is no exception in this respect. The term *li* 力 exhibits the typical connotations of terms for *force* in other cultures. The term *force* has an anthropomorphic root designating human physical strength. The experience that the human force can be used to produce an effect leads to the belief that any change in our environment is the result of some force applied by an agent, the magnitude of which corresponds somehow to the intensity of the effect it produces. Accordingly, as the review shows, also the term *li* 力 was used – as early as in the Pre-Qin period – not only to designate human physical strength but also forces such as those of animals and of muscles and bones, of eyes and ears, or of the water of the sea carrying the ships. In particular, as forces in other cultures, *li* 力 is considered to be necessary in order to raise a weight or, in the context of the interculturally shared “motion-implies-force model,” to set a body into motion.

These universal mental constructions become the elements of mental models which are no longer universal, but which may still be universally understandable. This is the case, for instance, for the mechanical foundations of some cosmological models analyzed by TIAN Miao in her contribution on mechanical reasoning in ancient Chinese sources dealing with the motion of celestial bodies. The ancient Chinese legend for example that says that the heaven was resting on eight pillars until the mythical creature *Gong Gong* 共工 furiously smashed his head against one them and crushed it with the consequence that the heaven was tilted and the celestial bodies began to move downwards is surely specific to the ancient Chinese culture. However, the mechanism of this model is understandable independent of experiences in this culture since broken pillars and tilted planes have the same consequences in all cultures.

3. The Specific Impact of the Chinese Culture on Intuitive Mechanical Knowledge

The study of the meanings of the term *li* 力 in the literature of the Pre-Qin period does, however, not only reveal its universal basis in culturally independent human activities. It also shows that the term has certain connotations that are specific to the ancient Chinese culture. Once the experience of the physical strength of the human body as a cause of effects is projected onto other agents, there is no longer a natural limitation of accountable causes that might be designated by the term *li* 力. Thus the term is attributed in the early Chinese literature not only as in other cultures to phenomena such as the mechanical forces of animals and physical bodies. The term *li* 力 also designates the force to overthrow ends of others. It was associated with high position and authority and also attributed to heaven and earth – forces exceeding human abilities. In the *Yan Zi Chunqiu* 晏子春秋 [*The Story about Master Yan*] the term was even attributed to the divination by interpreting dreams, explicitly stating that it is the *li* 力 of the divination rather than some kind of power of the diviner himself.

It is tempting to consider attributions of terms related to mechanical phenomena such as the Chinese term *li* 力 to objects without any mechanical implication simply as metaphors and to distinguish them from proper generalizations of mechanical experiences. But this would anachronistically impute the modern distinction between the use of the technical term *force* in mechanics and its metaphorical use independent of the realm of mechanical experience onto the Chinese term. Such a distinction can hardly make sense applied to a term that was in use long before any systematically structured body of mechanical knowledge had been developed. It seems more adequate to interpret differences in the semantic fields of terms designating *forces* in different cultures as they are indicated by the Chinese attribution of *li* 力 to entities such as heaven, earth, and authority as resulting from a fundamental influence of cultural conditions on the intuitive knowledge gained from basic human activities. It seems, at least, possible that the ideal of the Western Zhou dynasty in the politically unstable later Pre-Qin period influenced not only the motivations of the philosophers of the time but also the general understanding of what sets the conditions for human activities.

Another example of culturally specific modifications of essentially universal explanations of mechanical phenomena is provided by the use of the term *shi* 勢 for explaining why in certain situations *li* 力 as a cause has not the expected effect. The term may be translated as *circumstances*, but in contrast to this English term which leaves the reason for the unexpected result essentially open, the Chinese term *shi* 勢 is used as if it designates an alternative to *li* 力 as a cause of the outcome of a human activity. Here the term does not express a problem in finding an adequate mechanical explanation but rather introduces an alternative cause that inherently limits any attribution of mechanical causes.

The most obvious example of a generic term specific to the Chinese culture that is used for explaining certain phenomena related to motions is the term *qi* 氣 as it is analyzed in the contribution of TIAN Miao about mechanical knowledge in ancient Chinese cosmology (chapter 3). In spite of the fact that the term had the same theoretical function of explaining the motion of heavenly bodies as the fifth element had in Aristotelian philosophy, the two terms belonged to quite different semantic networks. The meaning of the Aristotelian term is constituted in the context of the Greek doctrine of the four elements *earth, water, air* and *fire*, adding to the interpretation of *natural motions* in the sublunar world by means of these *elements* an independent explanation for the motion of celestial bodies. In contrast, the term *qi* 氣 designates a generic state that can dissociate to form heaven and earth, and the interaction of heaven and earth act as *yin* 陰 and *yang* 陽, which determine the various motions in heaven and on earth. Accordingly, the term *qi* 氣 cannot simply be translated into a corresponding English term, but only be circumscribed. However, whether this untranslatable Chinese term is rooted in culturally specific, intuitive knowledge of relations between cause and effect or whether it is rather an outcome of reflection in the context of Chinese philosophy has to be left open here.

4. The Impact of Technology in ancient China and Greece

In contrast to knowledge gained through universal human activities, knowledge that is based on experiences with invented devices can become interculturally shared knowledge only if these devices themselves are used across cultures. A typical example of such a device is the balance. While

indigenous cultures as long as they do not belong to the trading area of civilizations using balances do not develop a quantitative concept of weight beyond the level of immediate sensory perception of heaviness, all civilizations such as ancient China which use this device establish weight standards and develop metrologies for weight with essentially the same hierarchical structure of sizes.



Using a shoulder pole in China.
18th century carpet.
Museum für angewandte Kunst, Vienna.



Carrying wine jars on a shoulder pole.
Floor medallion of a kylix. 500 B.C.
Athens Agora Museum.

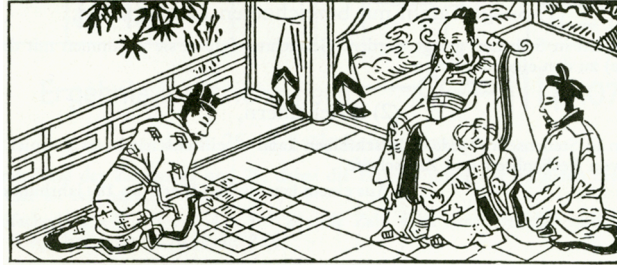
Another device developed even earlier in human history is the shoulder pole which facilitates the carrying of heavy loads. It is impossible to determine precisely when the Chinese shoulder or carrying pole (*biandan* 扁擔, in ancient times possibly called *gang* 杠) was invented, but it was probably used already in the 3rd century B.C. as the shoulder pole dance *dan wu* 擔舞 described in a later poem of the Tang dynasty suggests. In Greece, the use of the shoulder pole dates also back to early times; it is attested by a medallion depicting a man carrying wine jars at around 500 B.C. Equipped with a jar, a bucket, or a basket at each end and carried by a person on his shoulder this device provides immediate experience of the relation between the distribution of the load to the two containers and the adequate point to place the shoulder in order to keep the pole in equilibrium.



Moving the counterpoise of a Chinese steelyard.

Based on such experiences it was in both ancient cultures possible to invent or to adapt from another culture the balance with unequal arms which, once it was invented, provided precise experiences making it possible to understand the non-obvious possibility to compensate weight with distance using the movable counterpoise of such a balance. The use of the shoulder pole and of the balance with unequal arms thus provided in both cultures an intuitive knowledge about the mechanical function of these devices, knowledge which in modern theoretical mechanics is the basis of the *law of the lever*.

In the same way as balances initiated in different cultures the acquirement of similar practical knowledge about force and weight, certain weapons had the same function with regard to projectile motion. The contribution of YIN Xiaodong investigating the influence of western military technology on Chinese ballistics (chapter 10) shows how this shared practical knowledge, embodied for instance in aiming techniques, was the basis for merging traditional Chinese knowledge with knowledge about western military technology transferred to China in the 17th century by the Jesuits.



Chinese calculation board with rod numerals in an 18th century Japanese publication.

Source: D. E. Smith and Y. Mikami, *History of Japanese Mathematics*, Chicago 1914.

Another important example of knowledge gained from the handling of devices which were used in many cultures in a similar way is arithmetical knowledge related to calculation techniques. Such knowledge has its origins in the representation of objects by tallies which were handled instead of the objects themselves in order to control activities that change their quantities, activities like their distribution among people in a centralized state such as ancient China. As a rule, indigenous cultures did not invent and use any elaborate calculation technique. In contrast, virtually all early civilizations with a centralized administration developed arithmetical devices to control their economy.

In all these cultures, the arithmetical knowledge contained essentially the same basic number concept, the so-called *natural numbers*, which were abstracted from the representation by tallies, in ancient China the so-called *counting rods* (*chou* 筹), in Greece the *calculi* (ψῆφοι). Since, however, the sophisticated tools for performing calculations developed on this basic representation technology differed considerably between different cultures for a long time, the arithmetical knowledge acquired by using these tools also differed widely from civilization to civilization. In ancient China this knowledge was derived from a specific way of geometrically arranging counting rod numerals on the floor or on a calculation board and by the operations performed with the counting rods within this geometrical framework. The outcome was a set of arithmetical methods for solving various types of problems, which determined the specific scope of ancient Chinese mathematics represented by the *Jiuzhang suanshu*. The application of this specific Chinese knowledge to problems related to mechanics and the transformation of this knowledge resulting from the confrontation with western arithmetical knowledge and calculation technology after the advent of the Jesuits in China is the subject of TIAN Miao's contribution analyzing the mechanical knowledge in the *Jiuzhang suanshu* (chapter 5) and XIAO Yunhong's contribution concerning the knowledge about the lever in 18th century Chinese mathematics (chapter 13) of this volume.

5. The Origins of Theoretical Reflection in ancient China and Greece

The transformation of intuitive and practical knowledge on mechanical phenomena into theoretical knowledge in ancient Greece and ancient China by means of reflection on the underlying practical activities is investigated by Jürgen RENN and Matthias SCHEMMEL in their contribution about mechanics in the Mohist Canon and its European counterpart (chapter 4). Their comparison of the origins of theoretical mechanics in the two cultures exhibits common conditions under which theoretical knowledge emerges and reveals a core of shared theoretical knowledge related to mechanical phenomena within otherwise quite different theoretical frameworks.

The analysis concerns the earliest known texts of theoretical mechanics in ancient Greece and ancient China, that is, the Aristotelian *Mechanical Problems* and the *Mohist Canon*, both written independently

at about the same time around 300 B.C. They were both written under conditions that favor conscious reflection and commitment to the results by representing them in the permanent medium of written language. These conditions were provided in both cases in the context of a culture of disputations about practices. Such practices were discussed not with the aim of carrying them out but rather to understand the implicit knowledge embodied in the rules that guide the activities of the practitioners. As far as basic mechanical knowledge is concerned, the two texts struggle with the same problem. They share the same intuitive understanding of the relation between mechanical forces and their effects: a greater force or weight has a greater effect. Moreover, both texts are concerned with the same practical experience acquired by the use of mechanical devices such as the shoulder pole, the balance, and the lever: mechanical devices make it possible to produce greater effects with smaller forces or weights. The implicit contradiction between the intuitive mechanical knowledge, which was the same in ancient China and in ancient Greece, and the knowledge of practitioners who used basically the same mechanical devices in China and Greece triggered in both cultures the reflection on the implicit knowledge embodied in the rules for using mechanical devices. The reflection resulted in theories that eliminate the contradiction, thus creating an incipient theoretical mechanics.

The two theoretical attempts did not share the same future. In Greece before the decline of the ancient world the incipient theory of mechanics quickly developed into a comprehensive body of theoretical knowledge about mechanical devices, prototypically represented in its theoretical and practical aspects by the works of Archimedes and Heron respectively. In China, by contrast, the theoretical tradition of the Mohists was soon interrupted by the autocratic regime of the Qin dynasty and the long-lasting dominance of Confucianism.

This striking difference of the historical influence of early theoretical mechanics in Greece and in China points to another important aspect of the geographical and historical transmission and transformation of knowledge. As far as mechanics is concerned, the intuitive mechanical knowledge that is based on general human experiences is interculturally and historically stable and provides a solid basis for all processes of transformation and transmission. The practical knowledge that is gained from the use of mechanical devices is dependent on the survival of technologies in the course of the rise and fall of societies in history. The theoretical knowledge is also dependent on the historical persistence of conditions that guarantee their survival. But as long as the institutions that made scholarly work possible were fragile, the risk that such knowledge was irrecoverably lost with the doom of an intellectual culture was much higher than that of the disappearance of the inherent knowledge of basic practices. On the other hand, in contrast to the knowledge of practitioners, theoretical knowledge can survive over long historical periods in a state of latency as a treasure hidden in manuscripts and prints kept in the safe custody of archives and libraries until they are unearthed again by the renaissance of an ancient culture.

Another body of ancient Chinese theoretical knowledge, Chinese mathematics represented by the *Jiuzhang suanshu* mentioned above, had a better chance to survive in the intellectual climate of Imperial China. There are primarily two reasons for this historical stability. First, the theoretical knowledge gained by the use of rod numerals on the counting board was much closer connected to the problems that the official of the Chinese state administration faced than the philosophical topics of the Mohist school. Second, the form of knowledge representation by problems, rules and explanations in the *Jiuzhang suanshu* is far more robust against changes and further development of the represented theoretical knowledge than the representation in the language of philosophy. The *Jiuzhang suanshu* analyzed in the contribution of TIAN Miao (chapter 5) provides a historically influential Chinese example that unveils the advantages and disadvantages of the transmission of theoretical knowledge by this kind of knowledge representation.

A closer look at one of the problems of the *Jiuzhang suanshu* can make this clear. The problem provides an exercise of the *method of excess and deficiency* (*yingbuzu* 盈不足) and concerns the theory of specific weight. The weight and the side of a cube is given. The cube consists partly of stone and partly of jade. The problem is to find the amount of jade and of stone in this cube. The solution has to be calculated from the also given weights of two other cubes, one of them purely made of jade, the other made of stone, both of them with sides equal to the unit of length.

Any solution of this problem requires some theoretical knowledge about the relation between *weight*, *volume* and *specific weight*. A modern solution would make this knowledge explicit and derive the steps of the calculation from it by means of some algebraic formalism that allows for a formal representation of the given conditions and facilitates conclusions to be drawn from them. But at the time the *Jiuzhang suanshu* were composed, neither an explicit theory of the relation between the three basic theoretical concepts, nor an adequate formalism for the representation of the necessary inferences and the steps of the numerical calculation existed. What is recorded in the *Jiuzhang suanshu* is thus simply a record of the steps to be performed on the calculation board.

Since these steps lead to the correct solution, the written representation is robust against any doubts raised by possible discussions. However, the representation conveys the basic theoretical knowledge about specific weight only indirectly. A disciple will easily find out the rules according to which the steps of the numerical solution are organized. But he has to do many similar exercises in order to reconstruct in his mind the disguised theoretical knowledge from the steps of these rules of the calculation.

This weakness of the representation of theoretical knowledge by rules of what has to be done to solve given problems becomes particularly obvious in view of the attempt to explain the reasoning behind the steps of the solution by the ancient scholar Liu Hui in his commentary on the problem which is also discussed in the contribution to this volume about the *Jiuzhang suanshu* by TIAN Miao. The commentary of Liu Hui explains the steps of the calculation without reflecting about the role of the problem within the broader theoretical context of *weight*, *volume*, and *specific weight*. But since the simple calculation presented in the *Jiuzhang suanshu* as the solution uses tacitly the fortuitous condition in the given figures that the difference of the two specific weights equals 1, the crucial step cannot really be explained without making explicit how the difference of the two specific weights influences this solution. It seems that Liu Hui saw that the solution was obviously correct, but could not give any justification for its crucial step.

This example shows the reason for the historical robustness as well as the structural difficulty of the representation of theoretical knowledge by casuistically arranged procedures. The correctness of the application of a rule and of the outcome can be judged independent of the theoretical background. This makes the knowledge underlying the rule robust, but difficult to be reconstructed from the rules that are communicated and transmitted.

Chinese mathematics is a typical example of such a form of knowledge representation, Babylonian mathematics is a similar case, but the representation of mechanical knowledge in the Aristotelian *Mechanical Problems* is not. The problems in this treatise serve, in contrast to the problems in the other two examples, not as exercises for the applications of rules, but to justify an explicitly given theoretical principle: in circular motion, the same force causes a greater motion if it acts further from the center of the circle. Accordingly, the influence of this text was quite different from texts such as the *Jiuzhang suanshu*. On the one hand, the treatise initiated the rapid development of theoretical mechanics in ancient Greece. On the other hand, its theoretical principle was soon replaced by a more powerful one, the *law of the lever*.

6. The Cultural Transmission and Transformation of Mechanical Knowledge

Probably in all cultures, the historical transmission of knowledge is characterized by continuities and discontinuities, which may range from an interruption in the transmission of knowledge via its renewal to its reorganization. In the Western tradition the Renaissance is an example for both the renewal and the reorganization of knowledge. Evidently, the rediscovery of Greek science in early modern Europe was a reorganization because it was not limited to just reviving texts as storage of theoretical knowledge but also comprised a new integration of practical and theoretical knowledge triggered by the great technological challenges of the time.

This phase transition of knowledge is particularly visible in the conflicts between the Church adhering to a worldview merging Aristotelian and biblical ideas and the emerging group of engineer-scientists who were forced into the opposition to this worldview due to the freedom they took in theoretically interpreting the new practical experiences. But this conflict had consequences also within the powerful

hierarchies of the Church in its coalition with the secular powers. This is especially evident if we look at the role of the Jesuits in the transmission of mechanical knowledge to China, the subject of the chapters by ZHANG Baichun and TIAN Miao (chapter 6), by Peter DAMEROW and Urs SCHOEPLIN (chapter 7), and that by Rivka FELDHAY (chapter 8). Since the foundation of the order in the early 16th century, the Jesuits adopted more and more Renaissance achievements in their attempts to combine the Christianization of the world with spreading science, technology, and welfare. Their adaptation of, and their own contributions to, early modern science were thus distinguished by a marked practical orientation which was also helpful in maintaining a safe distance from the potentially dangerous implications of the new science for a new world view.



The Jesuits Matteo Ricci, Adam Schall von Bell, and Ferdinand Verbiest

Source: Johann Baptista du Halde, *Ausführliche Beschreibung des Chinesischen Reiches und der grossen Tartarey*, Rostock 1749, p. 93.

Any attempt to analyze the historical transmission and transfer of knowledge within Chinese culture must address the question of whether here a similar Renaissance of ancient knowledge occurred and to which extent such a Renaissance may have been triggered by external stimuli such as the Jesuit intervention.

An exemplary case to study this question is one of the most influential outcomes of the cooperation between Jesuits and Chinese scholars, the *Yuanxi qiqi tushuo luzui* 遠西奇器圖說錄最 [*The Record of the Best Illustrations and Descriptions of Extraordinary Devices of the Far West*] by Johannes Schreck (Terrentius) and Wang Zhang completed in 1627, which is the main example discussed in several contributions in this volume. What becomes evident from the investigation of this example is the array of contexts shaping the process of a massive transmission of European scientific and technological knowledge to China, as well as the surprising consequences for the transformation of knowledge that such a process can have. The most important condition of this transformation process was the confrontation of two quite different cultural contexts with their distinct knowledge systems. In the contribution of Peter DAMEROW and Urs SCHOEPLIN (chapter 7) the unreconciled tensions within the European knowledge background are analyzed as a precondition of the contribution of the Jesuits. Focusing on the example of salt mining, the chapter by Hans-Ulrich VOGEL (chapter 9) investigates some of the practical challenges of the Chinese society, thus making evident some of the potentials and resistances of this society with regard to adopting new technologies and theoretical ideas from the Western world.

The productive, but difficult coordination process resulting from the confrontation of the two contexts after the intervention of the Jesuits in China is precisely what becomes clear if one investigates in depth both the literary form and the contents of the *Qiqi tushuo*. The authors were forced, on the one hand, to bring together basic theoretical ideas such as Stevin's theory of the center of gravity and Ramelli's depictions of partly realistic and partly fictive complex machinery in a way that had no precedent in the European literature, with the possible exception of Jesuit manuals. On the other hand, they had to translate the results of this integration into categories understandable to Chinese scholars and practitioners. Under the condition that a few Jesuits had for a limited time influence on part of the ruling class, the available bandwidth of transmission was unavoidably limited. The necessity to make optimal use of this bandwidth led to radical simplifications and curious syncretisms joining such distant assets of European culture as the Bible and the works of Archimedes, for instance by merging the biblical story of the expulsion from Paradise with the early modern appreciation of mechanical instruments, to formulate the claim that the latter were invented by Adam and Eve as a means of survival outside the Paradise.

Nevertheless the ambitious goals of the Jesuits to transfer the religious, cultural, scientific, and technological heritage of the West to China for a time had a limited success. The contribution of YIN Xiaodong about the influence of Western military technology on Chinese ballistics (chapter 10) shows, for example, how Western knowledge about projectile motion was reformulated in Chinese terminology based on traditional techniques of shooting. The contribution of CHEN Yue (chapter 11) about the role of Galileo's theory on the strength of materials in Ferdinand Verbiest's book on astronomical instruments *Xin Zhi Ling Tai Yi Xiang Zhi* 新制靈台儀象志 [A Record of New-built Astronomical Instruments in the Observatory] shows how the latest Western scientific achievements were introduced into the community of Chinese scholars and practitioners. The Jesuit Verbiest describes in his book the six instruments which he built for the Beijing astronomical observatory. Exploiting Galileo's theory as a legitimation, he shrewdly used the superior quality of his instruments to propagate the excellence of European science and to gain acceptance for the alleged superior competence of the Jesuit scholars whose status in Chinese society nevertheless remained precarious.

Another example of the merging of Western and Chinese technology and scientific knowledge is the adaptation of Western surveying techniques. The contribution of FUNG Kam-Wing (chapter 12) offers an overview and a detailed analysis of the integration of both traditions, extending the thematic scope of this volume to include the case of Japan.

What were the preconditions for the Jesuits' limited success? One fortuitous condition was given by an internal problem at the borderline between science and administration. The Jesuit scholars would have hardly received as much attention from the Chinese court as they did, if there had been not the need of the Imperial Astronomical Bureau, motivated by reasons of state, for a more precise and reliable prediction of eclipses, a need which the Jesuits could – with some effort – satisfy, given that the Jesuit Christopher Clavius in Rome was the leading specialist in the field, responsible for realizing the Gregorian calendar reform. The success of the Jesuits in this matter stymied for some time their opponents at the court.

There were, however, also more systematic reasons for the fact that the Jesuit intervention was so powerful and yet so fragile. On the political side, the Jesuit strategy of gaining influence by convincing members of the political elite of their superior competence matched the hierarchical structure of Chinese society. On the epistemic side, the Jesuits' cultural and scientific baggage determined by the state of development of Western science corresponded to a culture of Chinese scholarship that had independently evolved so that mismatches were unavoidable, but displayed a comparable level of sophistication. The contribution by XIAO Yunhong deals with a telling example of a successful integration of Western knowledge into an autonomous Chinese tradition, namely the integration of Western methods including sophisticated applications of the law of the lever into the Chinese canon of problems in the tradition of the *Jiuzhang suanshu*, an integration that was aided by the fact that a balance with unequal arms, the steelyard, was known in both cultures. The law of the lever could be integrated without difficulty because of two preconditions, practitioners' knowledge and the availability of an appropriate mathematics. As a matter of fact, both mathematical traditions, the Western and the Chinese, were of comparable power to the point that the same problems could first be solved with

traditional Chinese methods of the *Jiuzhang suanshu* and then with the newly acquired Western techniques based on the theory of proportions.

In spite of the many striking parallels between Western and Chinese traditions of technology and scientific knowledge, they are nevertheless distinguished by a feature that was often considered to be the hallmark of Western science and the ultimate reason for its alleged superiority to its Chinese counterpart, that is, the deductive organization according to the model of mathematical proofs, which is absent even from those Chinese texts that are otherwise a more or less direct rendering of Western knowledge such as the *Qiqi tushuo*. What the authors Johannes Schreck and Wang Zheng had to accomplish was nothing less than to realize transformations such as that of the traditional European form of organizing scientific arguments, the deductive structure for which Euclid's *Elements* had offered the paradigm, into the traditional Chinese form of organizing a theoretical argument, the casuistic scheme whose paradigm is the *Jiuzhang suanshu* – without losing either the range of the imported Western knowledge or its force of conviction. For this reason such texts, written jointly by Chinese and Western scholars, offer a touchstone for analyzing the difficulties and the scope of the solutions found by them to transform Western into Chinese knowledge. They are also a fruitful source to study the difficulties that arise in the communication of knowledge between two cultural systems.

The Concept of *Force* (*li* 力) in Early China

ZOU Dahai

1. Preface

In ancient China we find no such category of learning as mechanics. Yet, Chinese made and used mechanical devices that require and thus give evidence of mechanical knowledge implicit in their practices. Apart from the possibility to infer such practical knowledge from the archaeological remains of their activities, it is possible by analyzing the ancient Chinese literature to find evidence of some concepts through which such knowledge was communicated or at least described. It is true that no classical Chinese source actually discusses this kind of knowledge in an easily accessible way using concepts that correspond to the categories of modern mechanics, but descriptions of natural phenomena and human endeavor entail attempts to conceptualize experiences which in modern mechanics are theoretically explained using concepts such as force, equilibrium, energy, and work. The use of language may represent structures of knowledge. The consistent use of terminology, the choice of subjects, and the focus of attention may give evidence of mental models by which knowledge about mechanical phenomena was structured and communicated. Whether the ancient Chinese texts handed down to us are lyrical or essayist, whether they deal with philosophical issues or document political, social or economic history as their main theme, they may be sources that can help us to understand the ancient Chinese modes of thinking about mechanical phenomena and how such mechanical knowledge was integrated into the common knowledge of the time.

The present paper investigates the usage and understanding of the term *li* 力 as representing some aspects of mechanical knowledge in early Chinese texts. This term was chosen after careful consideration with regard to its modern Chinese translation as “force”. In ancient Chinese literature *li* 力 has a quite broad connotation which has been studied by linguists and paleographers. While their work on the origin and later development of this term has turned out to be helpful for the present study, it does not sufficiently allow to draw conclusions about the structure and scope of the mechanical knowledge, which is the aim of this study. Scholars like Chen Cheng-Yih¹ and Dai Nianzu 戴念祖 see the concept of *li* 力 in its mechanical sense reflected in the Mohist canon, however, especially the text of the *Mohist Canon* poses considerable problems.² Tan Jiefu 譚戒甫³ and Qian Linzhao 錢臨照⁴ used Newton’s first law of motion to explain the section defining *li* 力, an explanation which was accepted by Needham⁵ and Chen Cheng-Yih. They impose, however, with their interpretation of *li* 力 as the Newtonian concept of force that causes acceleration, an essentially western idea to the traditional

1 Chen Cheng-Yih 程貞一, *Early Chinese Work in Natural Science: A Re-examination of the Physics of Motion, Acoustics, Astronomy and Scientific Thoughts*. Hong Kong University Press. 1996. p. 9.

2 See the groundbreaking study and translation by A. C. Graham, *Later Mohist Logic, Ethics and Science*, Hong Kong: Chinese University Press, 1978. The Mohists appeal to the distinction between knowing names and knowing objects. There is an ongoing controversy about the connection between the *Mohist Canon* and the Zhuang Zi text and its terminological relationship. For reference see, e.g., Chen Cheng-Yih, *Early Chinese Work in Natural Science: A Re-examination of the Physics of Motion, Acoustics, Astronomy and Scientific Thoughts*. Hong Kong University Press, 1996. At the Max Planck Institute for the History of Science in Berlin discussions on Mechanics in the *Mohist Canon* were organized which resulted in translations and explanations of some sections. See: Matthias Schemmel, *The Sections on Mechanics in the Mohist Canon*, Preprint 182 of the Max Planck Institute for the History of Science, 2001; William G. Boltz, Jürgen Renn, Matthias Schemmel, *Mechanics in the Mohist Canon and Its European Counterpart: Texts and Contexts*, Preprint 241 of the Max Planck Institute for the History of Science, 2003; see also chapter 4 in this volume.

3 Tan Jiefu 譚戒甫 (1887–1974), *Mobian Fawei* 墨辯發微 [*Discovery of the hidden meanings of Mo Bian*]. Zhonghua Shuju in Beijing, 1964, reprint in 1987. pp. 101-102.

4 Qian Linzhao 錢臨照 (1906-1999), “Shi Mojing Zhong Guangxue Lixue Zhu Tiao” 釋墨經中光學力學諸條 [“Explanations on the Sections of Optics and Mechanics in the Mohist Canons”], in *Li Shizeng Xiansheng Liushi Sui Jinian Lunwenji* 李石曾先生六十歲紀念論文集 [*Collected Essays in Memory of Professor Li Shizeng’s 60th Birthday*]. 1942. pp. 135-162.

5 Needham, Joseph (1900-1995). *Science and Civilisation in China*. Vol. 4, Part 1, Physics. London: Cambridge University Press. 1962. p. 19.

Chinese text. Qian Baocong 錢寶琮 disagreed to this and interpreted *li* 力 in the section as representing a general idea of physical strength⁶, an interpretation which was in accordance with the later one by Graham⁷. Dai Nianzu cited several scholars' opinions about this section without his own judgment. He also quoted several paragraphs concerning the word *li* 力 from the *Lun Heng* (論衡) by Wang Chong 王充 (27 A.D.-97 A.D.), and gave simple explanations⁸. However, Wang Chong cannot be considered a mainstream thinker but rather an exception to the rule. His text was hardly known in its own time and throughout the period of imperial China. Dai Nianzu's explanation moreover tends to neglect the context of Wang Chong's arguments which rarely aimed to explain physical phenomena, but rather concerned cosmological structure.

The agenda of the present paper is to investigate the meaning of *li* 力 and its relation to other important concepts or notions of mechanical knowledge, in particular concepts represented by the terms *zhong* 重 (*weight*), *gong* 功 (*effect*), and *shi* 勢 (*circumstances*), and the concept of motion. For analytical purposes the following semantical distinction will be used to describe the various meanings of the term *li* 力.

1. noun: physical strength or power of human body or animal; also used to refer to physical strength/ability of thinking organs (this does not mean that the *li* 力 was used as strength/ability of thinking itself)
2. noun: general ability or power/energy of human beings
3. noun: general strength, power, energy or ability of any nonliving as well as living things
4. verb or adverbial modifier: to endeavor to do, to make effort to do, to do with exertion
5. noun: effect, achievement, contribution
6. noun and verb: labor, work
7. noun (also as adverbial modifier or attribute): forcible power, strong arm, especially armed force, military strength.
8. noun: corvee

After a short survey into the early occurrences of the character *li* 力 on oracle bones, the paper will focus on the pre-imperial and early imperial period which will be briefly characterized in the following.

In its early days China was ruled by the western Zhou dynasty that based its power on a centralized state system. The following period of political fragmentation, the Spring and Autumn period (770-476 B.C.) and the Warring States period (475-221 B.C.), provides the backdrop against which China's elementary philosophies—for instance, the school of Confucianism which referred to the centralized organization of the western Zhou dynasty as a model state for the Confucian ideology—developed their distinct profiles. In various competitive feudal states a plethora of schools and ideologies arose that would hitherto found the basis and nourish Chinese intellectual thinking. The earliest cohesive texts (except from oracle bones), the text corpus that would later form the classics, stem from these periods.

The rise of the First Emperor of the Qin dynasty in 221 B.C. brought dramatic changes. While he gradually united the states between the Changjiang and Huanghe regions, he established a centralized autocratic empire, forbade free thinking, banned most schools, and burned many documents of the former feudal states. Although the Qin dynasty lasted merely 15 years it had a lasting influence on the later formation of Chinese thought: it meant to destroy the line of transmission vitally. The following Western Han (206 B.C. – 25 A.D.) and Eastern Han (25 A.D. – 220 A.D.) dynasties inherited a landscape of fragmentary bits of wisdom which could only be partially recomposed to their original forms. During the Han dynasties formerly independent, individual cosmologies were merged into new

6 Qian Baocong 錢寶琮 (1892-1974), "Mojing Lixue Jinshi" 《墨經》力學今釋 ["Modern Explanations on the Mechanics of Mohist Canons"], in *Kexueshi Jikan* 科學史集刊 [publication of *Collected Essays on History of Science*], No.8, 1965. pp. 65-72.

7 Graham, A. C., *Later Mohist Logic, Ethics and Science*, Hong Kong: Chinese University Press, 1978. p. 279.

8 Dai Nianzu 戴念祖, Lao Liang 老亮, *Zhongguo Wulixueshi Daxi Lixueshi* 中國物理學史大繫 力學史 [A Series of *History of Physics in China*, Vol. Mechanics]. Changsha: Hu'nan Jiaoyu Chubanshe, 2001. pp. 161-165.

systems of thought that from then on constituted an important epistemological backdrop for the description of natural phenomena and for the formation of a Confucian state system.

2. The Concept of *li* 力

The earliest occurrences of the Chinese character *li* 力 are found in the oracle-bone inscriptions. It was written as graphs (1) and (2) (ca. 13th century B.C.). The character also occurred in the inscriptions on bronze (13th century B.C. – 3rd century B.C.), where it was written as graphs (3)-(9). The graph (10) is from the *Shuowen Jiezi* 說文解字 [*Explaining Radicals and Analyzing Compound Characters*, 100 A.D.-121 A.D.] by the scholar Xu Shen 許慎 (ca. 58-ca. 125 A.D.). It is the earliest book trying to identify the original meanings of Chinese characters.



Xu Shen believed that the character *li* 力 is closely related to the compound character *jin* 筋 (tendon, muscle) and depicts the shape of a human muscle. According to his belief, *li* 力 (strength) would denote the function of *jin* 筋 (tendon, muscle)⁹ He considered other meanings of the character as derived from this basic meaning:

2.1 力，筋也；象人筋之形。治功曰力，能圍大災。) ¹⁰

li 力 [means] *jin* 筋, [it] depicts the shape of a human being's *jin* 筋. The achievement of administration that are based on laws is called *li* 力, [which] can protect against disaster.

Modern Chinese paleographers criticize Xu Shen's assumption about the original meaning of *li* 力. They argue that this assumption was based on the peculiar form of the graph as it was written in Xiao Zhuan 小篆 style, a style of writing characters used in the Qin dynasty. Most of them rather agree to the opinion of Xu Zhongshu 徐中舒 (1898-1991) which was based on an analysis of oracle-bone inscriptions. He came to the conclusion that the character *li* 力 originally depicted the shape of a *lei* 耒, a kind of farm tool for plowing soil.¹¹ Thus, Xu Shen's assumption about the relation between *li* 力 and *jin* 筋 was probably wrong. Nevertheless, he might have been right with his opinion that the character *li* 力 was originally used to designate force or strength. It is true that in the oracle-bone inscriptions the character *li* 力 was only used to designate a kind of sacrifice so that attributing to its origin any meaning related to human strength or human work seems unfounded. But later references to periods as early as the Shang dynasty indicate that the character may anyway have had the connotation of strength and power already at that time.

In the chapter *Pangeng* 盤庚 of *Shang Shu* 尚書, possibly dating back to the 13th century B.C., the King of Shang dynasty, Pangeng 盤庚 (dated back to the later period of 14th century B.C. to the early 13th century B.C.), is reported to have said to his people:

2.2 汝無侮老成人，無弱孤有幼。各長于厥居，勉出乃力，聽予一人之作猷。 ¹²

You should not show the cold shoulder to adults, [you] should not slight children. Everybody should long live in your new places, and make effort to use your *li* 力 (effort) and obey my order to act and rest.

9 Duan Yucai 段玉裁 (1735-1815) said: "The *jin* (tendon, muscle) is the substance, while the *li* is the function" See Duan Yucai's *Shuowen Jiezi Zhu* 說文解字注 [*Commentary on Explaining Radicals and Analyzing Compound Characters*]. Shanghai Guji Chubanshe, 1981. p. 699.

10 Xu Shen 許慎 (ca. 58-ca. 125 A.D.), *Shuowen Jiezi* 說文解字 [*Explaining Radicals and Analyzing Compound Characters*, 100-121 A.D.]. Beijing: Zhonghua shuju, 1978. p. 291.

11 Hsü Chung-Shu 徐中舒 (1898-1991): "Leisi Kao" 耒耜考 ["On Some Agricultural Implements of the Ancient Chinese"], in *Zhongyang Yanjiuyuan Lishi Yuyan Yanjiusuo Jikan* 中央研究院歷史語言研究所集刊 [*Academia Sinica Bulletin of The National Research Institute of History and Philology*], Vol. II, Part I, 1930. pp. 11-59.

12 Sun Xingyan 孫星衍 (1753-1818), *Shangshu Jin Ju Wen Zhushu* 尚書今古文注疏 [*Commentaries on the Modern and Ancient Texts of Shangshu*]. Beijing: Zhonghua shuju, 1986. p. 231.

In the poem *Jian Xi* 簡兮 of *Shi Jing* 詩經 (possibly not later than the middle of 6th century B.C.), a poet wrote a poem to admire a *wanwu* 萬舞 dancer:

2.3 碩人俣俣，公庭萬舞。有力如虎，執轡如組。¹³

How strapping the tall man is, when he is performing the *wanwu* dance in the ancestral temple. He has [so much] *li* 力 (strength) as a tiger, holds the horse reins (so skillfully) like (a weaver's) weaving.

These examples make it likely that, in spite of the limited use of the character *li* 力 in the oracle-bone inscriptions, the first of the meanings listed in the preface, physical strength, was the primary one which formed the starting point from which the ancient Chinese developed the various meanings of the term *li* 力. The later definition in the *Mohist Canon* that “*Li* 力 (strength) is that by which the body exerts itself (力，刑之所以奮也)”¹⁴ may well have preserved the essence of the original meaning of the term *li* 力.

The literature that came down to us from later times of the Pre-Qin Period provides much richer sources about connotations of the term *li* 力.

The *Chunqiu Zuo Zhuang* 春秋左傳 (ca. 5th century B.C.) records a sentence of Zhi Wu Zi 知武子 (564 B.C.).

2.4 君子勞心，小人勞力，先王之制也。

A gentleman uses his *xin* 心 (heart) to work hard [while] a petty man uses his *li* 力 to work hard. This is the rule of ancient kings.

The ancient Chinese thought thinking was a function of the heart. Thus, Mencius 孟子 (ca. 372 B.C. - ca. 289 B.C.) also juxtaposed *li* 力 with *xin* 心 (heart).

2.5 故曰：或勞心，或勞力；勞心者治人，勞力者治於人。¹⁵

Therefore [I should] say: some people [use their] *xin* 心 (heart) to work hard while others [use their] *li* 力 (strength) to work hard. The people [using their] *xin* 心 (heart) to work hard rule people, while the people [using their] *li* 力 to work hard are ruled by other people¹⁶.

(From the chapter *Gao Zi Shang of Mencius* 《孟子·告子上》)

2.6 心之官則思。¹⁷

The function of the heart is thinking.

The third of the meanings of *li* 力 listed above, general power/strength/energy, is not frequently used, especially compared with the first and second meanings. In the *Zhanguo Ce Jianzhu* 戰國策箋注 a proverb mentioning the *li* 力 of the warrior Meng Ben 孟賁¹⁸ is quoted in a discussion between Su Qin 蘇秦 and King Min of the state of Qi (齊閔王).

2.7 語曰：“麒麟之衰也，駑馬先之；孟賁之倦也，女子勝之。”夫駑馬、女子，筋骨力勁非賢於麒麟、孟賁也，何則？後起之藉也。¹⁹

13 Yuan Mei 袁梅, *Shijing Yizhu (Guofeng Bufen)* 詩經譯注 (國風部分) [Translation and Commentaries of Classic of Poetry (the Part of Songs of Various States)]. Jinan: Qilu Shushe, 1980. p156.

14 Mo Zi 墨子 [Master Mo], *Dao Zang* 道藏 [Depository of Taoist Scriptures] Edition. Photocopied by Shanghai Hanfenlou, 1924. Vol. 10, 1a. The sentence will be discussed later in this paper.

15 Yang Bojun, *Meng Zi Yizhu*. 1962. p. 270.

16 D. C. Lau presented a free translation. See: Lau, D. C., *Mencius: A Bilingual Edition*. Hong Kong: the Chinese University Press, 2003. p. 115.

17 Yang Bojun 楊伯峻 (1909-1992), *Meng Zi Yizhu* 孟子譯注 [Translation and Commentary of the Mencius]. Beijing: Zhonghua Shuju, 1962. p. 124.

18 Meng Ben 孟賁, sometimes written as 孟奔, is a famous warrior in Warring States Period. Re. Zhang Qingchan 張清常 (1915-1998), Wang Yandong 王延棟, *Zhanguo Ce Jianzhu* 戰國策箋注 [Commentaries of Strategies of the Warring States]. Tianjin: Nankai University Press, 1993. p. 130.

19 Zhang Qingchang 張清常 (1915-1998), Wang Yandong 王延棟, *Zhanguo Ce Jianzhu* 戰國策箋注 [Commentaries of Strategies of the Warring States]. Tianjin: Nankai University Press, 1993. p. 293.

The proverb says: “When a swift horse is exhausted, [then even] an inferior horse can surpass it; when Meng Ben is tired, [even] a woman can overcome him.” The extent of the inferior horse’s and the woman’s muscle and bone *li* 力 (strength) is not greater than the swift horse and Meng Ben. So how could this be? [This is because the inferior horse and the woman] take advantage of starting later [without having exhausted their *li* 力 (strength)]²⁰.

(Chapter 12 of *Zhanguo Ce* 《戰國策》卷十二)

The passage shows that *li* 力 may be attributed to muscles and bones.

2.8 聖人既竭目力焉，繼之以規矩準繩，以為方員平直，不可勝用也；既竭耳力焉，繼之以六律正五音，不可勝用也。²¹

Once a sage has reached the limit of his eyes’ *li* 力 (ability), then [he] turns to using a compass, carpenter’s square, level and plumb line to make a square, circle, plane and vertical line. The utility of [these tools] is inexhaustible. After [he] has reached the limit of his ears’ *li* 力 (ability), then [he] turns to using six bamboo pitch pipes to adjust the five tones. The utility of [these tools] is inexhaustible.²²

(Chapter *Li Lou Shang of Mencius* 《孟子·卷七·離婁上》)

Here, Mencius mentioned “the *li* 力 of the eyes” (目力) and “the *li* 力 of the ears” (耳力).

2.9 諸藉車皆鐵什。藉車之柱，長丈七尺，斤錐者四尺；夫長三丈以上，至三丈五尺，馬頰長二尺八寸，試藉車之力而為之困，夫四分之三在上。²³

The various *ji che* 藉車 are all made of iron. The *ji che*’s [vertical] post is 1 *zhang* 丈 7 *chi* 尺; the part [of the post] that is buried is 4 *chi* 尺 long. The *fu* 夫 (possibly a lever) is 3 *zhang* to 3 *zhang* 5 *chi*. The *ma jia* 馬頰 is 2 *chi* 8 *cun* long. To test the *li* 力 (force) of the *ji che*, make a *kun* 困 (possibly a fulcrum). Three-fourths of [the length of] the *fu* 夫 is above [the *kun*]²⁴.

(From chapter *Bei Chengmen of Mo Zi* 《墨子·備城門》)

In this paragraph, the Mohist mentioned the *li* 力 of a military device, “*li* 力 of *Ji Che*” (藉車之力)

Lü Shi Chunqiu 呂氏春秋 mentions “*li* 力 of land” (地力, fertility of land) and “*li* 力 of water” (水之力, force of water) :

2.10 管子復於桓公曰：“墾田大邑，辟土藝粟，盡地力之利，臣不若寧速。請置以為大田……”²⁵

Guan Zi said to Duke Huan: “[As for] cultivating fields and expanding cities, reclaiming land and planting millet, and fully using the benefit of the land’s *li* 力 (fertility), I am not as good as Ning Su. Please appoint [him] as Great Field Officer. ...”

(From chapter *Wu Gong of Lü Shi Chunqiu* 《呂氏春秋·勿躬》)

20 J. I. Crump, Jr. also presented a translation. See: J. I. Crump, Jr. *Chan-Kuo Ts’u*. Oxford University Press, 1970. p. 198.

21 Yang Bojun, *Meng Zi Yizhu*. 1962. p. 162.

22 D. C. Lau also presented a translation. See: Lau, D. C., *Mencius: A Bilingual Edition*. p. 149.

23 *Mo Zi. Dao Zang* Edition. 1924. Vol. 14, 9b. Re. collations of *Mo Zi Jiangu* by Sun Yirang. See: Sun Yirang 孫詒讓 (1848-1908), *Mo Zi Jiangu* 墨子問詁 [Explanations on Mo Zi], *Zhuzi Jicheng* 諸子集成 [Collected Books of Various Schools] Edition published by Shijie Shuju in 1937. Photocopied by Shanghai Shudian, 1992. pp. 317-318.

24 *Zhang* 丈, *chi* 尺 and *cun* 寸 are units of length. The translation is based on Cen Zhongmian’s, and Robin D. S. Yates’ explanations. The term *che* 車 means vehicle, *Ji Che* 藉車 is possibly a device like a wheeled catapult for throwing stones to attack enemies. The term *ma jia* literally means *horse jowl*; it is possibly a muzzle. See: 1. Cen Zhongmian 岑仲勉, *Mo zi chengshou gepian jianzhu* 墨子城守各篇簡注 [Concise Commentary on the Military Chapters of Mo Zi]. Beijing: Guji chubanshe, 1958. pp. 35-36. 2. Yates, Robin, D. S., “Siege engines and late Zhou military technology,” in Hu Daojing (ed.), *Explorations in the History of Science and Technology in China*. Shanghai: Shanghai Chinese Classics Publishing House. pp. 416-418.

25 Zhang Shuangdi 張雙棣, Zhang Wanbin 張萬彬, Yin Guoguang 殷國光, Chen Tao 陳濤, *Lüshi Chunqiu Yizhu* 呂氏春秋譯注 [Translation and Commentary of the Story of Lü School]. Changchun: Jilin Wenshi Press, 1987. p. 566.

2.11 禹通三江五湖，決伊闕，溝回陸，注之東海，因水之力也。舜一徙成邑，再徙成都，三徙成國，而堯授之禪位，因人之心也。²⁶

Yu 禹 linked the three rivers and the five lakes, made a channel through the Yi Que mountain, and dredged a connection to the Hui Lu [lake], [so that] the water could flow into the East Sea. [He] did this to follow the *li* 力 (force) of water. First Shun 舜 moved and established a small city. Second [he] moved again and established a big city. Third [he] moved and established a state. Yao then abdicated the throne to him. [Yao] did this to follow the will of people.

(From chapter *Gui Yin of Lü Shi Chunqiu* 《呂氏春秋·貴因》)

In the following paragraph, *Zhuang Zi* 莊子 does not directly give the expressions “*li* 力 of water” (水力) and “*li* 力 of wind” (風力), but from the context it is clear that the writer thought of water and wind as having *li* 力.

2.12 且夫水之積也不厚，則其負大舟也無力。覆杯水於坳堂之上，則芥爲之舟；置杯焉則膠，水淺而舟大也。風之積也不厚，則其負大翼也無力。故九萬里則風斯在下矣，而後乃今培風；背負青天而莫之夭闕者，而後乃今將圖南。²⁷

Furthermore, [if] the accumulated water is not deep, then it will not have *li* 力(force) to support a big ship. [If] a cup of water is poured into a depression [in the floor] of a hall, then a blade of grass [will float] like a boat [on the surface of the water]; [but if] a cup is put [on the water], then [it] will stick to [the floor] because the water is shallow but the boat is big. [If] the amassed wind is not deep, then there is no *li* 力 (force) to support the large wings [of peng 鵬]²⁸. Therefore when at a height of ninety thousands *li* 里, with the wind beneath [its wings], [the bird] can harness 培 the wind. With the blue sky to its back and nothing in its way, then [the bird] can plan [to fly] south²⁹.

(From chapter *Xiaoyao You of Zhuang Zi* 《莊子·逍遙游》)

The following example is from *Heguan Zi* 鶡冠子：

2.13 故聖人者，後天地而生，而知天地之始，先天地而亡，而知天地之終；力不若天地，而知天地之任。³⁰

Therefore a sage is born later than heaven and earth, but [he] knows their beginnings; [he] dies before heaven and earth, but knows their end; [his] *li* 力 (force, power) can not compare to that of heaven and earth, but [he] knows their undertakings.

(From chapter *Neng Tian of He Wan Zi* 《鶡冠子·能天》)

The author said that a sage’s “*li* 力 (force, power) can not compare to that of heaven and earth” (力不若天地), which means that the ancients thought that the heaven and the earth have *li* 力.

Han Fei Zi 韓非子 reports that *Mo Zi* 墨子 used *li* 力 to express the strength of a carriage-crossbar, i.e. the splint connecting drawbar and yoke:

26 Zhang Shuangdi, Zhang Wanbin, Yin Guoguang, Chen Tao, *Lüshi Chunqiu Yizhu*. 1987. p. 479.

27 Guo Qingfan 郭慶藩 (1844-1896), *Zhuang Zi Jishi* 莊子集釋 [Collected Explanations on Zhuang Zi], in *Zhuzi Jicheng* 諸子集成 [Collected Books of Various Schools] Edition published by Shijie Shuju in 1937. Photocopied by Shanghai Shudian. 1991. p. 4.

28 According to the former text of the same chapter, the peng 鵬 was a large bird whose back was several thousand *li* 里. *Li* 里 is a unit of length, at that time, 1 *li* 里 = 415.8m ~554.4m.

29 There were many scholars who translated this paragraph into English, Such as: 1. Legge, James, *The texts of Taoism: the Tao Te Ching and the writings of Chuang Tzu*, with an Introduction by D. T. Suzuki. New York: The Julian Press, 1959, p. 213. 2. Burton Watson, *The Complete Works of Chuang Tzu*, New York and London: Columbia University Press, 1968, p. 29. 3 William G. Boltz, ‘The Structure and Interpretation of “Chuang tzu”’: Two Notes on “Hsiao yao yu”’, *Bulletin of the School of Oriental and African Studies*, Vol.43, no.3 (1980), pp. 532-543. 4. A. C. Graham, *The Seven Inner Chapters and Other Writings from the Book Chuang-Tzu*. London: George Allen & Unwin (Publishers) Ltd, 1981. p. 44.

30 Heguan Zi 鶡冠子 [Master Heguan], *Zhuzi Baijia Congshu* 諸子百家叢書 [Series of Various Schools], Shanghai: Shanghai Guji Press, 1990. p. 50.

2.14 墨子爲木鳶，三年而成，蜚一日而敗。弟子曰：“先生之巧，至能使木鳶飛。”墨子曰：“吾不如爲車輓者巧也。用咫尺之木，不費一朝之事，而引三十石之任致遠，力多，久於歲數。今我爲鳶，三年成，蜚一日而敗。”惠子聞之曰：“墨子大巧，巧爲輓，拙爲鳶。”³¹

Mo Zi made a wooden glider. [He spent] three years to finish [it]. [But it] flew for [only] one day and broke. [His] disciples said: “Master’s skill has reached [the level] to be able to make [even] a wooden glider fly.” Mo Zi said: “I am not as skillful as a carriage-crossbar maker. Using [only] a piece of wood one *zhi* (咫, a unit of length) or one *chi* 尺 long, [a carriage maker] spends less than one morning’s work [to make a crossbar] which can carry a load of 30 *dan* 石 (a unit of weight) long distance. [The crossbar] has so much *li* 力 that the time [over which it can be used] is counted in years. Now I have made a glider [and spent] three years to finish [it]. [But it] flew only one day and broke.” Hui Zi heard this and said: “Mo Zi was greatly skilled. [He realized that] Making carriage-crossbars [requires] skill, [but] making gliders is artless.”

(From chapter *Wai Chu Shuo Zuo Shang of Han Fei Zi* 《韓非子·外儲說左上》)

As physical strength or power, *li* 力 may not only be a power that is being executed or has been executed, but also a potential power that a body possesses, but has not yet executed. This possibility is still kept when the meaning of the term is extended to general power, strength, energy, ability, etc. For example, *Han Fei Zi* 韓非子 said:

2.15 夫馬之所以能任重引車致遠道者，以筋力也。萬乘之主、千乘之君所以制天下而征諸侯者，以其威勢也。威勢者，人主之筋力也。今大臣得威，左右擅勢，是人主失力。人主失力而能有國者，千無一人。³²

The reason why a horse is able to carry weight and draw a carriage a long distance is that it uses [its] muscle’s *li* 力 (strength). A ruler of ten thousand chariots or a sovereign of a thousand chariots uses his power and influence (*wei shi* 威勢) to control everything under the heaven and wage campaigns against other states. The power and influence is [like] the ruler’s muscle *li* 力 (strength). When ministers receive their authority (*wei* 威) [from the ruler] and courtiers arrogate power (*shi* 勢) [to themselves], this [implies that] the ruler loses his *li* 力 (authority). Among rulers who have lost their *li* 力 (authority) yet still control their state, out of one thousand there is not [even] one.

(From chapter *Ren Zhu of Han Fei Zi* 《韓非子·人主》)

The first occurrence of *li* 力 in the above quote can be understood as that what is being executed when a horse is doing work. The second and the third occurrences of *li* 力 express *wei shi* 威勢 (power and influence) which are associated with high position and authority. The *wei shi* is what a monarch uses to overawe his liegemen, but the *Han Fei Zi* takes it as a special kind of *li* 力 of the muscle or tendon of a monarch. In spite of this, *wei shi* can be taken away by others, and as a privilege, it is not a power to do material work, but a power to make others feel fear and to get them to perform or not to perform certain actions. So the term should be regarded as expressing potential power. Even though *shi* 勢 is reduced to *li* 力 in the above example, in most cases, *shi* 勢 and *li* 力 are distinguished, the first in contrast to the second having the connotation of a potential.

The sources quoted above belong to the Pre-Qin period.³³ They show that *li* 力 could express general power or ability, either of living beings or of nonliving things. This made it possible that it became a concept of mechanics used for the explanation of mechanical phenomena. But the present investigation shows that the early Chinese did not have strong intentions and did not make substantial efforts to use it as a universal concept for explaining mechanical motions or mechanical operations. As mentioned above, the *Han Fei Zi* 韓非子 reports that Mo Zi 墨子 used *li* 力 to express the strength of a carriage-crossbar (see quotation 2.14), but in the *Mohist Canon*, when the breaking of hairs is explained, this is not attributed to their having not enough *li* 力:

31 Chen Qiyou 陳奇猷 (1917-), *Han Fei Zi Xin Jiaozhu* 韓非子新校注 [New Collations and Commentaries of the Master Han Fei]. Shanghai: Shanghai Guji Chubanshe, 2000. p. 670.

32 Chen Qiyou 陳奇猷 (1917-), *Han Fei Zi Xin Jiaozhu* 韓非子新校注 [New Collations and Commentaries of the Master Han Fei]. Shanghai: Shanghai Guji Chubanshe, 2000. p. 1162.

33 Only Xu Shen’s *Shuowen jiezi* was not written in the Pre-Qin period but during the mid Eastern Han Dynasty. However, it is generally accepted that the book discusses the original meanings of Chinese characters.

2.16 [經]：均之絕不，說在所均。³⁴

[經說]：均髮均縣。輕而髮絕，不均也。均，其絕也莫絕。³⁵

[Canon]: Under equal conditions, whether or not [something] breaks, the explanation lies in what is equalized.³⁶

[Explanation]: Equal hairs [should] suspend equal weights. When the weight are light some hairs break, [it is because the weight distributed among all] the hairs is not equal. [If the weight] were equal, then the hairs would not have broken.³⁷

The situation in the chapter *Tang wen* 湯問 of *Liezi* 列子 is also the same as in the *Mohist Canon*:

2.17 均，天下之至理也，連於形物亦然。均髮均縣輕重。而髮絕，髮不均也。均也，其絕也莫絕。³⁸

Equality is the highest principle under the heaven. This is also true for things having form. Equal hairs [should] suspend equal weights. But if some hairs break, [it is because the weight distributed among all] the hairs is not equal. [If the weight] were equal, then the hairs would not have broken.

(From chapter *Tang Wen of Lie Zi* 《列子·湯問》)³⁹

The *Mohist Canon* discusses the ascent and descent of a device that is possibly a lever:

2.18 [經]：挈與收反，說在權（？）⁴⁰

[經說]：挈，有力也；引，無力也。繩制挈之也，若以錐刺之。挈：長重者下，短輕者上；上者愈得，下者愈亡；繩直，權重相若，則正矣。不正，所挈之止於施也收：上者愈喪，下者愈得，上者權重盡，則遂。⁴¹

34 *Mo Zi, Dao Zang* Edition. 1924. Vol. 10, 3a.

35 *Mo Zi, Dao Zang* Edition. 1924. Vol. 10, 14b. In *Mo Zi* 墨子 [Master Mo] (5th-3rd century B.C., by Mo Di 墨翟 who lived between 460's B.C. and 380's B.C. and his believers), the chapter 40 *Jing Shang* 經上 [Former Part of Canons], chapter 41 *Jing Xia* 經下 [Latter Part of Canons], chapter 42 *Jing Shuo Shang* 經說上 [Explanation on the Former Part of Canons], chapter 43 *Jing Shuo Xia* 經說下 [Explanation on the Latter Part of Canons], are called together *Mojing* 墨經 [Mohist Canons]. *Mohist Canons* consists of a number of relatively independent sections. The latter two chapters are respectively the explanations of the former two chapters. The Explanation and the Canon of the same section have one or a few similar Characters. Contrasting with the similar sentence in the *Lie Zi*, we know that the first character *jun* 均 of the Explanation is not just a mark that the later sentence is the explanation of the corresponding canon with the same first character, but should be read together with the later characters as well.

36 Refers to Graham, A. C., *Later Mohist Logic, Ethics and Science*, Hong Kong: Chinese University Press, 1978. pp. 420-421.

37 Refers to Graham, A. C., *Later Mohist Logic, Ethics and Science*, Hong Kong: Chinese University Press, 1978. pp. 420-421. But our translation has some differences.

38 Yang Bojun 楊伯峻, *Lie Zi Jishi* 列子集釋 [Collected Explanations on Master Lie]. Beijing: Zhonghua Shuju, 1991. pp. 171-173.

39 Graham gave a translation of these sentences. See: Graham, A. C., *The Book of Lieh-tzŭ*, New York: Grove Press Inc. 1960. P105. But my understanding has some differences.

40 *Mo Zi, Dao Zang* Edition. Vol. 10, p. 4a. The *quan* 權 is original written as *bo* 薄, which makes the context incomprehensible. Sun Yirang 孫詒讓 thought it should be revised as *quan* 權, because the Explanation text does not contains the meanings of *bo* 薄. See: Sun Yirang 孫詒讓 (1848-1908), *Mo Zi Jiangu* 墨子問詁 [Explanations on Mo Zi], *Zhuzi Jicheng* 諸子集成 [Collected Books of Various Schools] Edition published by Shijie Shuju in 1937. Photocopied by Shanghai Shudian, 1992. p. 200. His opinion is reasonable. From the sentence pattern of the *Canon* text, the word after *shuo zai* 說在 (the Explanation lies on) should be the key word of the explanation. Instead of *bo* 薄, the key word in the *Explanation* text is *quan* 權.

41 *Mo Zi, Dao Zang* Edition. Vol.10, p. 11b. The above text is primarily on the basis of Zhang Huiyan and Qian Baocong. See: 1. Zhang Huiyan 張惠言 (1761—1802), *Mo Zi Jing Shuo Jie* 墨子經說解 [Explanation of the Chapters Canons and Canon Explanations in Mo Zi]. Collected in the *Congshu Jicheng Xubian* 叢書集成續編 [Second Collection of Series of Books]. Shanghai Shudian Press, 1994. pp. 903-927. 2. Qian Baocong, *Mojing Lixue Jinshi*.

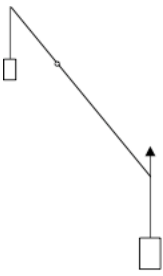


Figure 1a



Figure 1b

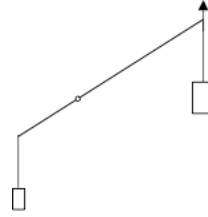


Figure 1c



Figure 1d

[Canon] To pull (*qie* 挈) and to let go (*shou* 收) are opposites which can be explained through *quan* 權 (?).

[Explanation] Pulling [something] up (*qie* 挈) requires *li* 力 (force) ; letting [something] fall (*yin* 引) does not require *li* 力 (force). Using a rope [to draw a lever] for pulling [something] up, is like using an awl to pierce [something]. To pull up (*qie*): when the long and heavy [side of a lever] falls the short and light side goes up. The upper [side] gains [*quan* 權] and the lower side loses [*quan* (effectiveness)] more and more (figure 1a). [When] the rope is perpendicular [to the lever], the amount of *quan* [on both sides] is equal; then [the lever] is is balanced (figure 1b). [Proceeding to pull the lever up, the lever] is not balanced (figure 1c), what is being pulled up will stop at an incline. To let fall: The side going up loses [*quan*], the more the side going down gains [*quan*]. [When] the amount of the *quan* of the side going up is exhausted, then [the process] will be finished (figure 1d).

Here only the first sentence of the *Explanation* mentions *li* 力: the ascending and descending of the device should be related to *li* 力. But even though the *Mohist Canon* gives a definition of the concept *li* 力, this paragraph does not use it to describe or explain the cause of the movement of the device, and does not say anything about the magnitude of the *li* 力 or how *li* 力 is applied. Instead of *li* 力, the quoted text uses three verbs, *qie* 挈 (pulling up), *yin* 引 (letting go down), and *shou* 收 (letting go down) which only express specific actions. In fact, the use of specific terms instead of the more general *li* 力 can often be observed also in other documents.

In the Warring States, there were some scholars who were intensively concerned with natural phenomena and with logic. Reports about their work indicate that they raised theoretical questions and may even have developed theories to answer them. But only very few sources that document these questions and theories have been preserved. In particular, it is unknown whether any of these theories was based on *li* 力 and other concepts in order to explain mechanical phenomena. The meanings of the term *li* 力 (1)-(3), (5)-(7) listed at the beginning are all related to mechanical notions, but studied as isolated terms this is not immediately revealed. Therefore, later, in the following three sections of the paper, the relations between *li* 力 and other concepts and notions will be discussed.

Summarizing the above discussion indicates that the Pre-Qin scholars used *li* 力 with a wide range of meanings related to mechanical phenomena. But according to my thorough examination of extant Pre-Qin literature, it seems unlikely that any scholar of that time deliberately used *li* 力 as a universal concept to explain mechanical phenomena. In contrast to the use of *li* 力 in that period, however, a conceptual change in the literature of the following Han Dynasties is indicated by the fact that *li* 力 is more frequently used as a concept to explain motion and change.

In *Huainan Zi* 淮南子 (139 B.C., written by dependants of the King Huainan 淮南王) *li* 力 appears 144 times. Several passages contain *li* 力 as a concept concerning mechanical knowledge. For example:

2.19 故積力之所舉，則無不勝也；衆智之所爲，則無不成也。.....夫舉重鼎者，力少而不能勝也，及至其移徙之，不待其多力者。(《淮南子·卷九·主術訓》)⁴²

42 Zhang Shuangdi 張雙棣, *Huainan Zi Jiaoshi* 淮南子校釋 [Collations and Explanations of Master Huainan]. Beijing: Beijing University Press, 1997. p. 912.

Therefore, with the collective *li* 力 (strength) [of many people] to lift something, there is nothing [they] cannot overcome; with the collective knowledge of many people to do something, there is nothing they cannot complete. ... When lifting a heavy vessel, if [a person's] *li* 力 (strength) is small then he will not be able to succeed; [but] to just move it does not require much *li* 力 (strength).

2.20 輻立三十輻，各盡其力，不得相害。使一輻獨入，衆輻皆棄，豈能致千里哉！（《淮南子·卷十七·說林訓》）⁴³

The hub [of a wheel] is set up with thirty spokes, each spoke exerts its *li* 力 (force), and do not negate each other. If there is only one spoke in [the hub], and all the other spokes are lost, how would [the vehicle] be able to reach a thousand *li* 里？

A wheel with thirty spokes is also mentioned in *Wen Zi* 文子 [Master Wen] and *Lao Zi* 老子 [Master Lao]. It is a controversial question whether or not *Huainan Zi* is earlier than *Wen Zi*, but *Lao Zi* was written before the middle of 4th century B.C., much earlier than *Huainan Zi*. The graph *li* 力 appears in *Wen Zi*, but does not appear in *Lao Zi*. This may indicate that *li* 力 was more frequently used in later times.

2.21 夫矢之所以射遠貫堅者，弩力也；其所以中的剖微者，人心也。（《淮南子·卷二十·泰族訓》）⁴⁴

What an arrow uses to shoot to a distant place and go through a solid is the *li* 力 (force) of a crossbow; what it uses to hit the target and split the exiguity is a person's mind.

(From chapter *Taizu Xun of Huainan Zi*)

The following is an example from *Shi Ji* 史記:

2.22 (韓)安國曰：“.....且彊弩之極，矢不能穿魯縞；衝風之末，力不能漂鴻毛。非初不勁，末力衰也.....”⁴⁵《史記·卷一百八·韓長孺列傳》

[Han] Anguo 安國 (韓安國，?-127 B.C.) said: "... Moreover, at the end of [the flight of] a strong crossbow arrow, the arrow is not able to puncture the silk of Lu⁴⁶; at the end of the charging wind, [its] *li* 力 (force) is not able to float a goose feather. [This] is not [because] they are not strong at first, [but because] their *li* 力 at the end is decreased [to its weakest extent]."⁴⁷

(From chapter *Han Changru Liezhuan of Shiji, Biograph of Han Changru of Records of History*)

In Eastern Han Dynasty, Wang Chong 王充 (27-97 A.D.) wrote a book *Lun Heng*, in which *li* 力 frequently appears, namely 265 times. In this book virtually everything (bone 骨, tendon 筋, hand 手, ear 耳, eye 目, people 人, talent 才, cattle 牛, ant 蟻, medicine 藥, bow 弓 and crossbow 弩, arrow 矢, stick 杖, land 地, mountain 山, pen and ink 筆墨, etc) can have *li* 力.

For example, Wang Chong took *li* 力 (force) of a bow to refute the story that at the time of ancient Emperor Yao ten suns rose simultaneously and Yao shot down nine of them:

2.23 夫人之射也，不過百步，矢力盡矣。日之行也，行天星度，天之去人，以萬里數，堯上射之，安能得日？使堯之時，天地相近，不過百步，則堯射日，矢能及之；過百步，不能得也。⁴⁸

When a person shoots [an arrow], [it] will not reach more than 100 *bu* (步, a unit of distance) [by the time] the arrow's *li* 力 is exhausted. The sun moves by heaven's measurement of the stars. The

43 Zhang Shuangdi, *Huainan Zi Jiaoshi*. 1997. p. 1787. The three characters *xu er zhong* 虛而中 (empty and middle) are added according to the opinion of Yu Yue 俞樾 (1821-1907), which is quoted in *Huainan Zi Jiaoshi* by Zhang Shuangdi.

44 Zhang Shuangdi, *Huainan Zi Jiaoshi*. 1997. p. 2045.

45 Sima Qian 司馬遷 (145 or 135 B.C.- early period of the first century B.C.), *Shi Ji* 史記 [Records of History]. Beijing: Zhonghua Shuju, 1985. p. 2861.

46 The silk of Lu was very thin.

47 A free translation was presented by Watson, Burton in *Records of the Grand Historian of China: Translated from the Shih-chi of Ssu-ma Ch'ien*. Vol. II. New York and London: Columbia University Press, 1961. pp. 135-136.

48 Huang Hui 黃暉, *Lun Heng Jiaoshi* 論衡校釋 [Collations and Explanations of Discourses Weighed in the Balance]. Beijing: Zhonghua Shuju, 1995. pp. 227-228.

distance from heaven to a person, is counted in 10000 *li* 里. [When] Yao shot upwards it, how could [the arrow] reach the sun? If in the time of Yao, the heaven and the earth were near and [the distance] were not more than 100 *bu*, then [when] Yao shot the sun the arrow could reach it. [If the distance from the heaven to the earth] was more than 100 *bu*, [then the arrow] was not able to reach [the sun].⁴⁹

Furthermore, Wang Chong deliberately used *li* 力 as a universal concept to explain mechanical phenomena, as well as mental and social phenomena. He wrote a treatise entitled *Xiao Li* 效力 (investigating *li* 力) in which not only physical strength but also intelligence, talent, and ability are expressed by *li* 力. For example:

2.24 或伐薪於山，輕小之木，合能束之。至於大木十圍以上，引之不能動，推之不能移，則委之於山林，收所束之小木而歸。由斯以論，知能之大者，其猶十圍以上木也，人力不能舉薦，其猶薪者不能推引大木也。⁵⁰

Somebody cut firewood on a mountain. The light and small trees could be bound by enclosing [them with hands]. As for a big tree of more than ten *wei* 圍⁵¹, [he] pulled it [but] was not able to move [it]; [he] pushed it [but] was not able to shift [it]; then [he] abandoned it in the mountain forest and collected the small trees that he had bound, and went back. With this as a basis for discussion, a person of great intelligence and ability is like a tree bigger than ten *wei*, a man whose *li* 力 (ability) is not able to recommend [a person of great ability to the court], is like a lumberjack who is not able to push or pull a big tree.

Wang Chong considered the strength for moving trees and the ability of intelligence both to be *li* 力, and he thought that an ordinary man who was not able to recommend a man of great ability to the court can be compared with a lumberjack whose strength was not great enough to move a large tree. Thus, using a term that is usually used to designate physical strength, he explained a mental and social phenomenon. In the chapter *Xiao Li*, Wang Chong's intention is to discuss intelligence and to demonstrate that physical strength (筋骨之力, *li* 力 of tendon and bone) is not as powerful as virtue and morality (benevolence and rectitude), he does not go into detail as concerns physical strength.

Later, Zheng Xuan 鄭玄 (127-200 A.D.) discussed the force of a bow in a quantitative way when he commented on the passage of *Kao Gong Ji* 考工記 (Critical records on handicrafts) on making a bow. The passage in *Kao Gong Ji* reads:

2.25 材美，工巧，爲之時，謂之參均。角不勝幹，幹不勝筋，謂之參均。量其力有三均。均者三，謂之九和。九和之弓，角與幹權，筋三侷，膠三鈺，絲三邸，漆三斛。⁵²

[When making a bow if] the materials are excellent, the workmanship skillful and the production at the proper time, this is called tripart balance (*san jun* 參均). The inner part of the bow made of horn (*jiao* 角) should not surpass the middle part made of wood (*gan* 幹); the wood part (*gan* 幹) should not surpass the outer part made of tendon (*jin* 筋)⁵³; this is [also] called tripart balance (*san jun* 參均).

Measuring the *li* 力 (force) of a bow also has tripart balance (*san jun* 三均). [Hence] there are three tripart balances [collectively] called the nine harmonies (*jiu he* 九和). [In] a bow of nine harmonies the part made of horn (*jiao*) is aligned with (*quan* 權) the part made of wood (*gan*); there are three *mou* 侷 of tendon, three *lue* 鈺 of glue, three *di* 邸 of silk, [and] three *yu* 斛 of lacquer.⁵⁴

Zheng Xuan (鄭玄, 127-200 A.D.) commented this passage as follows:

49 Re. Alfred Forke (tr.), *Lun Heng* part II. New York: Paragon Book Gallery, 1962. p. 171.

50 Huang Hui, *Lun Heng Jiaoshi*. Beijing: Zhonghua Shuju, 1995. p. 585.

51 *Wei* 圍 is unit for measuring circular bodies. It means hand span or arm span.

52 Sun Yirang 孫詒讓 (1848-1908), *Zhouli Zhengyi* 周禮正義 [Proper Meanings of Rites and institutions of Zhou Dynasty]. Beijing: Zhonghua Shuju, 1987. pp. 3557-3559.

53 *Sheng* 勝 means surpass. It has a broad connotation. Yang Fuxi 楊福喜, a craftsman making traditional bow said, in a bow the *jiao* is not longer than *gan* in length, and the *gan* is not longer than the *jin*. He did not know the description in *Kao Gong Ji*. (According to a telephone talk between Prof. Zhang Baichun and Yang Fuxi on August 9, 2006).

54 *mou* 侷, *lue* 鈺, *di* 邸, *yu* 斛 are all units of measurements. The magnitudes of *mou* 侷, *lue* 鈺 and *di* 邸, are unclear. Wen Renjun 聞人軍 deduced that *yu* 斛 is a little less than 3.6 ml. See: Wen Renjun 聞人軍, *Kao Gong Ji Yizhu* 考工記譯注 [Commentary and Translation of *Kao Gong Ji*]. Shanghai: Shanghai Guji Chubanshe, 1993. p. 110.

2.26 “有三” 讀為 “又參” 。“量其力又參均” 者，謂若幹勝一石，加角而勝二石，被筋而勝三石，引之中三尺。假令弓力勝三石，引之中三尺，弛其弦，以繩緩擐之，每加物一石，則張一尺。

The words “having three” (*you san* 有三) should be read as “also three” (*you san* 又參). “Measuring the *li* 力 (force) of a bow also has tripart balance (*san jun* 參均)” (量其力又參均) means that if the wood part of a bow is able to bear one *dan*, adding horn [the bow] will be able to bear two *dan*; and covering it with tendon [the bow] will bear three *dan*; when drawn [the distance] between [the bow and its string] is three *chi*. Supposing that the *li* 力 (force) of a bow can bear three *dan* when it is drawn to three *chi*. Now slacken the bowstring. Using a rope tied to the bowstring, every time something weighing one *dan* is added the bow is drawn one *chi* wider.

In the *Kao Gong Ji*, the *li* 力 of a bow is not clearly described in quantitative ways while Zheng Xuan gives a quantitative description of the relation between the *li* 力 of a bow and the extent to which it is drawn. He assumes that the force required to draw the bow is proportional to the distance in the middle between the bow and the bowstring when drawn. This assumption is not in accordance with the facts. It provides an example of a combination of the mathematical notion of proportionality with the intuitive knowledge about the drawing of a bow and the force with which it shoots the arrow. *Jun* 均 means balance, harmony, equality. In *Kao Gong Ji*, the former two *jun* mean balance, harmony, but it is not clear whether the latter two *jun* just mean balance, harmony or mean equality as well, but Zheng Xuan understood the latter two as equality. Combining the intuitive knowledge that the more open a bow is drawn, the more *li* 力 is needed, he actually produced a theoretical proposition that the extent of a bow’s opening is proportional to the force that is needed to draw the chord of the bow, though he did not give a universal expression of this proposition.

In the above examples, the *li* 力 of a bow or a crossbow, the *li* 力 of a flying arrow, the *li* 力 being used to lift or move something, the *li* 力 of liquid for floating something, the *li* 力 of the wind, the *li* 力 of water making the current go smoothly, the *li* 力 of a carriage-crossbar, etc., are related to measurable mechanical phenomena. By contrast, the *li* 力 of ears or eyes, the *li* 力 of land, which was first formulated before the Qin Dynasty and further cultivated during the Han Dynasty, the *li* 力 of medicine, the *li* 力 of pen and ink, the *li* 力 of people, etc., are not closely related to measurable mechanical phenomena.

However, the ancient Chinese did not always distinguish these two kinds of meaning. Wang Chong, for instance, deliberately used *li* 力 as a general concept not only for explaining mechanical phenomena, but also for expressing various other kinds of ability or power, although even in these cases the use of the term may have some mechanical connotations. Therefore, even the study of these other usages of the term *li* 力 can help reconstruct the mechanical thinking in ancient China.

In the case of the first occurrence of the term *li* 力 in quotation 2.15, the force is exerted by the horse’s muscle, and the objects on which it is exerted are the weight that is carried or the carriage that is drawn. In the case of the use of *li* 力 in quotation 2.19, the objects on which the force is exerted are the heavy vessel or some other heavy objects, and that what exerts the force is a person. In the case of the occurrence of *li* 力 in quotation 2.21, what exerts the force is a crossbow while the object on which the force is exerted is the arrow and the pierced target which in the present case is a solid body. In quotation 2.23, Wang Chong argued that when a person shoots an arrow, the arrow will not reach more than 100 *bu* 步 because then the arrow’s *li* 力 is exhausted. He further argued that for the same reason, if the distance from the heaven to the earth were more than 100 *bu*, then an arrow would not be able to reach the sun. These arguments indicate that Wang Chong believed that as long as an arrow travels, there must always be *li* 力 to make the arrow move, and that when the *li* 力 disappears, the arrow will stop.

These implicit assumptions in the way the term *li* 力 was used reflect a conceptual structure of the notion of force which seems to be a universal component of the intuitive mechanical knowledge, which is universally acquired by any human being. An agent achieves an effect by directly applying a force or by transferring a force to an object which is then capable of achieving the effect as the agent itself or himself. However, the given examples of texts documenting the use of the term *li* 力 in ancient China do not show any attempt to formulate the relation between the actor, the force, and the effect in a generalized way, let alone that they document any attempt to make explicit the geometrical relations

between a force, where it is applied, and the resulting effect. The examples as well as further ones discussed below are typical for ancient Chinese sources using the term *li* 力.

3. *Li* 力 (Force) and *Zhong* 重 (Weight)

A further clarification of the implicit assumptions of the use of the term *li* 力 in ancient China can be achieved by investigating its relation to other semantically close terms. In particular, it is useful to study the relation between weight and force.

In ancient Chinese, weight is usually designated by the term *zhong* 重. The term is used in several ways. It may be used as a noun designating weight in a general sense. As an adjective it mostly indicates that something heavy. It sometimes also designates a heavy object. As far as it is known, there is no definition of the term in the ancient Chinese literature, but the *Mohist Canon* gives a good description that reflects in a general way its intuitive meaning:

3.1 凡重，上弗挈，下弗收，旁弗劫，則下直。地，或害之也。⁵⁵

Speaking generally about *zhong* 重 (weight), if not pulling from above, not drawing back from below, or not controlling from the side, then [it] falls straight down. If [it falls] obliquely, [it is because] there is something interfering with it.⁵⁶

(From chapter *Jing Shuo Xia* of Mo Zi 《墨子·經說下》)

This passage which is part of an explanation of a complex mechanical device concerns the motion of a heavy body which is falling down. Ancient craftsmen used *zhun* 準 (a leveling instrument) to determine the horizontal and *sheng* 繩 (a rope-made instrument) to determine the perpendicular, which could be regarded as the experiential background of Mohists's understanding of perpendicularity.

In the above quotation, the actions of *qie* 挈, *shou* 收 and *yin* 引 require force, but actually the term *li* 力 is not used to describe the interference of these actions with weight. It seems that the ancient Chinese did not try to determine the precise relations in such interferences between *li* 力 and *zhong* 重, though in the *Mohist Canon* it reads:

3.2 [經]：力，刑之所以奮也。（經上第四十）⁵⁷

[經說] 力，重之謂，下與重，奮也（經說上第四十二）⁵⁸

[Canon] *Li* 力 (strength) is that by which the body exerts [itself].

[Explanation] *Li* 力 (strength), refers to *zhong* 重 (weight). Lifting a *zhong* (weight) from below is exertion.

(From chapters *Jing Shang* and *Jing Shuo Shang* of *Mohist Canons* 《墨經·經上》、《墨經·經說上》)⁵⁹

55 *Mo Zi. Dao Zang* Edition. 1924. Vol. 10, 12a.

56 Here we modify the translation of William G. Boltz, Jürgen Renn, and Matthias Schemmel, *Mechanics in the Mohist Canon and Its European Counterpart: Texts and Contexts*, Preprint 241 of Max Planck Institute for the History of Science, 2003; see also chapter 4 in this volume.

57 *Mo Zi. Dao Zang* Edition. 1924. Vol. 10, 1b.

58 *Mo Zi. Dao Zang* Edition. 1924. Vol. 10, 6b. The character *fen* 奮 of the Explanation text was written as *jiu* 舊 (old, past) in the *Dao Zang* edition. But it is written as *fen* 奮 in Bi Yuan's 畢沅 *Mozi Zhu* 墨子注, Sun Yirang's *Mozi Jiangu* 墨子間詁, and Wu Yujiang's 吳毓江 *Mozi Jiaozhu* 墨子校注. However, only Wu said "in old editions the *fen* 奮 is written as *jiu* 舊." The *jiu* 舊 makes the context incomprehensible, it is proper to collate it as *fen* 奮, because they are similar in shape and the *fen* 奮 occurs in the corresponding Canon text. Graham add a character *ye* 也 after the *wei* 謂 of the Explanation. I think it is not necessary. The chapter *Xiao Xing* 孝行 of *Lü Shi Chunqiu* 呂氏春秋 said "suowei ben zhe, fei gengyun zhongzhi zhi wei, wu qi ren ye" [所謂本者，非耕耘種殖之謂，務其人也] (what I call basis, does not refer to tillage or planting, but refers to the people). Here, there is no *ye* 也 after the second *wei* 謂. Furthermore, Graham only gave two examples of this sentence pattern, of which only one originally has *ye* 也, the other is just this example in which the *ye* 也 is not the original text but was added by Graham. See: 1. Bi Yuan 畢沅 (1730-1797), *Mo Zi (Zhu)* 墨子(注) [*Master Mo (with Commentary)*]. Shanghai Shudian, 1989. p. 79. 2. Sun Yirang, *Mozi Jiangu*, 1992. p. 204. 3. Wu Yujiang 吳毓江 (1898-1977), *Mo Zi Jiaozhu* 墨子校注 [*Collations and Commentaries of Mo Zi*]. Chongqing: Xi'an Normal University Press, 1992. p. 402. 4. Graham, A. C., *Later Mohist Logic, Ethics and Science*. 1978. pp. 279, 146-147.

59 The translation is based on Qian Baocong's explanation and Graham's translation. See: 1. Qian Baocong, 'Mojing

This explanation of *li* 力 in the *Mohist Canon* asserts a close relation of the concept to *zhong* 重 without making explicit how precisely this relation was conceived. As the following example shows, this is the case also in other parts of the *Mohist Canon* which somehow concern the relation of these two concepts, but they make clear that to raise *zhong* 重 requires *li* 力.

3.3 不，舉重不與箴，非力之任也。爲握者之顛倍，非智之任也。若耳目。⁶⁰

[To decide] to lift a heavy weight (*zhong* 重) but not lift a needle, is not the duty of *li* 力 (strength) (i.e. the duty of intelligence). The fall of a person who is holding [a heavy weight (*zhong*)], is not the duty of the intelligence (i. e. the duty of *li* 力). [These] are like ears and eyes [having different functions].

(From the chapter *Jing Shuo Shang of Mohist Canons* 《墨子·經說上》)

The statement that if somebody raises a heavy load but not a needle is a matter of intelligence (*zhi* 智) and not a matter of force (*li* 力) implies that in general a force is needed to raise a weight. Similarly is this assertion implied in the following passages of about the same period.

3.4 故能小而事大，辟之是猶力之少而任重也，舍粹折無適也。⁶¹

Therefore, [a person's] having small ability but doing great things, is like having small *li* 力 (strength) but bearing a heavy weight (*zhong* 重). There are no results other than breaking his body.⁶²

(From *Xiao Ru of Xuan Zi* 《荀子·儒效》)

3.5 然則一軍之中必有虎賁之士，力輕扛鼎，足輕戎馬，拳旗取將。⁶³

Then, it is certain that there are warriors in an army corps. [Their] *li* 力 (strength) [makes them feel that] it is easy to lift a *ding* 鼎 (a kind of heavy copper vessel); [and] their feet [make them feel that] it is easy [to chase] war horses, [and they] are able to capture [the enemy's] standards and to kill [the enemy's] generals.

(From *Liao Di* chapter of *Wu Zi* 《吳子·料敵》)

3.6 且夫水之積也不厚，則其負大舟也無力。覆杯水於坳堂之上，則芥爲之舟；置杯焉則膠，水淺而舟大也。風之積也不厚，則其負大翼也無力。⁶⁴

Furthermore, [if] the accumulated water is not deep, then it will not have *li* 力 (force) to support a big ship. [If] a cup of water is poured into a depression [in the floor] of a hall, then a blade of grass [will float] like a boat [on the surface of the water]; [but if] a cup is put [on the water], then [it] will stick to [the floor] because the water is shallow but the boat is big. [If] the amassed wind is not deep, then there is no *li* 力 (force) to support the large wings [of *peng*].

(From chapter *Xiaoyao You of Zhuang Zi* 《莊子·逍遙游》)

These text passages imply the tacit proposition that the heavier a weight, the more *li* 力 is needed to raise it up and vice versa. Although in the passages 3.5 and 3.6 the word *zhong* 重 does not explicitly

Lixue Jinshi, in *Kexueshi Jikan*, No.8, 1965. pp. 65-72. 2. Graham, A. C., *Later Mohist Logic, Ethics and Science*. 1978. p. 279.

60 *Mo Zi*. *Dao Zang* Edition. 1924. Vol.10, 10a. The first *bu* 不 located before *ju* 舉, I take Liang Qichao's 梁啟超 (1873-1929) opinion to exchange them. See: Liang Qichao 梁啟超 (1873-1929), *Mojing Jiaoshi* 墨經校釋 [Collations and Explanations of Mohist Canons], collected in Yinbinshi Zhuanji 欽冰室專集 [Collected Essays of Yinbinshi]. Beijing: Zhonghua Shuju, 1989. pp. 59-60. In the part *wei wo zhe zhi dian bei* 爲握者之顛倍, the *dian* 顛 was originally written as *hong*, a character combined from the two components 角 and 頁, which is rarely seen in the literature and makes the passage *wei wo zhe zhi hong bei* 爲握者之角頁倍 difficult to understand. Wu Yujiang amended it as *dian* 顛 and read *bei* 倍 (usually means twice, with back towards, or betray), as *bo* 踏 (usually means fall), which can be accepted before a proper collation is made. See: Wu Yujiang, *Mo Zi Jiaozhu*. 1992. p. 447.

61 Liang Qixiong 梁啟雄 (1900-1965), *Xun Zi Jianshi* 荀子簡釋 [Simple Explanations of Master Xun]. Beijing: Zhonghua Shuju, 1983. p. 86.

62 A free translation was presented by John Knoblock in *Xun Tzu A Translation and Study of the Complete Works* Vol. II. Stanford, California: Stanford University Press, 1990. p. 75.

63 *Wu Zi* 吳子 [Master Zi], in *Zhuzi Jicheng* 諸子集成 [Collected Books of Various Schools] Edition published by Shijie Shuju in 1937. Photocopied by Shanghai Shudian. 1991. p. 4.

64 Guo Qingfan, *Zhuang Zi Jishi*. 1991. p. 4.

appear in the text, it seems evident that objects such as a boat, a *ding* 鼎, a big ship, and large wings can all be considered as heavy so that they need great *li* 力 to be raised up. Furthermore, the text passage 3.6 argues that the deeper the layer of liquid, the more *li* 力 the liquid has and the heavier loads it can bear.

The results of the investigation of the use of the term *li* 力 in early sources presented so far can be summarized in the following way. Human beings perceived and knew their body's physical strength, their *li* 力, and perceived the weight (*zhong* 重) of an object by using their strength to lift or to move it. So the first item of the list of meanings of *li* 力 given at the beginning is the basis of human perception of weight. Humans are easily able to distinguish between very strong *li* 力 and very weak *li* 力 by the effect of their actions. But when the differences between the efforts to produce certain effects are small, i.e. two *li* 力 are near in magnitude, they cannot easily be distinguished. It is difficult to construct a metrological standard of *li* 力 solely on the basis of such human perceptions. However, weights can easily be measured by balances and standard weights. The intuitive notion "more *li* 力 can raise up more weight" and the reverse notion "the more heavy a weight is, the more *li* 力 is needed to raise it up" constitute a correspondence between force (*li* 力) and weight (*zhong* 重) so that *li* 力 becomes measurable by standards of weight. Accordingly, we can find many examples documenting that the ancient Chinese related *li* 力 to *zhong* 重 or to something expressing *zhong* 重, *li* 力 not only conceived as human physical strength but also in a more general way as power or force. We can, in particular, find that *li* 力 was measured by the units of weight such as *jun* 鈞, *dan* 石, *jin* 斤, and *liang* 兩 (1 *jun* 鈞 = 30 *jin* 斤, 1 *dan* 石 = 4 *jun* 鈞 = 120 *jin* 斤, 1 *jin* 斤 = 16 *liang* 兩).

3.7 [孟子] 曰：“有復於王者曰：‘吾力足以舉百鈞，而不足以舉一羽；明足以察秋毫之末，而不見輿薪’。則王許之乎？”曰：“否。”⁶⁵

[Mencius] said: “[If] there were a person who said to the king: ‘My *li* 力 (strength) is enough to raise up one hundred *jun* 鈞, but is not enough to raise up a feather; my eyesight is enough to scrutinize the tip of [a new] down in autumn, but is not enough to see a cartload of firewood.’ Would the king believe it?” [The king] said: “No.”⁶⁶

(From chapter *Liang Hui Wang Shang of Mencius* 《孟子·梁惠王上》)

In this text passage, 100 *jun* 鈞 denotes a quantified weight that was used to describe *li* 力.

3.8 [蘇代] 對曰：“……今夫烏獲舉千鈞之重，行年八十而求扶持。”⁶⁷

[Su Dai 蘇代] replied: “…Now [though] Wu Huo 烏獲 can lift a weight of one thousand *jun* 鈞, but at age of eighty, [he] needs to be supported [by others].”⁶⁸

(Chapter 29 of *Zhangguo Ce* 《戰國策》卷二十九).

The term 1000 *jun* 鈞 is used here to describe the warrior's *li* 力, while the in following text passage the term 1000 *jun* 鈞 is used to describe a horse's *li* 力.

3.9 今為馬多力則有矣，若曰勝千鈞則不然者，何也？夫千鈞非馬之任也。⁶⁹

If [you] said that a horse had much *li* 力 (strength), then it would be a fact. But if [you] said [a horse] could bear one thousand *jun* 鈞, then it would be incorrect. Why? One thousand *jun* 鈞 is not what a horse is able to bear.

(From chapter 17 of *Zhangguo Ce* (Strategy of Warring States) 《戰國策》卷十七)

In the following text passage, Tai Gong 太公 (11th century B.C.) used another unit of weight to describe *li* 力.

65 Yang Bojun, *Meng Zi Yizhu*. 1962. p. 15.

66 D. C. Lau presented a free translation. See: Lau, D. C., *Mencius: A Bilingual Edition*. Hong Kong: the Chinese University Press, 2003. p. 17.

67 Zhang Qingchang 張清常 (1915-1998), Wang Yandong 王延棟, *Zhangguo Ce Jianzhu* 戰國策箋注 [Commentaries of Strategies of the Warring States]. Tianjin: Nankai University Press, 1993. p. 786.

68 A free translation was presented by J. I. Crump, Jr. in his *Chan-Kuo Ts'e*. p. 530.

69 Zhang Qingchang, Wang Yandong, *Zhangguo Ce Jianzhu*. 1993. p. 412.

3.10 太公曰：“選車士之法：取年四十以下，長七尺五寸以上，走能逐奔馬，及馳而乘之，前後左右、上下周旋，能束縛旌旗；力能彀八石弩，射前後左右，皆便習者，名曰武車之士，不可不厚也。”⁷⁰

Tai Gong 太公 (11th century B.C.) said: “the method for selecting chariot-soldiers: select [the soldiers who] are younger than 40 years old, taller than 7 *chi* 尺 5 *cun* 寸, and when running are able to chase galloping horses; when riding on the running horses, [they are able to] contend with [the horses by various ways like] moving forward, backward, left, or right, and are able to bind up banners and flags; [their] *li* 力 (strength) is able to draw a crossbow of 8 *dan* 石 to the full extent and shoot fore, rear, left or right [targets]. The soldiers who acquire all these skills, are named powerful chariot-soldiers, and must be given special treatment.

(From chapter *Quan Tao* of *Liu Tao* 《六韜·犬韜》)

In this text the *li* 力 of soldiers is characterized by saying they are able to draw a crossbow of 8 *dan* 石 to the full (力能彀八石弩). The unit *dan* 石, however, is a unit of weight. It is used here to describe a crossbow's force and accordingly to describe a soldier's *li* 力.

The intuitive understanding of the relation between *li* 力 and *zhong* 重 documented by such texts had also an influence on the ancient Chinese theoretical and philosophical literature. As has been mentioned already at the beginning of this section, the *Mohist Canon* makes this relation explicit as a theoretical statement, albeit without allowing to reconstruct the precise meaning of this relation from the texts that survived.

One of the problems in the *Jiuzhang suanshu* 九章算術 [*Nine Chapters of Mathematical Procedures*] provides another example referring to the relation between *li* 力 and *zhong* 重. In this problem, horses drag a load up a slope, the success depending on the force of the horses.

3.11 今有武馬一匹，中馬二匹，下馬三匹，皆載四十石至坂，皆不能上。武馬借中馬一匹，中馬借下馬一匹，下馬借武馬一匹，乃皆上。問武、中、下馬一匹各力引几何？答曰：武馬一匹力引二十二石、七分石之六，中馬一匹力引十七石、七分石之一，下馬一匹馬引五石、七分石之五。術曰：如方程，各置所借，以正負術入之。⁷¹

Now given that there is one strong horse, [a team of] two average horses and [a team of] three weak horses. Each [of these three teams] carts 40 *dan*, to a slope, which none is able to ascend. If the strong horse is teamed with one average horse, or the [two] average horses are teamed with one weak horse; or the [three] weak horses are teamed with the strong horse, then all [the teams] would be able to ascend [the slope]. Question: How much [weight] can the *li* 力 (strength) of a strong horse, or an average horse, or a weak horse draw?

Answer: the *li* 力 (strength) of a strong horse can draw [a weight of] 22 and 6/7 *dan*; the *li* 力 (strength) of an average horse can draw [a weight of] 17 and 1/7 *dan*; and the *li* 力 (strength) of a weak horse can draw [a weight of] 5 and 5/7 *dan*.

70 He Zhihua 何志華, *Liu Tao Zhuji Suoyin* 六韜逐字索引 [*Index for Every Character of Six Military Strategies*]. Hong Kong: Shangwu Yinshuguan's Hong Kong Ltd., 1997. p. 45.

71 Guo Shuchun 郭书春, *Jiuzhang Suanshu Huijiao* 《九章算術》彙校 [*Nine Chapters on Mathematical Procedures, with Collations based on all kinds of Editions*]. Shenyang: Liaoning Education Press, 1990. pp. 394-395.

Procedure: According to the *fangcheng* [method]⁷², lay down [counting rods representing] the teams, and use the method of positive/negative to solve [the problem].

(From the *Fangcheng* chapter of the *Nine Chapters of Mathematical Procedures* 《九章算術·方程》)

As in the text passage 2.19 discussed above, the problem implicitly assumes a relation between the number of humans or animals supplying their *li* 力 and the resulting effect. In the present problem it is assumed that raising up and drawing a weight needs different *li* 力 and that therefore under different conditions the same *li* 力 can move different weights. In fact, dragging heavy bodies up a slope is related to several factors, especially the weight, the magnitude and the directions of the *li* 力, and the gradient of the slope. Such knowledge was surely part of the intuitive knowledge of practitioners of the time. The abstraction process leading from such knowledge to the problem of the *Jiuzhang suanshu*, however, reduces the relation to the tacit assumption that on the same slope the magnitude of *li* 力 is proportional to the *zhong* 重 (weight) needed to drag it up the slope.

In the philosophical literature reflecting the relation between force and weight such simple assumptions are called in question because they seem to be in conflict with actual or assumed experiences such as the legendary force of the warrior Wu Huo 烏獲 and his alleged inability to lift his own body up.

3.12 孔子曰：“由志之，吾語女：雖有國士之力，不能自舉其身，非無力也，勢不可也。...”⁷³

Confucius said: “You 由 remember it, I tell you: even if [a person] has the *li* 力 (strength) of a national warrior, [he] is not able to raise up his own body. [This] is not because he does not have [enough] *li* 力 (strength), [but because] the *shi* 勢 (posture) [makes it] impossible....”(From *Zi Dao* chapter of *Xun Zi* 《荀子·子道》)

3.13 天下有信數三：一曰智有所有不能立，二曰力有所有不能舉，三曰彊有所有不能勝。故雖有堯之智，而無衆人之助，大功不立。有烏獲之勁，而不得人助，不能自舉；有賁、育之強，而無法術，不得長生。故勢有不可得，事有不可成。故烏獲輕千鈞而重其身，非其身重于千鈞也，勢不便也；離朱易百步而難眉睫，非百步近而眉睫遠也，道不可也。故明主不窮烏獲以其不能自舉，不困離朱以其不能自見。因可勢，求易道，故用力寡而功名立。⁷⁴

Under heaven there are three believable principles: the first is that there is something that intelligence is not able to establish, the second is that there is something that *li* 力 (strength) is not able to raise up, the third is that there is something that sturdiness is not able to overcome. Therefore, [even a person who] has the same intelligence as that of Yao 堯, but [if he] does not have help of others, [he] will not be able to establish great gong 功 (exploits). [Even a person] has the same *jing* 勁 (strength) as that of Wu Huo 烏獲, but [if he] does not get others' help, [then he] will not be able to raise himself up. [Even a person] has the same sturdiness as that of Ben 賁 or Yu 育, but if [he] has no *fa shu* (法術, law and tactics), [then he] will not be able to live a long life. Therefore, there are unreachable *shi* 勢 (posture), there are unattainable things. Therefore Wu Huo 烏獲 [felt that]

72 “*fangcheng* 方程” is an important method of ancient Chinese mathematics to solve problems through a matrix-like expression and its operation. It is quite similar to today's extended matrix method or system linear equations. For this problem, the ancient set up a *fangcheng* 方程 as:

$$\begin{array}{r} 1 \quad 1 \\ \quad 2 \quad 1 \\ \quad \quad 3 \quad 1 \\ 40 \quad 40 \quad 40 \end{array}$$

They used to multiple each number of one row with one of the three former numbers of another row, then they extracted the later row from the former for several times. Finally, they made every row only contain two numbers, of which one is the last one, and they got:

$$\begin{array}{r} 7 \\ \quad 7 \\ \quad \quad 7 \\ 40 \quad 120 \quad 160 \end{array}$$

73 Liang Qixiong, *Xun Zi Jianshi*. 1983. p. 394.

74 Chen Qiyou 陳奇猷 (1917-), *Han Fei Zi Xin Jiaozhu* 韓非子新校注 [New Collations and Commentaries of the Master Han Fei]. Shanghai: Shanghai Guji Chubanshe, 2000. p. 522.

1000 *jun* (鈞, a unit of weight) was light but his body was heavy. [This] is not [because] his body was heavier than 1000 *jun*, [but because] the *shi* 勢 (posture) made it unsuitable [for he to do it]. Li Zhu 離朱 [felt that] it was easy [to see clearly the tip of a new down away from] 100 *bu* (步, a unit of distance) but [felt] it was difficult [to see his] eyebrows and eyelashes; [this is not because] 100 *bu* 步 was near while his eyebrows and eyelashes were distant, [but because] the Way⁷⁵ made it impossible. Therefore, a wise ruler would neither make Wu Huo embarrassed just because he was not able to raise up himself, nor make Li Zhu 離朱 abashed just because he was not able to see his own [eyebrows and eyelashes]. [He] takes advantage of practicable *shi* 勢 (circumstances, including posture) and pursue easy Way, then [he] can use less *li* 力 (ability, including strength) but establish exploits and reputation (gong ming 功名)⁷⁶.

(From the chapter *Guan Xing of Han Fei Zi* 《韓非子·觀行》)

These two passages both claim that a strong man is not able to lift himself, and provide essentially the same explanation. In order not to abolish the mental model based on the assumption of a proportionality between forces and their effect, an additional condition is introduced, designated by the term *shi* 勢 (circumstances). If the effect does not correspond to what could be expected, the *shi* 勢 is being considered not to be suitable for the application of *li* 力 (烏獲輕千鈞而重其身，非其身重于千鈞也，勢不便也). Such an explanation still implies that the greater the *li* 力 is, the more weight it can raise up. By introducing the abstract concept *shi* 勢 it is made possible that the conclusion goes beyond the assumption of a simple proportionality. *Shi* 勢 designates a complex situation, which includes all the concerning factors, and preserves the connotation of inevitability and potential. The term *shi* 勢 represents an abstract concept, which leaves open the precise meaning of the assumption that *shi* 勢 is not suitable when the *li* 力 does not yield the expected effect.

Not only Xun Zi but also other later scholars dealt with such problems in a similar way. Examples are provided by Han Ying's 韓嬰 *Han Shi Wai Zhuan* 韓詩外傳 (2nd century B.C.) who, similar to Xun Zi, refers to Confucius, and by Wang Chong (27-97 A.D.), who instead of using the concept *shi* 勢 to explain the riddle, used the problem as an example to explain that an able person must rely on others to realize his abilities.

[孔子曰：] 雖有國士之力，不能自舉其身。非無力也，勢不便也。⁷⁷

[Confucius said:] [A person] who even has the *li* 力 (strength) of a national warrior is not able to raise up his body. [This] is not because of not having [enough] *li* 力, but because that the *shi* 勢 (posture) is not suitable [for the use of *li* 力].

有人於斯，其知如京，其德如山，力重不能自稱，須人乃舉，而莫之助，抱其盛高之力，竄於閭巷之深，何時得達？鼻、育，古之多力者，身能負荷千鈞，手能決角伸鉤，使之自舉，不能離地。智能滿胸之人，宜在王闕，須三寸之舌，一尺之筆，然後自動，不能自進，進之又不能自安，須人能動，待人能安。⁷⁸

Here is a person, whose knowledge rises as high as a peak, and whose virtue is like a mountain. Though [his] *li* 力 (strength) is great, [he] is not able to raise himself up. There must be other people [to raise him] then [he] can be raised up. But if no one help him, [he] should [have to] take his vigorous *li* 力 (energy), and scuttle in the deep lanes. [Then] when will [he] be able to occupy an eminent position? Ao and Yu were two men of great *li* 力 (strength) in ancient times. Their bodies were able to carry a thousand *jun*, and their hands were able to break horns and to stretch hooks. [But if you] made them raise themselves from the ground, [they] would have been unable to detach themselves from it. Men whose bosoms are filled with wisdom and ability should [work] in the palace. They require a tongue of 3 *cun* 寸 and a brush pen of 1 *chi* 尺, and then they can assert themselves. [But] they are not able to promote themselves. [Even if they] are able promote

75 Way (*dao* 道), is a concept of Chinese philosophy, especially of Taoist School. It means way, principle, doctrine.

76 A free translation was presented by W. K. Liao in his book *The Complete Works of Han-Fei-Tzu: A Classic of Chinese Legalism*. London: Arthur Probsthain, 1959.p. 259.

77 Han Ying 韓嬰 (2nd century B.C.), *Han Shi Wai Zhuan* 韓詩外傳 [Explanations Beyond the Original Words of the Classic of Poetry Handed down by Han], Collected in *Han Wei Congshu* 漢魏叢書 [Series of Books From Han to Wei]. Changchun: Jilin University Press, 1992. p. 61.

78 Huang Hui, *Lun Heng Jiaoshi*. Beijing: Zhonghua Shuju, 1995. pp. 584-585.

themselves, [they] are still unable to keep themselves steady. [They] have to get [the help from] others to get high positions, and rely on others to keep their positions.⁷⁹

4. *Li* 力 (Force) as a Cause of Motion

It is a general human experience that forces are required to move heavy bodies. Thus it does not come as a surprise that this experience has left its traces also in the early Chinese literature. *Li* 力 is applied by a running man and a galloping horse. It is applied equally by horses and men to move a load. This close connection between force and motion sometimes even led to the connotation in Chinese literature that the death of animals and humans is nothing but the exhaustion of their *li* 力.

4.1 人有畏影惡迹而去之走者，舉足愈數而迹愈多，走愈疾而影不離身，自以爲尚遲，疾走不休，絕力而死。不知處陰以休影，處靜以息迹，愚亦甚矣！⁸⁰

There was a person who feared his shadow and hated his footprints and ran away to leave them, [but] the more frequently [he] took steps, the more footprints [he] made; [though he] ran faster and faster, the shadow did not leave his body. [However he] himself thought [this was because he ran] still too slowly, [so he] ran faster without pause, [and finally] exhausted his *li* 力 (physical strength) and died. [The person] did not know to stay under shade to avoid [his] shadow, and to be still to avoid making footmarks, what a fool he is!

(From chapter *Yu Fu of Zhuang Zi* 《莊子·漁父》)

4.2 夫馬之所以能任重引車致遠道者，以筋力也。(From chapter *Ren Zhu of Han Fei Zi* 《韓非子·人主》，see passage 2.15.)

The reason why a horse is able to carry weight and draw a carriage a long distance is that it uses [its] muscle's *li* 力 (strength).

One of the consequences derived from the intuitive knowledge of force as a cause of motion is the dependence of the space traversed by a moving body on the amount of *li* 力 that caused its motion. The first chapter of *Yan Zi Chunqiu*, for instance, suggests that the distance a man is able to run indicates his amount of *li* 力.

4.3 昔夏之衰也，有推侈、大戲；殷之衰也，有費仲、惡來，足走千里，手裂兕虎，任之以力。⁸¹

In the past, when the Xia 夏 Dynasty declined there were Tui Chi 推侈 and Da Xi 大戲; when Yin 殷 Dynasty declined there were Fei Zhong 費仲 and E Lai 惡來⁸². [They could] run one thousand *li* 里 on foot, split a *si* 兕 (a kind of rhinoceros) or tiger with their hands. [The kings] appointed them for their *li* 力 (strength).⁸³

There are other references to *li* 力 in the ancient Chinese literature which are less easy to understand. The chapter *Nei Ye* of *Guan Zi* 《管子·內業》，for instance, suggests that the application of forces to gases does not follow the same rule as that to humans, animals, and rigid bodies.

4.4 是故此氣也，不可止以力，而可安以德；不可呼以聲，而可迎以意。⁸⁴

79 Re. Alfred Forke (tr.), *Lun Heng* part II. New York: Paragon Book Gallery, 1962. p. 91.

80 Guo Qingfan, *Zhuang Zi Jishi*. 1991. p. 446.

81 Sun Xingyan 孫星衍 (1753-1818), Huang Yizhou 黃以周 (1828-1890), *Yan Zi Chunqiu* 晏子春秋 [The Story of Yan Zi]. Shanghai: Shanghai Guji Press, 1989. p. 3.

82 Tui Chi 推侈, Da Xi 大戲, Fei Zhong 費仲 and E Lai 惡來 were very strong men.

83 Re. W. Allyn Rickett (tr.), *Guanzi: political, economic, and philosophical essays from early China: a study and translation* Vol. II. Princeton, N. J.: Princeton University Press, 1998. p. 71-72.

84 Zhao Shouzheng, *Guan Zi Zhuyi*, Book II. 1987. p. 77.

Therefore this *qi* 氣⁸⁵ can not be stopped by *li* 力, but can be made tranquil by *de* 德 (virtue). [It] is not able to be summoned by one's call, but can be welcomed by *yi* 意 (intention).⁸⁶

Another example indicating a broader conceptual network into which the concept of *li* 力 is integrated is provided by the chapter *Xing Shi Jie* of *Guan Zi* 《管子·形勢解》.

4.5 造父，善馭馬者也。善視其馬，節其飲食，度量馬力，審其足走，故能取遠道而馬不罷。明主，猶造父也。善治其民，度量其力，審其技能，故立功而民不困傷。故術者，造父之所以取遠道也，主之所以立功名也。⁸⁷

Zaofu was an excellent charioteer. [He] was good at inspecting his horses, and regulating their food and drink. [He] measured the horses' *li* 力 (strength) and judged their pace. Therefore [he] was able to take distant roads without tiring the horses. The enlightened ruler is like Zaofu. [He] is skilled in governing his people. [He] measures their strength and estimates their capabilities. Therefore [he] has achievements without distressing or injuring the people. It is [by mastering their respective] techniques that Zaofu could take distant roads and the ruler can have achievement and fame.⁸⁸

The passage shows that among other factors the *li* 力 is important for making horses run. But a wise man such as Zao Fu knew that things are not so simple. His inspection of horses had further aims than just judging their *li* 力 and rulers to be wise should follow him by not basing their decisions on too simple arguments but looking also on such qualifications as *shu* 術.

A further example warning against a one-dimensional understanding of *li* 力 being the cause that determines the resulting motion is provided by the chapter *Qing Zhong Jia* of *Guan Zi* 《管子·輕重甲第八十》，which in a realistic way describes the difficulty to get a carriage moving if the conditions are not favorable for that.

4.6 管子對曰：“杠、池平之時，夫妻服輦，輕至百里。今高杠、深池，東西南北不相睹，天酸然雨，十人之力不能上；廣澤遇雨，十人之力不可得而恃，夫舍牛馬之力所無因。牛馬絕罷，而相繼死其所者相望，皮、幹、筋、角徒予人而莫之取”⁸⁹

Guan Zi 管子 (Master Guan, named Guan Zhong 管仲, ? – 645 B.C.) replied [Duke Huan of Qi State, 齊桓公]: “when the bridges and pools were flat, a husband and wife could easily drive a handcart a hundred *li* 里. Now the bridges are high and the pools are deep, [people] of the east, the west, the south and the north are not able to see each other. When it drizzles, the 力 (strength) of ten persons are not able to raise [a carriage to a bridge]. When it rains in a broad marsh, the *li* 力 (strength) of ten persons are not able to be depended on, there is nothing to be depended on except for the employment of cattle's or horses' *li* 力 (strength). The cattle and horses become extremely tired and die one after another where they are; [this] can be seen from one place to another. [Therefore] no one fetches hides, ribs, sinews and horns even for free.⁹⁰

The above examples show that the reflection of the intuitive knowledge by ancient Chinese scholars and philosophers did not lead to the outcome to generalize such intuitive knowledge that the amount of *li* 力 used to the amount of motion produced to abstract propositions. Such reflection rather led to qualifying different phenomena which seemingly support the intuitive understanding of the relation between force and motion but on closer inspection do not simply follow an abstract law.

This should warn us to interpret the theoretical insights of the *Mohist Canon* dealing with mechanical problems too narrowly from the viewpoint of the western mathematical theory of mechanics. For instance, some scholars thought that this Chinese source is close to develop the concept of acceleration

85 *Qi* 氣, is a term of Chinese philosophy. It is the ethereal substance of which everything is composed. The *qi* 氣 mentioned in this passage is a subtle *qi* 氣 named *jing* 精, which can stay in everything and make it be vital. The author discussed how to develop special techniques for obtaining and keeping this *qi* in order to achieve physical longevity and spiritual power. Re. Qiu Xigui 裘錫圭, *Jixia daoia jingqishuo de yanjiu* 稷下道家精氣說的研究 [Research on the Ideas of Jixia Taoists about Jingqi], Shanghai: Shanghai yuandong chubanshe, 1996.pp16-58.

86 Re. W. Allyn Rickett (tr.), *Guanzi: political, economic, and philosophical essays from early China: a study and translation* Vol. II. Princeton, N.J.: Princeton University Press, 1998. p. 39.

87 Zhao Shouzheng, *Guan Zi Zhuyi*, Book II. 1987. p. 181.

88 Re. W. Allyn Rickett (tr.), *Guanzi*. Vol.I. p. 70.

89 Zhao Shouzheng, *Guan Zi Zhuyi*, Book II. 1987. p. 342.

90 Re. W. Allyn Rickett (tr.), *Guanzi*. Vol.II. p. 459.

of early modern mechanics in the European tradition.⁹¹ The “definition” of *li* 力 as that by which a body is aroused to move may suggest that the modern notion that the body is accelerated is implicitly contained in the *Mohist Canon*. But looking at the mechanical parts of the *Mohist Canon* within the context of Mohist philosophy, it is difficult to find any other evidence for such a narrow interpretation.

5. *Li* 力 (Force) and *Gong* 功 (Effect)

The use of *li* 力 always produces some kind of effect or result. The examples from the ancient Chinese literature documenting that *li* 力 was considered as being able to lift weights and cause motions can be regarded as special cases of a more general notion that *li* 力 produces effects. The term *gong* 功 represents such a general concept that describes effects or results. Sometimes *gong* 功 is used to express work, achievement, contribution and exploits, which, of course, can be taken as special effects. The following examples show that the Chinese concept of *li* 力 was in fact related to the concept of *gong* 功, at some places implying that the more *li* 力 is used, the more *gong* 功 will be produced.

5.1 禹之力獻功，降省下土方。⁹²

Yu 禹’ s *li* 力 [was used to] delicate *gong* 功 (achievement) [to Emperor Yao 堯] , [so Emperor Yao 堯] dispatched [him] to survey the subordinate regions.

(From chapter *Tian Wen* of *Chu Ci* 《楚辭·天問》)

5.2 故曰：君子以德，小人以力。力者，德之役也。百姓之力，待之而後功。⁹³

Therefore [people should] say: a gentleman uses virtue [and] a petty man uses *li* 力 (physical strength). *Li* 力 (physical strength) is the servant of virtue. People’s *li* 力 (physical strength) depends on it (i.e gentlemen’s virtue) and then makes *gong* 功 (achievements)⁹⁴...

(From chapter *Fuguo* of *Xuan Zi* 《荀子·富國》)

5.3 謀成在周長，有功在力多。⁹⁵

A successful design consists in considerate and long-range plans; an attainment of *gong* 功 (achievement, effect) consists in much *li* 力 (primarily physical strength)

(From chapter *Wang Pei Jie* of *Yi Zhou Shu* 《逸周書·王佩解》)

5.4 桓公九合諸侯，不以兵車，管仲之力也。⁹⁶

Without using armies and chariots, Huan Gong 桓公 (Duke Huan of Qi State 齊桓公, fl.685 B.C. – 643 B.C.) assembled various dukes and princes nine times. [This] is Guan Zhong’s 管仲 (=Guan Zi 管子, ?-645 B.C.) *li* 力 (achievement).⁹⁷

(from *Lun Yu* 《論語》)

5.5 故國有德義未明於朝者，則不可加於尊位；功力未見於國者，則不可授以重祿；臨事不信於民者，則不可使任大官。

(《管子·立政第四》)⁹⁸

91 For example, Qian Linzhao (1906-1999), *Shi Mojing Zhong Guangxue Lixue Zhu Tiao*. 1942. Needham, Joseph. *Science and Civilisation in China*. Vol.4, Part 1, Physics. 1962. p. 19. Chen Cheng-Yih. *Early Chinese Work in Natural Science: A Re-examination of the Physics of Motion, Acoustics, Astronomy and Scientific Thoughts*. 1996. p. 9.

92 Wang Siyuan 王泗原, *Chuci Jiaoshi* 楚辭校釋 [Collations and Explanations of the Poetry of Chu]. Beijing: Renmin Jiaoju Press, 1995. p. 90.

93 Liang Qixiong, *Xun Zi Jianshi*. 1983. pp. 123-124.

94 Re. Knoblock, John, *Xun Tzu A Translation and Study of the Complete Works*. Vol. II. Stanford, California: Stanford University Press, 1990. p. 126.

95 Huang Huaixin 黃懷信, *Yizhou Shu Jiaobu Zhuyi* 逸周書校補注譯 [Collations, Complementarities, Commentaries and Translations of the Redundant books on Zhou Dynasty]. Xi’an: Xibei University Press, 1996. p. 409.

96 Yang Bojun 楊伯峻, *Lunyu Yizhu* 論語譯注 [Translation and Commentary of the Analects]. Beijing: Zhonghua Shuju, 1962. p. 158.

97 Re. Legge, James, *The Chinese Classics*, Vol. I, Hong Kong: Hong Kong University Press. 1960. p. 282.

98 Zhao Shouzheng, *Guan Zi Zhuyi*, Book I. 1987. p. 26.

Therefore, [if] there are those in the state who have not demonstrated their virtue to the court, then [they] should not be given positions of honor. [If there are] those who have not shown to the state their *gongli* 功力 (achievement, contribution), then [they] should not be given large salaries. [If there are] those who, in overseeing affairs, are not trusted by the people, then [they] should not be employed in high officers.⁹⁹

(From chapter *Li Zheng of Guan Zi*)

The merging of the terms *gong* 功 and *li* 力 in the last of the above examples make particularly clear that it is problematic to identify the two terms with modern conceptual distinctions. In modern physics, *li* 力 is used to designate the concept of force and *gong* 功 is used to designate the concept of work, which is defined by force times the distance traversed in the direction of the force. However, the ancient Chinese primarily paid attention to the general effect or result of the use of force rather than analyzing how the *li* 力 was applied and how it took effect. When, for instance, *li* 力 was used, sometimes the actor and the person or object to whom the *li* 力 was applied are made explicit, but the descriptions neither considered the exact position where the *li* 力 was applied nor the course by which the *li* 力 produces an effect. It was the *gong* 功 that the ancient Chinese scholars were primarily interested in, and *li* 力 was not the only factor that influenced *gong* 功. For example, in the passage 5.2 quoted above, we can see that though the *li* 力 produces *gong* 功, how much *gong* 功 is produced also relies on the virtue (*de* 德). Similarly the following text passages show that *gong* 功 is determined by various factors.

5.6 引而使之，民不敢轉其力；推而戰之，民不敢愛其死。不敢轉其力，然後有功；不敢愛其死，然後無敵。¹⁰⁰

[If the king] inducts the people and uses them, [then] they dare not grudge their *li* 力 (strength); [if the king] pushes them and makes them fight, [then] they dare not take pity on their deaths. [If people] dare not grudge their *li* 力, then [they] will gain *gong* 功 (achievement); [if they] dare not take pity on their lives, then [they] will not have rivals.¹⁰¹

(From chapter *Fa Fa of Guan Zi* 《管子·法法》)

5.7 子貢南游於楚，反於晉，過漢陰，見一丈人方將為圃畦，鑿隧而入井，抱瓮而出灌，搨搨然用力甚多而見功寡。子貢曰：“有械於此，一日浸百畦，用力甚寡而見功多，夫子不欲乎？”為圃者仰而視之曰：“奈何？”曰：“鑿木為機，後重前輕，挈水若抽，數如沃湯，其名為槩。”為圃者忿然作色而笑曰：“吾聞之吾師，有機械者必有機事，有機事者必有機心。機心存於胸中，則純白不備。純白不備，則神生不定。神生不定者，道之所不載也。吾非不知，羞而不為也。”子貢……曰：“……吾聞之夫子，事求可，功求成。用力少，見功多者，聖人之道。……”¹⁰²

Zi Gong 子貢 (a disciple of Confucius) traveled south to Chu 楚. On [his] way back to Jin 晉 he passed south of the Han River and saw an elderly man about to plant his fields. [The man] had dug a channel to a well and was carrying an earthen jar to go out and irrigate [his fields]. [Although] he arduously toiled expending much *li* 力 (effort), the observable *gong* 功 (results) [of his labor] were small. Zi Gong said: “There is a device (*xie* 械) that can flood [an area] of 100 *qi* 畦 (a unit of area) in one day using very little effort (*li* 力) but with great result (*gong*). Wouldn't you like to try it?” The farmer, raising his head to look at [Zi Gong], said: “What's the use?” [Zi Gong] replied: “Carve a piece of wood to make a mechanism (*ji* 機) which is heavy in the back and light in the front. [It] can raise water by drawing it up with such great speed as water boiling over. Its name is a ‘counter-balanced bailing lever’ (*gao* 槩).”

The farmer became [red] with anger and [sardonically] laughed, “I have heard of it from my teacher. Those who have crafty devices (*ji xie* 機械) must do crafty things (*ji shi* 機事); and those who do crafty things must have crafty hearts (*ji xin* 機心). If one harbors a crafty heart in his bosom, then

99 Re. Rickett, W. Allyn, *Guanzi: political, economic, and philosophical essays from early China: a study and translation*. Vol. I. p. 101.

100 Zhao Shouzheng, *Guan Zi Zhuyi*, Book I. 1987. p. 142.

101 Re. Rickett, W. Allyn, *Guanzi*. Vol. I. p. 258.

102 Guo Qingfan, *Zhuang Zi Jishi*. 1991. pp. 74-75.

one's purity is not complete. If one's purity is not complete, then one's spirit becomes unsettled. Those whose spirit becomes unsettled do not possess the Way (*dao* 道). It is not that I do not know about [a counter-balanced bailing lever], but that I am ashamed to use it."

Zi Gong...said: "...I have heard from my teacher that in affairs [one must] pursue what can be done and in achievements [one must] pursue completeness. To use little *li* 力 (effort) to see great *gong* 功 (effects) is the Way of the sages...."

(From chapter *Tiandi of Zhuang Zi* 《莊子·天地》)

This Taoist polemic against Confucianism in particular clarifies a theoretical interest of Chinese philosophers in problems related to mechanics and, at the same time, shows similarities to the interest of ancient Greek philosophers in mechanics. It attributes to the disciples of Confucius the knowledge of the practitioners' experience that machines can produce a greater effect applying less force, an experience which, in ancient Greece, was a topic of the Peripatetic *Mechanical Problems*. However, there are no indications in the Chinese literature, that the Chinese scholars tried to answer questions such as: How is *li* 力 used and what effect is caused by *li* 力? Why can the machine save *li* 力 and gain *gong* 功? How can one construct more effective machines? The practitioners constructing machines surely considered such kind of questions. The quoted text is a representative example showing that ancient Chinese scholars did not pay attention to them. Except for a brief account of the machine's appearance and a description of its use, the writer neither tries to explain its functioning nor describes details of the conditions of its effect. His primary intention was to pursue the *dao* 道 which he considered to be obstructed by machines. From a modern point of view, the text seems to demonstrate that the attitude of the Taoist may have had a bad influence on ancient intellectuals' studying mechanical phenomena.

The following last text passage quoted here points at a necessary condition for any progress towards a theoretical understanding of mechanical phenomena: the conceptual differentiation between *li* 力 and *gong* 功.

5.8 舉事而材自練者，功分明；功分明，則民盡力；民盡力，則材自練。¹⁰³

Why the personnel were practised spontaneously, when the enterprises were undertaken, was because [their] *gong* 功 (achievements) were explicitly distinguished (分明). [Once] the *gong* 功 (achievements) [of different persons] are explicitly distinguished, then people will exhaust their *li* 力 (strength and ability) [to work]. [Once] people exhaust their *li* 力 (strength and ability) [to work], then the personnel were practised spontaneously.¹⁰⁴

(From chapter *Cuofa of Shangjun Shu* 《商君書·錯法》)

Though this passage primarily discusses matters of political administration, it postulates that the *gong* 功 (achievements) and the *li* 力 of different persons should be distinguished, which means to separate different persons' *li* 力 and to assign the corresponding *gong* 功. This kind of thought can be easily extended to a general distinction of *li* 力 (power, force) and *gong* 功 (effect, result). Such a chain of thought would have been helpful in analyzing mechanical phenomena and to make theoretically explicit the principles of the functioning of machines. It is only the example from the *Jiuzhang suanshu* 九章算術 [*Nine Chapters of Mathematical Procedures*] quoted above which indicates such a kind of thought: different *li* 力 have their corresponding effects (dragging the corresponding weights up a slope). Here the *li* 力 of different horses and the corresponding *gong* 功, which can be measured by the weight of the load, are clearly distinguished. The solution of the problem depends on the theoretical assumption that the *li* 力 of the horses are in proportion with the weights that the horses are able to drag up the slope¹⁰⁵.

103 Jiang Lihong 蔣禮鴻 (1916-1995), *Shangju Shu Zhuizhi* 商君書錐指 [*Explanations on the Book of Sir Shang*], Beijing: Zhonghua Shuju, 1986. p. 63.

104 Re. J. J. L. Duyvendak, *The Book of Lord Shang*. Arthur Probsthain, 1963. p. 239.

105 The composition of the *Nine Chapters* was finished probably in 1 century B.C. But most of its contents might appear in Pre-Qin Period. No matter when this example was designed, it can explain the consistency to the chain of thought of isolation of *li* 力 and *gong* 功.

6. Conclusions

The concept of *li* 力 has anthropomorphic origins. It is, at least, likely that its original meaning was closely related to the physical strength of the human body. The character *li* 力 may represent a farming tool that was related to physical strength, possibly a plough. But whatever the original meaning of *li* 力 may have been, the term was used in a much broader sense as early as in the Pre-Qin Period. As the text passages quoted in this study show, it was indeed used in this period to express the power or force of a great variety of things ranging from human beings and animals to rigid bodies, fluids, and gases. However, no instances were found where it served as a general, well-defined concept for the explanation of mechanical phenomena.

In the Han dynasties, the term *li* 力 was even more frequently used and became a basic concept for the explanation of various phenomena. In the Eastern Han dynasty, Wang Chong was the first who deliberately conceived of *li* 力 as a dedicated concept for explaining mechanical phenomena, but used it as well for mental and social phenomena, whereby the mechanical meaning of *li* 力 was the basis of this usage. Although he distinguished between different quantities of *li* 力, he did not explicitly formulate a relation between *li* 力 and its effects.

When *li* 力 was used to express the force of a flying arrow, the buoyancy of liquid, the strength of a crossbar, the physical strength for lifting or moving a weight, the elasticity of a bow or crossbow, the force of a mountain or of sand supporting small stones, etc., it had mechanical meanings. In modern physics, *force* is conceived of as a vector determined by its magnitude, its direction and its point of action. The mechanical knowledge of ancient Chinese scholars represented by the term *li* 力 also included these three aspects. They had a clear notion of magnitude and also had a notion of direction of *li* 力 but were unaware of its relevance. They intuitively assumed these two aspects when they used weight (*zhong* 重) as an effect that allowed to measure the quantity of *li* 力. The ancient Chinese scholars did not conceive of the direction of *li* 力 as an abstract geometrical line, but they had the notions of an agent of *li* 力 and the notion of what *li* 力 is applied on. As far as the point of action of a force is concerned, I have not found that they made any effort to formulate explicitly the point at which a *li* 力 is applied. Ancient Chinese scholars knew that two *li* 力 of the same magnitude but acting in different directions may cause different effects. They also knew that two *li* 力 of the same magnitude but being applied at different points may cause different effects. However, while they had an explicit concept of magnitude, they were unaware of the relevance of the question of where the force was applied and they seem never to have formulated an explicit notion for the point of action.

Thus the mechanical significance of the ancient Chinese concept of *li* 力 can be seen from the following aspects of intuitive knowledge to which ancient Chinese scholars tacitly adhered.

1. *Li* 力 is able to raise up weight; its magnitude is in proportion with the weight that it is able to raise up.
2. *Li* 力 produces motion, the more *li* 力 is used, the more motion is produced. Furthermore, they had the notion that if a *li* 力 makes an object move, then the agent of *li* 力 is always applying a force to the moving object.
3. *Li* 力 produces effects, the more *li* 力 is used, the more effect is produced.

None of these assumptions was expressed in the Chinese literature as an explicit proposition. When applied, these assumptions would have to be specified according to the actual conditions, but how the conditions determine the effects was not treated by the Chinese scholars.

When the ancient Chinese encountered the puzzle that nobody is able to raise up himself/herself, they introduced a concept *shi* 勢 (*circumstances*) to explain the different effect of *li* 力: the effect of *li* 力 depends on the whether the *shi* 勢 is convenient or not. But we cannot find any ancient Chinese definition of *shi* 勢 and no description that allows to reconstruct how they considered the process according to which *shi* 勢 influences the effect of *li* 力. Because *shi* 勢 is more difficult to be defined than *li* 力, its function in the explanation of mechanical phenomena was not effective.

The *Jiuzhang suanshu* 九章算術 [*Nine Chapters of Mathematical Procedures*] uses the tacit assumption that *li* 力 is proportional to the weight it can drag up a slope. The author neither expressed

this assumption as a proposition, nor did he show interest in the fact that the effect of *li* 力 is to produce motion. In fact, I have not found that any ancient Chinese scholar had the means to specify how *li* 力 is related to motion or other effects.

To sum up the results of the present paper, the ancient Chinese developed a certain knowledge about forces represented by the term *li* 力 and they could consciously or unconsciously use it to understand or explain various mechanical phenomena. They used *li* 力 with a wide range of meanings related to mechanical phenomena, which made the term useful and flexible for explaining such phenomena. But on the other hand, the ancient Chinese did not deliberately organize the knowledge into a theoretical structure of mechanical knowledge. We cannot find in the ancient Chinese literature any precise formulation of relations between *li* 力 and other concepts or notions, although ancient Chinese scholars seem to have been aware of some of them. On the basis of such a kind of mechanical knowledge it must have been difficult if not even impossible for them to derive more elaborate consequences of mechanical experiences.

Mechanical Knowledge in Ancient Chinese Cosmology

TIAN Miao

Ancient Chinese people paid great attention to celestial phenomena. Observational results concerning motion of the sun, the moon and the stars have been collected continuously in Chinese historical records¹. Based on these observations, ancient Chinese philosophers and astronomers produced their cosmological theories. Historians of astronomy dealing with ancient China have systematically sorted out source materials and outlined the development of cosmological theories in ancient China.² Previous research thus provided a solid base for the research here presented. In this paper, the author focuses on the process of reasoning concerning the motion of the heavens and the celestial bodies in ancient China, distinguishing different kinds of knowledge underlying the theories concerned. The author furthermore tries to reconstruct the development of the relevant theories in connection with that of mechanical reasoning in ancient China.³

1. Mechanical Reasoning Relating to the Origin of Motion of Celestial Bodies in an Ancient Chinese Legend

An ancient legend recorded both in the *Lie zi* 列子 [Master Lie, the 4th- 3rd century B.C.?] and the *Huainan zi* 淮南子 [Master Huainan, c. 2nd century B.C., not later than 139 B.C.] provides us precious information about the way in which the motion of celestial bodies was conceptualized in Chinese antiquity:

昔者共工與瑞瑣爭爲帝，怒而觸不周之山，天柱折，地維絕，天傾西北，故日月星辰移焉。地不滿東南，故水潦沈埃歸焉。

In the past, Gong Gong struggled with Zhuan Xu for the empire. Angered, [he] smote the Buzhou Mountain. A heaven's pillar broke and an earth's cord ruptured. Heaven leaned over to the north-west. Hence, the sun, moon, stars and plants move to there. And earth became incomplete in the south-east. Hence, water and dust turned over to there.⁴

The cosmological idea contained in this legend may be shortly explained as follows: Heaven has a material existence and covers the Earth. There are pillars on the earth to support the heaven. The heaven should be horizontal and the celestial bodies should originally be stationary. However, by smiting one mountain, a legendary person, Gong Gong broke one of the eight pillars which are supporting the heaven. Consequently, one corner of heaven drops, and thus begins the motion of celestial bodies.

1 See, in particular, Zhuang Weifeng 莊威風, Wang Lixing 王立興. ed., *Zhongguo gudai tianxiang jilu zongji* 中國古代天象記錄總集 [A complete collection of records of celestial phenomena in ancient time China]. Nanjing: Jiangsu Kexue Jishu Chubanshe, 1988.

2 About Chinese historians research on history of ancient astronomy in China, see: Xi Zezong 席澤宗, Chen Meidong 陳美東, Ershi Shiji Zhongguo Xuezhe de Tianwen Xue Shi Yanjiu 20 世紀中國學者的天文學史研究, Chinese Scholars Research in the 20th century on the history of astronomy]. *Guangxi Minzu Xueyuan Yuebao*, 2004. V. 10. Issue. 1. pp. 6-11. About the outline of the development of cosmological theories in Ancient China, See: Zheng Wenguang 鄭文光, Xi Zezong 席澤宗, *Zhongguo lishi shangde yuzhou lilun* 中國歷史上的宇宙理論 [Cosmological theories in history of China]. Beijing: Renmin Chubanshe, 1975. Anonymous author, *Zhongguo tianwen xueshi* 中國天文學史 [History of Astronomy in China]. Beijing: Kexue Chubanshe.

3 In their paper, "Mechanics in the Mohist Canon and its European Counterpart," Jürgen Renn and Matthias Schemmel formulate their research approach on parallels between the development of mechanical knowledge in different cultural traditions: they distinguish three kinds of shared knowledge, namely, intuitive, practical and theoretical knowledge, underlying mechanical reasoning (see also the introduction to this preprint). This paper is inspired by their arguments. Jürgen Renn, Matthias Schemmel. Mechanics in the Mohist Canon and its European Counterpart. Berlin: Max-Planck Institute for history of science, Preprint, Issue: 241. pp. 14-23.

4 Anonymous author, *Lie zi* 列子 [Master Lie]. Accomplished in the 4-3rd centuries B.C. Chapter 5. P. 209. The same story was recorded in *Huainan Zi* 淮南子 [Master Huainan], chapter 3. For an English translation with few variations, see: Joseph Needham, *Science and Civilization in China*. Cambridge: Cambridge University press, 1959. Vol 3.

The emergence of such a cosmological model can be reconstructed without much difficulty. Most ancient Chinese philosophers believed that the heaven has a material existence and is stable.⁵ They thought that “if the heaven is not stable, there is nothing which holds the sun and stars”.⁶ From experiential knowledge, ancient people knew that if a thing can stay above the earth in a stable way, it must be supported by pillars. In *Huainan Zi*, eight mountains serve as the pillars.⁷ Ancient people might have formed this idea intuitively, as earth-bound observations suggest that the only thing able to reach the heaven are high mountains. In such a system, the heaven should be horizontal. From experience one knows, on the other hand, that things placed on a horizontal plane should be stationary; the heavenly bodies, which are believed to be attached to the heaven, should thus be stationary. In reality, however, the heavenly bodies, including the sun, the moon and the stars move, and furthermore, the sun and moon rise from the east and set down from the west. There should hence be a reason for that. From experience, ancient people also knew that if one pillar supporting the roof of a house collapses, the roof should bend towards the corner where the pillar supported; everything attached to the roof would then move toward this corner and drop from there. Again from experience, if one pillar collapses, is natural to assume that there should be a cause responsible for it. In the legend quoted above, an ancient half divine figure by the name of Gong Gong was chosen to accomplish this task. He smites the Buzhou Mountain, the northwest pillar between the earth and the heaven, and the collapse of this pillar then results in the bending of the heaven toward the northwest. From experiential knowledge, things slide to lower places. This explains why the heavenly bodies move towards the north-west. The legend also offers a solution to a geographical problem. In China, most main rivers flow to east. From experience, ancient people knew that water flows from higher to lower places. The reason why rivers flow toward the east is thus formulated as follows: When the pillar collapsed, the cord of the earth was also broken off, and, as a consequence, the corner of the south-west of the earth ascended. Therefore, all the rivers flow toward the south-east.

Two underlying mental models of mechanical thinking may have contributed to the forming of the cosmological model in the above quoted legend. One is the model according to which motion is caused by a force, the second model implies that things move from higher to lower places.

Both of them have their origin in intuitive knowledge. In ancient China, force is commonly expressed by the character *li* 力, which is already found in the oracle-bone inscriptions (16th–11th century B.C.).⁸ According to the literature, the first definition of this character was given in the *Mozi*,

力，刑之所以奮也⁹

There are two different interpretations of this definition, which could be rendered into English as:

Li 力[force] is that by which the body exerts [itself].¹⁰

Li [force] is that which causes shaped things to move.¹¹

5 The *Xuanye* cosmology, however, denied the existence of a material heaven.

6 Anonymous author, *Wenzi*. Siku quanshu edition. Chapter 1. P. 15a.

7 In the *Zhuixing xun of Huainan zi*, it is mentioned that the heaven has eight poles: “The northeast [pole] is the Fangtu Mountain, which is the Cang Gate [of the heaven]; the east [pillar] is the Dongji Mountain, which is the [heaven’s] Kaiming Gate; the southeast [pillar] is the Pomu Mountain, which is the [heaven’s] Yang Gate; the south [pillar] is the Nanji Mountain, which is the [heaven’s] Shu Gate; the southwest is the mountain of Bianju, which is the Bai Gate; the west is the mountain of Xifang (west direction), which is the [heaven’s] Luhe Gate; the northwest [pillar] is the Buzhou mountain, which is the [heaven’s] Youdu Gate”. In the Tianwen Chapter, referring to Gonggong striking the Buzhou mountain with the result that the pillar collapsed and the heaven bent toward the northwest, the Buzhou mountain was regarded as the northwest pillar between the Earth and the heaven. So, it is reasonable to assume that the eight mountains mentioned above were nothing but the eight pillars which support the heaven. Liu An. *Huainan zi*, Chapter 4, *Siku Quanshu* edition. 5b-6a.

8 For a detailed and systematic discussion on the term *li* 力, see the contribution of Zou Dahai, The Concept of force (*li* 力) in Early China, to this volume (chapter 2).

9 *Mo Zi* 墨子 [*Master Mo*], *Dao Zang* 道藏 [*Depository of Daoist Scriptures*] Edition. Photocopied by Shanghai Hanfenlou, 1924. Vol. 10, 1a.

10 Qian Baochong, Qian Baocong 錢寶琮 (1892-1974), ‘*Mojing Lixue Jinshi*’ 《墨經》力學今釋 [*Modern Explanations on the Mechanics of Mohist Canons*], in *Kexueshi Jikan* 科學史集刊 [publication of Collected Essays on History of Science], No. 8, 1965. pp. 65-72. Graham, A. C., *Later Mohist Logic, Ethics and Science*. 1978. English translation quoted from Graham, A. C., *Later Mohist Logic, Ethics and Science*. 1978. p. 279.

11 English translation quoted from Joseph Needham, *Science and Civilisation in China*. Cambridge: Cambridge

The two interpretations present two aspects of the relation between force and motion. The former means that force, as an inner factor, can cause a thing to move. The latter stresses that force, as an outside factor, can move things. Based on the context in the *Mozi*, Zou Dahai argues that the first interpretation is in better accordance with the original meaning. However, this does, of course, not mean that ancient Chinese were not also familiar with the second conception. In fact, in the explanation of this sentence which was possibly written by Mo Di's disciples, the above sentence was interpreted as *li* 力 (force), refers to weight. Lifting a weight from below is exertion".¹² Exertion was thus defined as an ability of lifting weight. Combining the original definition and the explanation attached to it, the idea is that force is an inner factor of a thing, and relying on this factor, that a thing can move weight.

In the above quoted legend in *Liezi*, the concept of force was not explicitly mentioned. Nevertheless, in the *Huainan zi*, in order to show the reader the philosophy according to which one has to obey the way of nature and cannot conquer an empire by physical or martial force, the author quoted the story of Gong Gong and commented: "In the past, Gong Gong's force smote the Buzhou Mountain. This caused the earth bent toward southeast."¹³ Placed in this context, the passage shows that Liu An interpreted Gong Gong's action as exerting a force on a pillar of the heaven. Hence, it is reasonable to assume that in the cosmological model mentioned above, it is a force that causes the motion of the celestial bodies.

The model suffers, however, from some obvious weaknesses. First, if the heaven is rigid, three pillars are enough to support it; if, however, it were not rigid, it would not be able to be firmly supported by pillars, no matter how many they are. Secondly, if this theory is true, then, there must be a whole set of celestial bodies newly generated each day, as no design is made to let them get back to the places where they should be the next day according to this model. A similar cosmological system was mentioned in a famous poem by the third century B.C. poet Qu Yuan (339 B.C.-278 B.C.), the *Tianwen* 天問 (Question the heaven):¹⁴

斡維焉繫，天極焉加？八柱何當，東南何虧？

On what does the circumrotation of [heaven] depend? Where should the pole of the heaven be located? How do the eight pillars work? Why is the southeast incomplete?

何闔而晦？何開而明？角宿未旦，曜靈何藏？¹⁵

How does [the heaven] close and the night [begin]? How does [it] open and the day [begin]? When the east¹⁶ is not bright, where does the sun hide [its light]?

康回憑怒，地何故以東南傾？¹⁷

Kang Hui rages, why is the earth bent toward the southeast?

Qu Yuan's poem shows that the cosmological model described in the above mentioned legend was widely spread in ancient China. In the poem, Qu Yuan expresses his doubts about problems such as that about the position of the pole of the heaven, or the way in which the 8 pillars of the heaven work, and especially concerning the question of where the sun stays during the night so that it can hide its light. Qu Yuan's questions may well be rooted in intuitive mechanical thinking. However, in the *Tianwen*, he presented a whole system of cosmological questions. Even if he did not answer those questions, this elaborate part without any doubt belongs to the realm of theoretical thinking.¹⁸ In the second century

University press, first published in 1962, reprinted version, 1989. Vol. IV: 1, p. 19.

12 English translation quoted from 2. Graham, A. C., *Later Mohist Logic, Ethics and Science*. 1978. p. 279. For details, see the contribution of Zou Dahai, The Concept of force (*li* 力) in Early China, in this volume (chapter 2).

13 Zhang Shuangdi 張雙棣, *Huainan zi jiaoshi* 淮南子校釋. [Collations and Explanations of *Huainan zi*]. Beijing: Beijing daxue chubanshe, 1997. P. 59.

14 According to Wang Yi 王逸 (89?-158), the *Tianwen* was wrote short before Qu Yuan's death in 278 B.C. Wang Yi, *Chuci Zhangju* 楚辭章句 [the Sentences and Phrases of *Chuci*], completed around 114-120 A.D.

15 Qu Yuan. *Tianwen*. 183.

16 Jiaokang is the name of a star at the east part of the heaven.

17 Qu Yuan. *Tianwen*. 188. Later commentators of the *Tianwen* believe that Kang Hui is identical with Gong Gong.

18 Zheng Wenguang mentioned this legend in his *Zhongguo Tianwen Xue Yuanliu*. He connected the model in this legend with the Gaitian theory. See: Zheng Wenguang 鄭文光, *Zhongguo Tianwen Xue Yuanliu* 中國天文學源流 [the origin of astronomy in China]. Beijing: Kexue Chubanshe, 1979. Pp. 31-34. I think that the heaven could only be flat in the model of the universe in this legend, and there is no information about the shape of the heaven given.

A.D., a Chinese scholar Wang Chong 王充 provides a systematic refutation of the cosmological model in the legend in his *Lunheng* 論衡 [*Discussion on balance*]:

與人爭為天子不勝，怒觸不周之山，使天柱折，地維絕，有力如此，天下無敵。以此之力與三軍戰，則士卒螻蟻也，兵革毫芒也，安得不勝之恨，怒觸不周之山乎？且堅重莫如山，以萬人之力，共推小山，不能動也。如不周之山，大山也，使是天柱乎，折之固難使非柱乎，觸不周山而使天柱折，是亦複難信。顓頊與之爭，舉天下之兵，悉海內之眾，不能當也，何不勝之有！且夫天者，氣邪？體邪？如氣乎，雲煙無異，安得柱而折之？¹⁹

After having fought with a man for the empire, a fight that he did, however, not win, [Gong Gong] knocked against the Buzhou Mountain with anger, and caused the pillar of heaven to break, and the cord of the earth to be smashed. If one has a force like that, he would have no opponent under the heaven. With such a force [he could] engage three armies, and the soldiers would be [to him] like ants, and their weapons like blades of grass. Why should [he], resenting his defeat, strike against the Buzhou mountain? There is nothing harder and heavier than a mountain. The force of ten thousand men pushing would not be able to move even a small mountain, and the Buzhou Mountain is a big mountain. If it was really the “Pillar of heaven,” it would be a difficult thing to break it, if it was not, then, it cannot be admitted that by knocking against the Buzhou Mountain the “Pillar of heaven” was broken. Zhuan Xū in his fight against Gong Gong might have mustered all the soldiers under the heaven and all the multitudes peopling the land within the seas, he would not have been a match for him. How should Gong Gong not have been victorious? Moreover, is heaven *qi* 氣 or a body?²⁰ If it be *qi*, it cannot be different from clouds and mist. Then there could be no pillar which might be broken. Since Nü Wa repaired it with stones,²¹ it must be a body. If it be so in fact, then it is something like gems and stones. The substance of stones is heavy, a single pillar would not be a sufficient support for a thousand li. Not even the peaks of the five Mountains could prop heaven as pillars.²²

Wang Chong’s refutation shows us that such experiential mechanical knowledge as it is represented by the insight that a non-rigid thing cannot be supported firmly by pillars and that a thing would break if its heaviness supersedes its strength were already common knowledge by his time, as he used them as the unquestioned basis for his argument. For the question as to where the sun hides during the day, Wang Chong’s answer is that the sun never drops under the horizon. He argued that when the sun is not seen during the night, this is so because it is out of people’s eyesight. This idea is identical with the theory of the *Gaitian* cosmological school. We will discuss this theory in the next section.

In fact, Wang Chong clearly reduced the action of Gong Gong to exerting a force acting on a mountain. Obviously, the force of a normal human being could not achieve such a task. It seems therefore that astronomers did not take the model of the universe expressed in the legend seriously. In fact, most Chinese thinkers would not be satisfied with a theory depending on a half divine force. The third century B.C. Chinese philosopher Zhang Zhou 莊周 inquired:

天其運乎？地其處乎？日月其爭於所乎？孰主張是？孰維綱是？孰居無事推而行是？意者其有機緘而不得已邪？意者其運轉而不能自止邪？

19 Wang Chong, *Lunheng*, Siku quanshu edition. Chapter 11. Pp. 1a-2b.

20 Qi is a crucial term in ancient China. It was also regarded as a factor producing the motion of the earth and the celestial bodies in ancient cosmos theories in ancient China. We will discuss this problem in detail in the following text.

21 In ancient Chinese legend, Nü Wa is a goddess of creation. It is her who created the human with clay. In the *Lie Zi*, a legend was recorded that in the remote antiquity, the heaven is not complet. Nü Wa melt five-colored stones to mend the heaven. Wang Chong connected the two legends, the about Nü Wa’s mending of heaven and the one of Gong Gong, together in his *Lun Heng*. See: Wang Chong, *Lunheng*. Chapter 11. P. 1a.

22 For an English translation with few variations, see: Alfred Forke. *Lun-Heng*. Second edition. New York: Paragon Book Gallery, 1962. 250-251.

Does heaven move? Does the earth stay?²³ Do sun and moon compete for a place [to shine]? Who masterminds all this? Who pulls the strings? Who resting inactive himself, gives the push that makes it go this way? I wonder, whether there is some mechanism that operates it and won't let it stop? I wonder if it just moves and turns and can't bring itself to a halt?²⁴

Zhuang Zhou's questions show us that another crude model of the cosmos existed in the 3rd century B.C., according to which the heaven is tied by a cord and turns like a ball tied by a rope revolving around the diameter through the point where it was tied. Here, again, a figure with super human powers is needed for holding the cord and giving a push to the motion of heaven. The questions imply that Zhuang Zhou doubted the existence of such a figure. In the last two questions he provided us instead with his own idea about the subject, according to which there should be a mechanism that makes the heaven move and keeps it from stopping. Zhuang Zhou's idea corresponds to the second argument we discuss in this paper which is commonly accepted as the reason for the motion of heaven and celestial bodies, that is, the round shape making things move by themselves.

2. Mechanical Reasoning Underlying the Tianyuan Defang Theory

The *Tianyuan Difang* 天圓地方 theory [that the heaven is round and the earth is square] existed in China from very ancient times. However, the *Tianyuan difang* theory does not necessarily mean that the heaven is actually round in shape and that the earth is square in shape. As a matter of fact, the *Zhoubi suanjing* 周髀算經 [*Mathematical Canon of the Gnomon of the Zhou Dynasty*] contains a piece of dialog between Zhou Gong 周公 (11century B.C.) and Shang Gao 商高 claims that “roundness belongs to the heaven and squareness belongs to the Earth”.²⁵ Shang Gao does, however, not claim that the shape of heaven is round and the shape of the earth is square. A piece of dialog between Shan Juli 單居離 and Zheng Can 曾參 (505 B.C.- ?) recorded in the *Dadai liji* 大戴禮記 provides us with more information concerning the understanding of the tianyuan defang theory in ancient China: Shan Juli objected: “ If it is really true that the heaven is round and the earth is square, then the four corners (of the earth) could not be covered (by the heaven)”. Zheng answered, “Can 參²⁶ heard from the Fuzi 夫子²⁷ that the way of heaven is round and the way of the earth is square”²⁸. Then, why did Zheng Sen insist that the way of heaven is round, even though the roundness in the shape of heaven had been refuted? An answer is provided in the *Lushi Chunqiu* 呂氏春秋: “Why do we say that the way of heaven is round? ... The circumference of a circle is curved?, there is [thus] no obstruction [in its motion].”²⁹ That means that the heaven should have identical Dao with the one of a round shaped thing, so that it can move without obstruction. Another text, the *Yin Wen Zi* 尹文子 [*Master Yin Wen, Yin Wen, 350-285 B.C.*], which can also be traced back to the 4th century B.C., contains a more common argument concerning the motion of a round shaped thing: “A round shaped thing automatically moves, and it has to move. The squared thing is automatically stationary, and it has to be stationary.”³⁰ Even though the shape of the heaven and the earth is not discussed in *Yin Wen Zi* 尹文子 [*Master Yin*

23 Burton Waston translated *yun* 運 as *turn*. In the *Erya* 爾雅 [the oldest extant Chinese lexicon, 3rd century B.C. to the 2nd century B.C.], *yun* 運 was defined as *xi* 徙, which means to move from one place to another place. *Yun* 運 also means to turn or to revolve in ancient China. This definition was given in *Guangya* 廣雅 [3rd century A.D.]. As the character *zhuan* 轉, which was used in a later part of this paragraph, definitely means *turn*, the *yun* 運 used in other parts could only be interpreted as *to move*. So, I prefer to translate the character as *to move*. The character *chu* 處 was translated as *to sit still* in Burton Watson's translation. Now, I translate it as *to stay*, which is just in contrast to *yun* 運, *to move*.

24 Zhuang Zhou, *Zhuang Zi*, Chapter14, 1a. in *Zhuangzi Jijie* 莊子集解 (Wang Xianqian 王先謙, 1900) English translation with few variations, See: Burton Watson, *The Complete Works of Chuang Tzu*. New York: Columbia University Press, 1968. P. 154.

25 Anonymous author, *Zhoubi suanjing* 周髀算經. [*Mathematical Classic of Zhoubi*]. Beijing: Wuying dian Juzheng edition, 1774. Chapter 1, 14b.

26 Sen is the given name of Zeng Sen.

27 Fuzi means Confucius.

28 Dai De 戴德. *Dadai liji* 大戴禮記. Siku quanshu edition. Chapter 5. P. 14a-b.

29 Lu Buwei 呂布韋. *Lushi Chunqiu* 呂氏春秋 [*Lü's Spring and Autumn Annals*]. written in 239 B.C. Re printed by Bi Yuan 畢沅, 1875. Chapter 4.

30 The original text reads: “圓者之轉，不得不轉也。方者之止非能止而止，不得不止也。因圓之自轉，使不得止。因方之自止，使不得轉”。 Anonymous author, *Yin Wen Zi* 尹文子 [Master Yinwen], *Zhuzi Jicheng* edition. Shanghai: Shanghai Shudian, 1991. reprinted. P. 4

Wen], the text makes it evident that in the 4th and 3rd centuries B.C., the theory that a round shaped thing moves by itself and that a square thing is stationary was common knowledge among Chinese philosophers. And we believe that this is the reason why they also claimed that the Dao, the way, of the heaven is round and the one of the Earth is square.

The conclusion that a less curvy shaped thing is more stable than a curvier shaped thing is clearly based on intuitive experiential knowledge, while the insight that circular and spherical things are the most mobile and that square things are the most stationary goes back to a reflection of practical experience. The quotations from *Yinwen zi* and *Lushi Chunqiu* clearly display theoretical reasoning. In summary, the source materials show that both of the two influential cosmological schools in ancient China, the Gaitian School and the Huntian School, held that the heaven is of round shape.

2.1. The Mechanical Reasoning of the Gaitian Cosmological School

The basic theory of the Gaitian school is that the heaven is like a canopy which covers everything under it including the earth. This idea easily follows from intuitive knowledge, as whenever one looks up, one finds that one is covered by the heaven just as is everything else on Earth. The Chinese philosopher *Zhuang Zi* 莊子 used the statement that “there is nothing that heaven doesn’t cover, nothing that earth doesn’t bear up”³¹ in his arguments. Such knowledge was obviously widely spread among Chinese scholars. An implication of this theory is that heaven does not move to a position under the surface of the earth.³² Chinese astronomers and thinkers who advocated this theory engaged themselves in establishing cosmological theories based on the heavenly phenomena observed. According to the *Jinshu* 晉書 [a book of the Jin Period, 646], they created at least two different models of the cosmos before the first century.

The *Jinshu* records that the two types of *Gaitian* cosmology were named *Zhoubi* 周髀 [Gnomon of Zhou Dynasty], and *Zhoubi Jia* 周髀家 [The school of Gnomon of the Zhou Dynasty], respectively. Both of them refer to the following basic celestial phenomena: The Sun rises each morning from the east and descends each evening in the west. Days in the summer are longer than days in other seasons; the length of day reaches its extreme in midsummer. In contrast, days in the winter are shorter than they are in other seasons; the length of day reaches its shortest in midwinter. Through analyzing the documents accumulated until the time when the Zhou Bi cosmological theories were formulated, astronomers established that the heavenly phenomena have regularity, working out a crude mathematical relationship among these phenomena. Their cosmological theories were intended to incorporate an explanation of all the observed results as well as of the two theories we mentioned above.

The most authoritative source about Zhou Bi cosmology is the *Zhoubi suanjing* 周髀算經 [*Mathematical canon of Zhoubi*].³³ This book provides us with the first explicit mathematical model of the cosmos in ancient China. However, except for mentioning the theory of the way of heaven being round and the way of the earth being square, it includes no information about the reason for the motion of the heaven and of the celestial bodies.

The *Zhoubi Jia* theory was far less important than the theory of *Zhoubi* in both the history of astronomy and that of mathematics in China. However, it includes interesting information concerning the contemporary thinking about the motion of the heaven and the celestial bodies. It claims that:

周髀家雲：“天員如張蓋，地方如棊局。天旁轉如推磨而左行，日月右行，隨天左轉，故日月實東行，而天牽之以西沒。譬之於蟻行磨石之上，磨左旋而蟻右去，磨疾而蟻遲，故不得

31 Zhuang Zhou, *Zhuangzi*, Chapter 1, in *Zhuangzi Jijie* 莊子集解 (Wang Xianqian 王先謙, Chapter 1, 32. English translation is quoted from: Burton Watson, *The Complete Works of Chuang Tzu*. New York: Columbia University Press, 1968. P. 70.

32 It was quite plausible for the ancient people to assume that the heaven covers the earth, and it would have been difficult for them to assume that the heaven could move under the earth? This insight probably originates from intuitive knowledge.

33 About cosmology in the *Zhoubi suanjing* and modern historical research about this topic, see: Christopher Cullen, *Astronomy and Mathematics in Ancient China: the Zhou bi suan jing*. Cambridge: Cambridge University Press, 1996; Qu Anjing 曲安京, *Zhoubi suanjing xinyi* 周髀算經新議 [*Recent discussion on the Zhoubi suanjing*], Xian: Shanxi Renmin Chubanshe, 2002.

不隨磨以左迴焉·天形南高而北下，日出高，故見；日入下，故不見·天之居如倚蓋，故極在人北，是其證也·極在天之中，而今在人北，所以知天之形如倚蓋也·日朝出陽中，暮入陰中，陰氣暗冥，故沒不見也·夏時陽氣多，陰氣少，陽氣光明，與日同輝，故日出即見，無蔽之者，故夏日長也·冬天陰氣多，陽氣少，陰氣暗冥，掩日之光，雖出猶隱不見，故冬日短”³⁴。

The Zhoubi school asserted: “the heaven is round [in shape] like an opened umbrella, [while] the earth is square like a chessboard. The heaven rotates sideways towards the left in the same manner as a turning mill-stone. The sun and the moon [both] move towards the right, [but at the same time they] have to follow the heavens, which rotates towards the left. Hence, though in reality they are moving east, they are dragged along by the turning of the heavens, so that they [appear to] set in the west.³⁵ Compare this with the motion of ants on a mill-stone. The mill-stone rotates towards the left, while the ants move toward the right; the mill-stone is fast, while ants are slow. Hence, they have to follow the mill-stone to rotate towards the left. The shape of the heaven is that [it] is high in the south and low in the north. The sun rises above, then, it is seen; the sun sinks down, then, it is unseen. The position of heaven is as an inclined canopy. Therefore, the pole is north of [the place of] human. This is its proof. The pole is at the center of the heaven, but now it is north of human, so, we know that the shape of heaven is like an inclined canopy. The sun rises from Yang in mornings, and sinks in Yin in the evenings. The Yin Qi is dark and obscure, therefore, [when a thing] sinks into [it], [it] cannot be seen. In the summer, there is more Yang Qi 陽氣, and less Yin Qi 陰氣. Yang Qi is bright and can shine together with the sun. Therefore, the sun can be seen when it rises, there is nothing to block it, so the day in the summer is long. In the winter, there are more Yin Qi and less Yang Qi. Yin Qi is dark and obscure, it blocks the light of the Sun. Even though [the sun] has already risen, it is still blocked and cannot be seen. Therefore, the day in the winter is short.

The *Jinshu* was compiled by scholars in the 7th century, when the Zhoubi Jia theory was no longer the dominant cosmological theory for some time. The *Jinshu* provides us in fact only with a short introduction of its main idea. However, we find this introduction very precious. It is obvious that the Zhoubi Jia tries to explain all the heavenly phenomena in terms of its model. According to this model, the heaven is physically round shaped, but it is not mentioned whether it is a circular plane or a spherical segment. It rotates sideways around its pole. The inspiration for such a model may have come from the practical knowledge that an umbrella with circular rim cannot stay in a stable position. This mechanism also explains why the pole of heaven, the North Pole, is not in the zenith. Both Gaitian schools believed that the north pole is the pole of heaven. This claim is based on the fact that the North Star appears stationary from the earth and on the practical knowledge that when a round shaped thing revolves, its pole does not move. The model also covers the observational result that the motion of celestial bodies varies in speed. In order to explain this circumstance, the Zhoubi Jia claims that the celestial bodies attached to the heaven move along with it, while moving by themselves at the same time. The sun and the moon move in a direction inverse to the direction of the motion of the heaven. But they move slower than the heaven. So they sink in the west, in accordance with the direction of the motion of the heaven. The Zhoubi Jia uses practical knowledge about the superposition of motions in order to explain this theory: when ants move on a revolving mill in the inverse direction of the motion of mill, and if they move slower than the mill, they are in fact moved in accordance with the motion of the mill. The Zhoubi Jia furthermore introduces Qi into the argument. It was assumed that there are Qi on the earth. When the celestial bodies sink to a place lower than the place where the Qi exists, they cannot not be seen by humans, while when they raise to a higher place above the earth, they can be seen. In the summer, there are less Qi and in the winter, there are more Qi. Therefore, the time in which the sun can be seen is longer than that in the winter.³⁶

There is no evidence of whether the Zhoubi Jia discussed the way in which the celestial bodies are tied to the heavens and no explicit reason is given for the proper motion of the celestial bodies. The mill-ant example is very visual. From this analogy it may be concluded that the Zhoubi Jia believed that

34 Fang Xuanling 房玄齡. *Tianwen zhi, Jinshu*, siku quanshu edition, chapter 11, 3b.

35 The translation from *Jinshu*, with few variations, is from Joseph Needham, with collaboration of Wang Ling, *Science and Civilisation in China*. Cambridge: Cambridge University Press, 1959. Vol. 3, p. 213.

36 About Qi, see the following for more detail.

heavenly bodies are alive so that they can move according to their own will or in accordance with some rule. But, as a matter of fact, celestial bodies are commonly regarded as non-living substances in ancient Chinese philosophy. Another weak point of this theory is whether the heaven touches the earth or whether it just hangs above the earth, a point that is not explicated in the extant material. In case of the former hypothesis, an additional difficulty occurs: as the earth is not absolute smooth, its motion should be affected by the friction between the heaven and the earth. In order to keep the heaven smoothly moving, there should thus be a force acting on the heaven. In case of the latter hypothesis, there occurs another hypothesis as what supports the heaven has not been made clear.

From the second century on, both versions of Gaitian cosmology are apparently no longer accepted. The Zhoubi theory was criticized for not being in accordance with the heavenly phenomena,³⁷ while no text specifically dedicated to the *Zhoubi Jia* theory is extant. Since the first century, the Huntian theory began to dominate in China. The Huntian cosmology also holds that the heaven is round and that the earth is square. As the theory is directly connected with another important factor concerning the nature of motion, to be discussed in the next section, we defer the analysis of Huntian cosmology to this section.

2.2. Mechanical Reasoning Concerning Qi As a Mover and Medium of Motion

Qi 氣 is a crucial philosophical term in ancient China.³⁸ It was rendered in publications in western languages as gas, ether, matter³⁹, air, vapour, material force, etc. However, none of these English terms renders the meaning of this unique term in ancient China properly, in particular as it can assume different meanings in different contexts. I therefore prefer to leave this term untranslated. In cosmological materials, qi may be regarded as the primordial substance of the universe. In relation with the Yin-yang theory of ancient China, Chinese scholars produced a theory about the origin of the universe. Parts of the relevant ideas are stated in *Tianwen xun* 天文訓 of the *Huainan zi* 淮南子 [Master Huainan, about 150 B.C.] explicitly:

天墜未形，馮馮翼翼洞洞灑灑，故曰太昭。道始於虛霽，虛霽生宇宙，宇宙生氣，氣有涯垠，清陽者薄靡而為天，重濁者凝滯而為地，清妙之合專易，重濁之凝結難，故天先成而地後定。天地之襲精為陰陽，陰陽之專精為四時，四時之散精為萬物，積陽之熱氣生火，火氣之精者為日，積陰之寒氣為水，水氣之精者為月。日月之淫為精者為星辰。天受日月星辰，地受水潦沈埃。⁴⁰

When the heaven had not formed, [there is a state which is] amorphous and chaotic. Therefore, it is called Taizhao 太昭 [the great brightness]. The Dao begins from empty extensiveness [Xukuo 虛曠], the empty extensiveness produced the cosmos, the cosmos produced Qi. Qi has its limits. The clear and light [*qi*] spread thin to become the heaven; the heavy and turbid [*qi*] coagulate to become the

37 Cai Yi (133-192) claimed, however, that the method and the calculations of Zhoubi remained. “Exam [it] with the phenomena in the heaven, there are lots of mistakes.” 周髀術數具存，考驗天狀，多所違失。See: Fang Xuanling, *Jinshu*. Siku quanshu edition. Chapter 11. P. 2b.

38 The character *qi* occurs in inscriptions on bones or tortoise shells of the Shang Dynasty [16th—11th century B.C.]. According to historians of philosophy in ancient China, not later than the fifth century B.C., Chinese scholars established a relationship between *qi* and the theory of *Yin* and *Yang*. In the *Guanzi* 管子 [Master Guan], *qi* was described as the primordial substance. Based on the parts concerning *qi* in the *Guanzi*, Graham claimed: *qi* is “adapted to cosmology as the universal fluid”, “out of which all things condense and into which they dissolve”. “It is like such words in other cultures as Greek *pneuma* ‘wind, air, breath’. It is the energetic fluid which vitalizes the body, in particular as the breath, and which circulates outside as the air”. But the Western terms Graham offers can actually not encompass the meaning of *qi* in the Chinese context. His discussion rather provides us with some understanding of the difficulties of translating *qi* into Western language, and I agree with him that it is better to leave this term not translated. See: A. C. Graham. *Disputers of the Dao, Philosophical argument in Ancient China*. Chicago: Open Court. P. 101. About the evolution of the concept of *qi*, See: Qiu Xigui 裘錫圭, *Jixian Daojia Jingqi de Yanjiu* 稷下道家精氣的研究 [Research on the Jingqi of the branch of jixian of Daoism]. in *Daojia Wenhua Yanjiu* 道家文化研究 [Research on the culture of Daoism]. 167-192; Zheng Zhengyu 曾震宇, *Zhongguo Qilun Zhaxue Yuanjiu* 中國氣論哲學研究 [Research on the history of qi in philosophy in China]. In this paper, we only focus on the materials concerning with motion of the heaven, the earth and the celestial bodies.

39 Fung Yu-lan, translated into English by Derk Bodde, *A History of Chinese Philosophy*, Princeton: Princeton University Press, 1983. P. 352.

40 Liu Wendian, *Huainan Honglie Jie*, Vol. 3, 1a-b. English translation with few variations, see: Fung Yu-lan, translated into English by Derk Bodde, *A History of Chinese Philosophy*. Pp. 396-397.

earth. The union of the clear and light was easy, [whereas] the coagulation of heavy and turbid was difficult. So that the heaven formed first and the Earth fixed later. The merged essences of the heaven and the earth formed the Yin and the Yang. The concentrated essences of the Yin and the Yang formed the four seasons, the scattered essences of the four seasons formed the myriad of things. The accumulation of the hot air produced fire, and the essence of fire formed the sun. The accumulation of cold air formed water, the essence of water formed the moon. The soaked essences of the sun and the moon forms the stars and planets. The heaven bears the sun, the moon and the stars; the earth bears the water and the dust.

According to the above text, *qi* is the primordial substance in the universe. *Qi* forms the heaven and the earth; the interaction of the heaven and the earth produced the Yin 陰 and Yang 陽⁴¹; the interaction of Yin and Yang produced the four seasons; the interaction of the four seasons produced myriads of things. The warm *qi* of the Yin and Yang forms fire, and the essence *qi* of the fire forms the Sun; the cold *qi* of Yin and Yang forms water, and the essence *qi* of water forms the moon. The interaction of the sun and the moon produces the stars. This procedure is clearly an analogy with the procreation of human beings. The intuitive knowledge that heavy things tend to move downwards and that light things tends to move upwardwards was clearly a further presupposition of this cosmological theory.

In the ancient Chinese context, *qi* continued to play an important role even after the formation of the universe. We already saw that the Zhoubi jia cosmology claimed that it is *qi* that causes the differences in the lengths of a day. In the Xuanye 宣夜 cosmology *qi* played a far more important role as the mover and medium of the motion of celestial bodies. The Xuanye cosmology was presented in very early times. In the 7th century, the compiler of the *Jinshu* stated that no sources of Xuanye theory existed in his time. The introduction of the theory given in *Jinshu* is based on an account of the second century A.D. historian Xi Ming 郗萌 who flourished in the Later Han, and may have been a younger contemporary of Chang Heng :

宣夜之書亡，惟漢祕書郎郗萌記先師相傳雲：“天了無質，仰而瞻之，高遠無極，眼瞶精絕，故蒼蒼然也。譬之旁望遠道之黃山而皆青，俯察千仞之深谷而窈黑，夫青非真色，而黑非有體也。星、日、月自然浮生虛空之中，其行其止皆須氣焉。是以七曜或逝或住，或順或逆，伏見無常，進退不同，由乎無所根繫，故各異也。故星西沒也。攝提、填星皆東行，日行一度，月行十三度，遲疾任情，而北斗不與，辰極常居其所。其無所繫著可知矣。若綴附天體，不得爾也。”⁴²

The books of the Xuanye system are lost. Alone that a librarian of Han [Dynasty] Xi Meng recorded what his deceased master had taught [him], [which] says: “the heavens were entirely void of substance. [When we] look up and see it, [it looks] limitlessly high and far away. The [human] eye is dim-sighted and the vision is exhausted, it is why [the heaven appears] deeply blue. It is like seeing yellow mountain far away sideways, then, [it] appears all blue, or when we gaze down into a valley of thousand ren⁴³ deep, then, [it appears] somber and black. While, the blue [of the mountain] is not its true color, and the black [of the valley] does not [mean] that [it] has a body. Stars, the sun, and the moon float and exist in the emptiness. Both their motion and stay depend on *qi*. Thus, [of] the seven luminaries,⁴⁴ some disappear and some appear, some move to the same direction and some move to the inverse direction. [They] have no regularities in disappearing and appearing, and are different in [moving] forward or retrograde. It is because that [they] are not rooted or tied to [a body] that [their movements] vary from each other. Hence, the stars sink to the west, but the Big Dipper does not

41 The *Yin* and *Yang* are two fundamental factors in philosophy in ancient China, which represent, respectively, female and male, darkness and light, soft and hard, inactivity and activity, etc. In Xu Shen 許慎 (25?-125?)’s *Shuowen Jiezi* 說文解字 (first compiled in 121 A.D., reprinted in 1873, photocopied edition in 1963, Zhonghua Shuju), Yin was interpreted as the north of a mountain and the south of a river. Except the meaning of female, Yin also means dark, shady, shadow, cold, moist, hidden or negative. In contrast, Yang originally means the south of a mountain and the north of a river. In the *Shuowen Jiezi*, Xu Shen interpreted Yang as high and bright. In addition to the meaning of male, it also means bright, worm, dry, sunny, etc.

42 Fang Xuanling, ed., *Tianwen*, in *Jinshu* 晉書 [Book of Jin Dynasty], Chapter 11, *Siku Quanshu* edition. 7a.

43 *Ren* 仞, an ancient measure of length. One *ren* is equal to eight *chi* in the Qin Dynasty, and seven *chi* in the Han Dynasty.

44 *Qi yao* 七曜 [Seven Luminaries] are the sun, the moon, the star of gold [the Venus], the star of wood [the Jupiter], the star of water [the morning star], the star of fire [the Mars], and the star of soil [the Saturn].

move with [them], and the pole star always stays at its place. The Sheli star and Tian star⁴⁵ all move eastwards. The sun moves one du⁴⁶ [a day] and the moon moves thirteen du [a day]. Their speed depends on [their individual] quality. It is known that they are not tied to anything. [For] if they were attached to a body of heaven, this could not be so.”⁴⁷

The Xuanye cosmology denied the existence of a material heaven. The above text presents detailed arguments for this theory. The basic argument is that if all celestial bodies were attached to a material heaven, their directions and velocities of motion could not be different from each other. This suggests that, before the Xuanye theories were developed, the existence of a material heaven was generally assumed. From the quantitative characterization of the motion of celestial bodies mentioned in the above text, we can infer that the Xuanye theory was based on observational results.

In this paper, we are mainly interested in the reasons given for the motion of celestial bodies in this text as well as in the process by which such a theory has been constructed. The Xuanye theory holds that all celestial bodies are floating in emptiness, and that they are moved by *qi*. The emptiness in the text does, however, not mean vacuum. In fact, the statement that the stillness and motion of celestial bodies rely on *qi* implies that the Xuanye School believed that the emptiness is full of *qi*. For the understanding of *qi*, Graham points out that “it is like such words in other cultures as Greek *pneuma* ‘wind, air, breath’”.⁴⁸ But even taken together, the three terms listed by Graham cannot encompass the full meaning of *qi*, although it is certain that ancient Chinese regarded these three kinds of things as belonging to *qi*, and although we do believe that the basic characteristics of *qi* were by analogy derived from the characteristics of wind, air and breath. The theory that *qi* can support things may thus have been developed from the intuitive knowledge that things can float on air.⁴⁹ Zhuang Zi claimed that “[if] the accumulation of wind is not thick, then it will not have enough force to support the large wings”.⁵⁰ Zhuang Zi implies that the force of supporting grows when the accumulation of wind increases. This idea may have been shared by the holders of Xuanye theory, which concluded that, as the heavenly bodies are clearly high above the earth, the accumulation of *qi* must be very thick in order to sustain them. As for why ancient Chinese thought that *qi* can move things, I think that this idea may have been derived from the intuitive knowledge that wind can move things, and also that by blowing one can drive things away. The doubt of whether heaven really has a material body might have arisen when the differences in the tracks and speeds of the motion of celestial bodies were observed. In order to solve the problems arising if heaven is not ascribed a material existence, where the sun, the moon and other celestial bodies are located, the Xuanye School introduced the concept of *qi* as the supporter of the celestial bodies. Their theory is, in some sense, also supported by intuitive knowledge.

The theory confronts, however, a serious problem. The extant Xuanye sources do not explain how the motions of celestial bodies can be regulated if they are only driven by *qi*. From experience it is indeed clear that things driven by wind do not move in a regular way. We do not know whether this was the reason why most Chinese scholars disapproved of the Xuanye theory. The historical fact is that the theory of the Xuanye school was rejected by Chinese scholars from early times, so that the materials of this school were all lost. Since the second century, the Huntian School became dominant in China.

Together with the Gaitian and the Xuanye school, the Huntian cosmology is usually described as one of the three main cosmological theories in ancient China. According to Li Chunfeng 李淳風, the two pieces which constitute not only the earliest but also the most important sources about Huntian theory,

45 Tian star is Saturn.

46 In ancient China, the heaven is divided in 365 1/4 du.

47 I consulted Needham’s translation of the text, but changed it according to my own understanding of it. See: Joseph Needham, with collaboration of Wang Ling, *Science and Civilisation in China*. Cambridge: Cambridge University Press, 1959. Vol. 3.

48 A. C. Graham. *Disputers of the Dao, Philosophical argument in Ancient China*. P.101.

49 Zhuangzi claimed that 風之積也不厚，則其負大翼也無力。[If] the accumulation of wind is not thick, then it will not have enough force to support the large wings. For detailed information about this material, see the contribution of Zou Dahai, *The Concept of force (li 力) in Early China*, in this volume (chapter 2).

50 Guo Qingfan 郭慶藩 (1844-1896), *Zhuang Zi Jishi 莊子集釋 [Collected Explanations on Zhuang Zi]*, in *Zhuzi Jicheng 諸子集成 [Collected Books of Various Schools]* Edition. Shanghai, Shijie Shuju, 1937. Photocopied by Shanghai Shudian. 1991. p.4. For a translation and a detailed discussion of this statement, see the contribution of Zou Dahai, *The Concept of force (li 力) in Early China*, in this volume (chapter 2).

the *Huntian Yizhu* 渾天儀注 [*Commentary on the Armillary Sphere*] and the *Lingxian* 靈憲 [the *Spiritual Constitution of the Universe*]⁵¹, can both be traced back to Zhang Heng 張衡(78-139)⁵². In the *Lingxian*, Zhang Heng claims:

天體於陽，故圓以動；地體於陰，故平以靜。⁵³

the body of heaven located in the Yang, so it is round in order to move, the body of the earth located in Yin, so it is flat in order to be still.⁵⁴

天如雞子，地如雞中黃，孤居於天內，天大而地小。天表裏有水，天地各乘氣而立，載水而行。周天三百六十五度。天道者，貴順也。近天則滯，遠天則速。⁵⁵

The heaven is like a hen's egg; the earth is like the yolk of the egg, and lies alone inside the heaven. Heaven is large and earth small. Inside the surface of the heaven there is water. Both the heaven and the earth are supported by *qi*, and they move holding water. The complete heaven has 365 du. The Dao of heaven likes [things] to follow it. When [celestial bodies] are close to it, [they] are slow. When they are far from it, [they] are quick.

And in his *Huntian Yizhu*, Zhang Heng argued:

天轉如車轂之運也。⁵⁶

The rotation of heaven is just like the motion of the wheels of vehicles.

These materials inform us that according to the *Huntian* theory there is a material heaven, attributing the motion of the heaven to its round shape and the stationary character of the earth to its flatness. This part does not differ from the *Gaitian* theory. The main difference between the *Huntian* theory and the *Gaitian* theory is that the *Huntian* theory claims that the heaven is a sphere, or rather an ellipsoid, and that it moves around the earth. In Zhang Heng's theory, *qi* plays the role of supporting the heaven and the earth. Zhang Heng does not really clarify the relation between the celestial bodies and the heaven. It is generally believed that he thought that the celestial bodies were somehow attached to the heaven. But, as it is mentioned that the distance between the celestial bodies and the heaven changes, it seems that Zhang Heng actually believed that the celestial bodies are independent of the heaven. But they are obviously nevertheless affected by the heaven. When they move to a place nearer to the heaven, they became slower, as the heaven wants them to move with it. It is recorded that Zhang He made an Armillary Sphere and used this instrument to verify the *Huntian* theory. The position of the celestial bodies on the Armillary Sphere corresponds to the positions of the real ones. After this verification, the *Huntian* theory was commonly accepted as the most accurate cosmology in China.

Chinese scholars never ceased to think about the functioning of the universe. In the Song Dynasty, Zhang Zai 張載 argued that *qi* was the mover the Earth. He asserted that only the fixed stars are tied to the heaven, and that they move with the heaven. The sun, the earth and the five stars all move in a direction inverse to that of the heaven. The earth stays in the *qi*.

As for the reason of motion of the heaven and the celestial bodies, Zhang Zai claims:

凡圓轉之物，動必有機；既謂之機，則動非自外也。⁵⁷

For all things revolving, there should be mechanism for [their] motion. Since it is designated as a mechanism, the motion cannot be caused from outside.

51 The translation is quoted from Joseph Needham. Vol. 3, 199.

52 The historians of astronomy in China commonly accepted that the *Huntian Yizhu* was written by Zhang Heng. Chen Jiujin argued that the book should be written by an anonymous writer of the third or fourth century. Chen Meidong argues, however, that Chen Jiujin's argument is not convincing. See: Cheng Jiujin, Chen Meidong.

53 Ban Gu, *Hanshu* 漢書 [*Book on Han Dynasty*]. Beijing: Zhonghua Shuju. P. 3216

54 Zhang Heng, *Lingxian*. In *han Shu*. Beijing: Zhonghua Shuju.

55 Fang Xuanling, *Tianti* 天體. in *Jinshu*. P. 280.

56 Fang Xuanling, *Tianti* 天體. in *Jinshu*. P. 282.

57 Zhang Zai 張載, *Zhengmeng* 正蒙. Chapter *sanliang*.

Although he did not explicitly explain the mechanism of the motion of the heaven and the celestial bodies, he clearly believed that *qi* plays a crucial role in it. His discussion of the motion of the earth deserves our attention:

地有升降，日有修短。地雖凝聚不散之物，然二氣升降其間，相從而不已也。陽日上，地日降而下者，虛也；陽日降，地日進而上者，盈也；此一歲寒暑之候也。至於一晝夜之盈虛，升降，則以海水潮汐驗之為信。⁵⁸

The earth sometimes heightens and sometime lowers, and the [length] of day is sometimes long and sometimes short. Although the Earth is a coagulated thing and will not disperse, the two kinds of *qi* nevertheless rise and lower between [the earth and the heaven], and [the earth and the heaven] follow [the *qi*] and do not stop.⁵⁹ [If] the heaven rises each day, and the earth lowers itself each day, then, looseness [is produced]; [if] the heaven lowers itself each day, and the earth rises each day, [then,] the surplus [is produced]. This is [why] there are [different] climates of winter and summer in one year. As for the looseness and surplus, and the heightening and lowering of the heaven and earth during one day and night, the [theory] could be verified by the tides of seawater.

Zhang Zai attributed the difference of the length of day in a year to the fluctuation of heaven and the earth, where the fluctuation of the heaven and the earth is caused by *qi*. He thought that the earth, as a solid, was floating on water, and that the space between the heaven and the earth and the water is full of *qi*. Although *qi* is shapeless, it maintains the balance of heaven and earth. If the heaven rises, and the earth lowers, the space between the heaven and the earth increases, and the *qi* will be thin, or in Zhang Zai's words, looseness will be produced. Then the *qi* would not be enough for maintaining the distance between the heaven and the earth. Consequently, the heaven would lower itself, and the earth would rise. The lowering of the heaven and the rising of the earth cannot last forever. As when the distance between the heaven and the earth is shorter, the space for the *qi* between them is reduced, and, as a consequence, the *qi* becomes thick, so that stress will be produced. The result is that the heaven would be held up and the earth would be pushed down. Thus begins another round of motion. Zhang Zai claimed that his theory can be proved by the motion of tides. In order to explain why people cannot feel the motion of the earth, he referred to the experiential knowledge that when one sails in a large ship, one does not feel the motion of the ship in spite of the boat's motion.⁶⁰

3. Conclusion

This paper intends to provide an account of the reasoning about motion in ancient Chinese cosmological sources, and to analyze the process by which such theories have been constructed. With the exception of the legend of Gonggong, Chinese scholars always tried to find the explanations of the heavenly phenomena and, in particular, of the motion of celestial bodies with reference to the inner characters or to the mechanism of the heaven, the earth and the celestial bodies. Clearly, their task was not an easy one to achieve. From the above discussion, we may infer the following main characteristics of reasoning about motion in ancient Chinese cosmological sources:

1. The theories about motion in ancient cosmological sources were based on elementary observations of celestial phenomena.
2. The theories are developed with reference to intuitive and practical knowledge, rooted in the daily life of the ancient Chinese society. Such knowledge was often explicitly introduced in cosmological texts as evidence or explanation of the theories.
3. There were heated debates about cosmological theories in ancient China. The refutation of an existing theories may have been based on newly obtained observational results, or on previously neglected intuitive and experiential knowledge. After an old theory has been refuted, a new theory was established. In most cases, as in the example of the explanation of the motion of the celestial bodies, what remained convincing in the old theory would be included within the new theory.

58 Zhang Zai, *Zhengmeng*.

59 Here, Yang means the heaven. Zhang Zai claimed that the heaven is a floating yang in the previous part of this text.

60 The theory that the earth moves in not an invention of Zhang Zai.

Mechanics in the *Mohist Canon* and Its European Counterpart

Jürgen RENN and Matthias SCHEMMEL

1. What to Compare?

This paper¹ is concerned with the sections of the *Mohist Canon* 墨經, dating from about 300 B.C., that have traditionally been classified as concerning mechanics. Given the fact that the term mechanics usually refers to a branch of knowledge in the Western scientific tradition, the classification of the Mohist sections as concerning mechanics immediately raises the question of their comparability to anything occurring in the history of Western science. In fact, such comparisons have been made. Joseph Needham, for example, concluded his analysis of one of the Mohist sections with the following assessment:²

The most important thing about this excerpt on the lever and balance is that it shows that the Mohists must have been essentially in possession of the whole theory of equilibria as stated by Archimedes.

However, on closer inspection, the comparison of the *Mohist Canon* with Archimedes' book on the equilibrium of planes turns out to be problematic. A crucial aspect of Archimedes' work is that it is deductively structured. It is, in fact, this aspect which is often taken as one of the main reasons for judging it a scientific work. The *Mohist Canon*, in contrast, is not structured in this way. A further crucial aspect of Archimedes' work is that it contains and even proves the law of the lever—the first law of mechanics in the history of Western science. The Mohist text again does not contain a formulation of this law, let alone its mathematical proof in the Western sense.

While it thus appears that there is little justification to compare the mechanical sections in the *Mohist Canon* to Archimedes' theory on the equilibrium of planes, the question remains whether any parallel exists between the independent traditions of mechanics in China and the West. In order to answer this question, one has to start by re-examining the Mohist sections on mechanics. The obvious starting point for such a re-examination is the thorough philological reconstruction of the *Mohist Canon* by A. C. Graham.³ On the background of Graham's work, two features of the *Mohist Canon* have emerged as being particularly striking:

1. A basic structure of reasoning is common to all sections. It is characterized by treating mechanical problems as puzzles and is reflected by a coherent use of technical terms for mechanical qualities, such as *zhong* 重 for “weight” or *quan* 權 for “positional advantage,” as Graham's translation reads. (On the basis of our analysis we have proposed the latter term to be translated as “effectiveness,” see below.) This basic structure of reasoning is quite independent of any specific interpretation of the sometimes obscure passages.
2. That structure of reasoning is largely shaped by the role of the text in representing knowledge in a culture of dispute, characteristic of the Chinese philosophical practice of the time.⁴

1 This article has previously been published as: Jürgen Renn and Matthias Schemmel, “Mechanics in the Mohist Canon and Its European Counterpart,” in: Hans Ulrich Vogel, Christine Moll-Murata, and Gao Xuan (eds.), *Studies on Ancient Chinese Scientific and Technical Texts: Proceedings of the 3rd ISACBRST*, Zhengzhou: Elephant Press 2006, 24-31.

2 Joseph Needham, *Science and Civilisation in China* Vol. IV: *Physics and Physical Technology Part 1: Physics*, Cambridge: Cambridge University Press, 1962, p. 23.

3 Angus Charles Graham, *Later Mohist Logic, Ethics and Science*, Hong Kong: Chinese University Press, 1978. A detailed re-examination of the sections on mechanics on the basis of Grahams work has been jointly undertaken in the context of the cooperation with the Partner Group by a working group at the Max Planck Institute for the History of Science including William G. Boltz from the University of Washington in Seattle.

4 See also William G. Boltz, “Mechanics in the Mohist Canon: Preliminary Textual Questions,” in: Hans Ulrich Vogel, Christine Moll-Murata, and Gao Xuan (eds.), *Studies on Ancient Chinese Scientific and Technical Texts: Proceedings of the 3rd ISACBRST*, Zhengzhou: Elephant Press 2006, 32-40.

These features suggest comparing the mechanical sections in the *Mohist Canon* not to Archimedes' writings on the equilibrium of planes to which Needham is referring but rather to another ancient text documenting the emergence of a science of mechanics in Europe. This is the so-called *Mechanical Problems* ascribed to Aristotle and his school. The text was written in about the same period as the *Mohist Canon* and represents the earliest European text on mechanics handed down to us. In contrast to Archimedes' work, *Mechanical Problems* is not deductively structured and does not, in its original form, contain the law of the lever, or at least does not ascribe any central status to it. On the other hand, the text possesses the features just ascribed to the *Mohist Canon*: it also treats mechanical problems as puzzles and is shaped by a coherent structure of argumentation. This structure is reflected by the use of technical terminology, and has its origins in a culture of dispute characteristic of the Western philosophical tradition of the time.

In this paper we address the question of whether this parallel between the mechanical sections in the *Mohist Canon* and *Mechanical Problems* merely concerns the formal structure of the two texts, or whether it also extends to their substance, namely the kind of knowledge on mechanics they embody. As will become clear in the following, such a question cannot be approached by purely philological means. Rather, one has to take into account that there are different kinds of shared knowledge underlying scientific reasoning. We systematically distinguish three kinds of knowledge. These are intuitive, practical, and theoretical knowledge, which are distinct with regard to their sources, to their modes of transmission, and to their inner structure. Intuitive knowledge is acquired in the process of ontogenesis. Since the physical conditions of ontogenesis are largely culture-independent, a great part of this knowledge may be considered universal. Another kind of knowledge may be termed practical knowledge. This is expert knowledge acquired in the handling of artefacts such as mechanical instruments, devices, and machines, and is therefore as culture-dependent and subject to historical change as these artefacts are. For example, the knowledge acquired when using a lever can be termed practical knowledge. Finally the third kind of knowledge may be called theoretical knowledge. Theoretical knowledge is characterized by the use of symbolic representations as provided by written language, giving rise to a drive for consistency and the emergence of abstract terms. All three kinds of knowledge are structured by mental models.⁵ For instance, according to a typical model of intuitive physics, the greater a force, the larger its effect. The model also figures in the knowledge of practitioners who experience this relation of force and effect when handling their instruments. Finally, the model may be a part of theoretical knowledge, as is the case in Aristotelian physics, where it amounts to a theoretical statement of universal validity.

In the following we give a brief account on how the knowledge on mechanics embodied in the two texts, the sections on mechanics in the *Mohist Canon* and in *Mechanical Problems*, can be analyzed and compared within the theoretical framework outlined above.

2. The European Case

Mechanical Problems is the earliest surviving text on mechanics in the Western tradition. It was influential in this tradition until the advent of classical mechanics in the age of Galileo and Newton. The text consists of an introduction and 35 sections called "problems," which are often merely one paragraph in length and almost always begin with the phrase "Why is it that ... ?" (διὰ τί ...). The first three sections are theoretical in character and introduce basic concepts and principles, indicating their connections to one another. Several of the subsequent problems apply these principles to provide an explanation for a number of phenomena resulting from the use of devices that allow, as the author writes, the weaker to master the stronger. His entire enterprise starts from the question:⁶

διὰ τί κινουῦσι μέγαρα βάρη μικραὶ δυνάμεις τῷ μοχλῷ [...];

Why is it that small powers [can] move big loads when using the lever [...];

5 For the notion of mental model and its application in describing historical forms of thinking, see Dedre Gentner and Albert L. Stevens (eds.) *Mental Models*, Hillsdale: Erlbaum, 1983.

6 Aristotele, *Mechanical Problems*, 850a 30.

In discussing various mechanical devices, the author attempts to reduce them to the lever, which in turn is reduced to the balance with unequal arms, functioning according to the general principle:⁷

ἀεὶ δὲ πλεόν βαρος κινεῖ, ὅσῳ ἄν πλεόν ἀφεστήκη τοῦ ὑπομοχλίου ὁ κινῶν τὸ βάρος.

The further that which moves the load is away from the fulcrum, the more it moves the load.

This pattern of argument, also recognizable in the text due to the consistent technical terminology associated with it, in fact relates the transformation of forces by mechanical devices to experiences which can be gained by varying the lengths of the arms of unequal-armed balances.

While *Mechanical Problems* thus reflects the basic knowledge of practitioners, the text is clearly not motivated by practical concerns. The literary form of *Mechanical Problems* reflects the Greek problemata tradition which probably emerged from a real dialogical situation.⁸ Obviously, the topic of mechanical instruments was addressed by its author merely because they constituted a provocation to the Aristotelian system of natural philosophy. While *Mechanical Problems* was thus a rather marginal component of the corpus of writings of Aristotle and his school, in hindsight one may nevertheless perceive in it the origin of theoretical mechanics.

In our interpretation of *Mechanical Problems*, then, the text addresses the challenges to Aristotelian physics that are presented by technical devices which produce beneficial effects that seem contrary to nature. As mentioned in the beginning, the mental model implying that a greater force produces a greater effect had, in the context of Aristotelian physics, turned into a universal statement. The text responded to the puzzles raised by mechanical devices producing effects which appeared to contradict this model. It did so by extending this model to a model which was itself based on practical experience attained by using unequal-armed balances that had only recently become commonplace in Greece. In the case of this kind of balance, the effect of a weight depends on the weight's position on the beam. The model of the balance and its theoretical justification could therefore become a general scheme accounting for the unexpected behavior produced by various devices of ancient mechanical technology.

3. The Chinese Case

We now turn to the earliest text on mechanics from the Chinese tradition, again beginning with a short recapitulation of its essential features. What we call a “section” of the *Mohist Canon* is made up from a Canon in the proper sense and an Explanation co-ordinated with it. The basic structure of the *Mohist Canon* as reconstructed by Graham⁹ is twofold (see the table below). The sections of the *Mohist Canon* cover “four branches of knowledge.” The first one may be called logic, though it is not a logic of syllogisms, but rather a reflection on language offering procedures for consistent description in order to avoid paradoxes. The second is on ethics. Of interest here is the third branch that may be referred to as being concerned with science, and in which the sections on mechanics are found. The last on the art of disputation. Each branch of knowledge is dealt with in two parts. In one part certain basic terms are defined, in the other place complex problems are dealt with.

The four branches of knowledge	Definitions	Propositions
1. Explaining how to relate names to objects	“Reason,” “unit,” “knowing”	Procedures for consistent description
2. Explaining how to act	Conduct and government	(Expounding the Canons)
(Bridging part: knowledge and change)	Spatial and temporal conditions of knowing	Spatial and temporal conditions of knowing
3. Explaining objects	Geometry	Problems in optics, mechanics, and economics
4. Explaining words	disputation (<i>bian</i> 辯)	Problems in disputation

Table: Structure of the *Mohist Canon*

7 Aristotele, *Mechanical Problems*, 850b14-16.

8 On the tradition of problemata see, for example, Hellmut Flashar (ed.), *Aristoteles Werke in deutscher Übersetzung*, Vol. 2: Problemata physica, Berlin: Akademie Verlag, 1991, pp. 297-303.

9 See Graham *op. cit.* pp. 30-2, 229-35.

The section on which Needham bases his far-reaching claim mentioned above has been only partially preserved. The first phrase of the Explanation reads:¹⁰

(衡) 。 加重於其一旁必捶， 權重相若也。

(The beam.) If you add a weight to its [i.e. the beam's] one side [this side] will necessarily hang down. This is due to the effectiveness [of the weight] and the weight matching each other.

The passage may be illustrated by imagining a practical situation in which a beam is suspended with the help of a noose in such a way that the noose can be moved along the beam's length. If a weight is attached to one side of the beam then this side will hang down. Here, the term "weight" (*zhong* 重) is complemented with another term, the *quan* 權. In the *Mohist Canon* we understand this term as designating an abstract measure of the effect the weight has. In the case at hand, the weight and its effectiveness (*quan* 權) match each other, i.e. the effect of the weight is as expected: the side where the weight is placed goes down. So far, this is in accord with our expectations and would not have required the introduction of a technical term. Now, however, as the explanation continues, things get more involved:¹¹

相衡， 則本短標長。 兩加焉重相若， 則標 (= 標) 必下， 標得權也。

Level [both sides] up with each other, then the base is short and the tip is long. Add equal weights to both sides, then the tip will necessarily go down. This is due to the tip having gained effectiveness [of the weight].

Now the beam with the weight attached to one of its sides is brought into the horizontal position again. To achieve this, the fulcrum, i.e. the point of suspension, has to be moved. The result is that one side of the beam, when counted from the fulcrum, is shorter than the other. The Mohist calls the side having the weight attached to it the "base" (*ben* 本), which is now short, and the other side the "tip" (*biao* 標), which is now long. After adding equal weights to both sides of the beam, something unexpected can be observed. While intuitively it seems to be clear that equal weights cause equal effects, the tip can now be observed to decline. This is what one would expect if the weight laid on the side of the tip were greater than that laid on the side of the base. It thus seems that the weight on the "tip"-side is somehow more effective than that on the "base"-side. This is expressed by the statement that the tip has gained in effectiveness. Most probably, the lost canon referred to this phenomenon which forced the Mohist to introduce the technical term, "effectiveness."

The central question addressed by this passage is as follows: how can it be that one and the same heavy body has, under certain circumstances, a different effect from the one it normally has? It is answered by introducing a pair of abstract terms, weight and effectiveness, that differentiate the term weight in order to account for its different behavior under certain circumstances.

From this reconstruction it becomes clear that the essential feature of the above passage, a feature that it shares with other mechanical sections, is the confrontation of the natural behavior of an object with the modified behavior it displays under certain artificial circumstances. The mechanical arrangements producing these circumstances can sometimes be reconstructed, as is the case for the section just discussed, while they remain obscure in other sections. In any case, their significance lies in their interference with the naturally expected course of things, which yields a puzzling outcome. We thus encounter a beam that does not bend although it is burdened with a weight, a curtain that comes down by itself although it has first to be pulled up with an effort, or something that leans and cannot be set upright. The natural tendency of a weight would be to move down vertically by itself. But the interference with an artifice, such as pulling a weight up or to the side or supporting it from below, prevents this from happening in the contrived circumstances introduced in the text. This general feature of the argumentation is expressed in the text itself by the following statement:¹²

凡重， 上弗挈， 下弗收， 旁弗劫， 則下直。 * 施， 或害之也。

10 *Mohist Canon*, Section B 25b. Here and in all following quotations from the Mohist Canon we follow Grahams transcription of the Taoist Patrology text under omission of his textual comments (Graham *op. cit.*).

11 *Mohist Canon*, Section B 25b.

12 This passage is found in *Mohist Canon*, Section B 27. The asterisk (*) indicates that the following graph is a variant of the one originally found in the text.

Speaking generally about weight, when you are not pulling it up, and when you are not letting it down, and when you are not pulling it to the side, it comes straight down. When it comes down on a slant this is because something is interfering with it.

In the course of the argument, the Mohist introduces a number of technical terms for mechanical qualities. There are, for example, terms for letting a weight down, for suspending it from above, or for pushing it from the side. Some of the technical terms are used in order to account for the non-natural behavior described above. In the case of the beam, which we have discussed, the “effectiveness of the weight” (*quan* 權) is such a term.

As was the case for the Aristotelian *Mechanical Problems*, the argumentative structure that has become clear from the preceding interpretation can be understood as resulting from a reflection on shared practical knowledge in the context of a culture of disputation. The artifices occurring in the mechanical sections represent mechanical devices that evidently played a role in contemporary technology, for example, in the techniques of military engineering in which the Mohists are believed to have excelled. While the knowledge documented by their texts thus has a practical background, the issues they raise are clearly not practical problems but a matter of theoretical reflection, drawing on the means offered by contemporary philosophical discussion. Following Graham’s argument,¹³ the Mohists opposed the derivation of ethics from natural tendencies, as was advocated by the Confucian tradition in reaction to the so-called Individualists, in particular Yang Zhu 楊朱. They rather strove to demonstrate that ethics could consistently be grounded in the sphere of human intentions and actions. In a similar way, the Mohists’ occupation with mechanical problems appears to have been motivated by the philosophical concern to show that, while mechanical processes induced by man may not occur as intuitively expected, they still remain rationally comprehensible.

On the basis of the mental model of intuitive physics representing the idea that a greater force has a greater effect, the structure of the mechanical sections may then be understood as follows. The model implies that equal weights have an equal effect. The practical experience gained in handling mechanical devices violates this model, for example, when equal weights laid on a beam have different effects. The Mohists’ theoretical reflection consolidates this conflict between practical knowledge about mechanical processes and intuitive knowledge about what would naturally occur by differentiating what should be considered a cause of the effect, thus enriching and thereby restoring the original mental model.

4. The Parallel Origins of Chinese and European Mechanics

In conclusion, let us come back to our original question: Does the structural parallel between the mechanical sections in the *Mohist Canon* and *Mechanical Problems* extend to the kind of knowledge on mechanics that is embodied in the texts?

We have argued here that this is indeed the case: Both texts were the result of a theoretical reflection on practical knowledge, induced in the context of specific cultures of disputation. The practical knowledge, which constituted the empirical basis of the texts, was in turn transformed by theoretical reflection. One consequence of this reflection was the drive for a consistency of reasoning atypical for intuitive or even practical knowledge and, as a result, the universality of the resulting argumentation.

In China as well as in Europe, neither the existence of a culture of disputation nor its specific concerns had, in the first place, anything to do with practical mechanical knowledge. In this sense, the emergence of theoretical knowledge on mechanics was a contingent historical event that was dependent on specific cultural circumstances. As it turned out, however, the similar discursive practices in both cultures shaped the reflection on practical mechanical knowledge in a similar albeit non-identical way. It resulted in fact in such different abstract concepts as the concept of “center of gravity” in the European tradition and the concepts of “effectiveness of weight” or “degree of fixed rigidity” in the Chinese tradition.

The specific character of theoretical mechanics at its origin was as transient as the historical context that brought it about. What remained as the substance of scientific mechanics in the long run was not a specific literary form shaped by this context but rather, first of all, the mental models of intuitive and

13 See Graham *op. cit.* pp. 15–25.

practical knowledge that remained stable over long periods of time due to the continuity of craftsmanship and engineering, and second, the mental models of theoretical knowledge and the abstract concepts associated with them, at least as long as they were handed down in a theoretical tradition dependant on the transmission of written texts. A continuous practical tradition existed in the Chinese as well as in the European case. But while, according to our interpretation, a theoretical mechanics emerged on this basis independently both in Europe and in China, it is the continuity of the theoretical tradition that was interrupted at a very early stage in China. Evidently, the conditions for the genesis of a scientific tradition are different from those for its long-term survival.

5. References

Boltz, William G. "Mechanics in the 'Mohist Canon': Preliminary Textual Questions," in: Hans Ulrich Vogel, Christine Moll-Murata, and Gao Xuan (eds.), *Studies on Ancient Chinese Scientific and Technical Texts: Proceedings of the 3rd ISACBRST*, Zhengzhou: Elephant Press 2006.

Flashar, Hellmut (ed.). *Aristoteles Werke in deutscher Übersetzung*, Vol. 2: *Problemata physica*, Berlin: Akademie Verlag, 1991.

Gentner, Dedre and Albert L. Stevens (eds.). *Mental Models*, Hillsdale: Erlbaum, 1983.

Graham, Angus Charles. *Later Mohist Logic, Ethics and Science*, Hong Kong: Chinese University Press, 1978.

Needham, Joseph. *Science and Civilisation in China* Vol. IV: *Physics and Physical Technology Part 1: Physics*, Cambridge: Cambridge University Press, 1962.

Mechanical Knowledge in the *Jiuzhang Suanshu*

TIAN Miao

“Mechanics” was not an independent branch of learning in ancient China. Thinking and knowledge concerning mechanics existed in scattered philosophical texts, technological texts, astronomical texts and mathematical texts. In this article, I will investigate problems relating to mechanical knowledge contained in the most influential Chinese mathematical work, the *Jiuzhang suanshu*¹, and evaluate the characteristics of that knowledge.²

The topic of this paper not only concerns mechanical knowledge, but also related algorithms and mathematical terms that were specially developed or existed in ancient China. Before getting into a detailed discussion of mechanical knowledge, I will first give a short introduction to the *Jiuzhang suanshu* and the basic character of mathematics at the time when it was compiled in ancient China. I will then discuss in detail the problems relating to mechanical knowledge in the *Jiuzhang suanshu*. In closing, after providing a summary of the whole paper, I will give my own analysis of the general attitude of ancient Chinese mathematicians towards problems relating to mechanical knowledge.

1. A Short Introduction to the *Jiuzhang Suanshu*

It is possible that readers who are familiar with mathematics and the relation between mathematics and mechanics in the ancient Greek tradition will be puzzled by the mathematical thought and knowledge of mechanics in ancient China. It is true that the system and mode of reasoning found in the *Jiuzhang suanshu* are different from the ones we associate with ancient Greek mathematics, and especially with Euclid’s *Elements*. If we agree that Greek mathematics was concerned primarily with the art of demonstration, Chinese mathematics was devoted primarily to the art of calculation. In fact, the original name for what we now regard as mathematics was *suanshu* 算術 or 筭術 in ancient China. The character *suan* 算 or 筭 means calculation³ and *shu* 術 originally meant road, but could also be used to connote “technique” in the sense of an “art.” Literally, *suanshu* may be translated as the art of calculation.⁴ We can imagine how great the differences between results might be when derived from two

1 Unless otherwise noted, all translations from the *Jiuzhang suanshu* are by the author. I have also relied at times on the recent translation and commentary by Karine Chemla and Guo Shuchun, *Les neuf Chapitres*. Paris: Dunod, 2004. An English translation is available by Shen Kangshen, J. N. Crossley, and A. W.-C. Lun, *The Nine Chapters on the Mathematical Art*. New York: Oxford University Press; Beijing: Science Press. 1999. Professor Guo Shuchun has also been very generous in discussing these translations with me, and has offered many valuable suggestions, for which I am pleased to express my sincere thanks.

2 Historians of ancient Chinese mathematics have been concerned largely with abstracting and analyzing the mathematical methods and thought to be found in ancient texts, but have paid much less attention to the concrete meaning of their content. Thus, problems we would associate today with mathematical mechanics have largely been ignored. In their paper “Commentary on Ancient Chinese Mathematical Books in the Light of Physics”, Dai Nianzu, Wei Zhong, and Wang Dakai do consider some problems concerned with mechanics in several ancient mathematical works. Their work represents a new point of view with respect to the study of mathematics in ancient China. See: Dai Nianzu 戴念祖, Weizhong 衛中, Wang Dakai 汪達開, Yi wulixue de guandian ping Zhongguo gudai Shuxue Zhuzuo 以物理學的觀點評中國古代數學著作 [Commentary on Ancient Chinese Mathematical Books in the Light of Physics]. *Ziran Kexueshi Yanjiu* 自然科學史研究 [Studies on the History of Natural Sciences]. Vol. 16, Issue 2, p. 161-173. However, these authors are interested primarily in checking and interpreting ancient texts on modern terms, and do not follow the methods in these texts and their origins closely enough.

3 Xu Shen 許慎 (58-147) defined the character 筭 as rod of calculation: “Suan: 6 *cun* in length, which is used to calculating calendar and numbers. [It] follows bamboo [and] follows manage” (長六寸，所以計算數者。從竹從弄。). 算 was defined as: “Suan is number. [It] follows bamboo [and] follows tool.” (數也，從竹從具). According to Xu Shen, 筭 is the counting rod, and 算 means calculation. Xu Shen, *Shuowen Jiezi* 說文解字 [Explaining simple characters and analyzing compound characters]. Finished between 100-121 A.D. Chen Changzhi printed edition, 1873. [Reprinted], chapter 5, p. 9a.

4 This does not mean that there are no proofs in ancient Chinese mathematical text. In order to establish the correctness of algorithms, some mathematicians added commentaries to argue the reason of them. Liu Hui’s commentaries on the *Jiuzhang suanshu* are the most important sources for research on the mode of mathematical thinking and proof in ancient China. About mathematical proof in ancient China, see: Karine Chemla, “What is at

such different bases. By understanding the relation between mathematics and mechanical knowledge, we may find some traces of such differences. In the *Mechanical Problems*, Aristotle (or possibly one of his followers) defined mechanical problems as those which “have a share in both mathematical and physical speculations, for the method is demonstrated by mathematics, but the practical application belongs to physics.”⁵ Chinese scholars, on the other hand, considered mathematics and numbers as follows: “numbers are one, ten, thousand, ten thousand. [Mathematics] is used to calculate and count things and put the principles of life in proper order.”⁶ The range of mathematics thus encompass the following aspects:

夫推历、生律、制器、规圆矩方，权重、衡平、准绳、嘉量，探赜索隐，钩深至远，莫不用焉。度长短者不失豪厘，量多少者不失圭撮，权轻重者不失黍粟⁷。

Calculating calendars, producing temperaments, making devices, [drawing] circles with compasses and squares with rulers, determining weights, judging the levelness [of things], [using] leveling strings and standard measuring vessels, probing the hidden mysteries and exploring deep and far, none of these does not use [mathematics]. [Using mathematics], one who measures length will not miss an iota (*haoli* 豪厘), one who measures amount [of thing] will not miss a drop (*guicuo* 圭撮), [and] one who measures weight will not miss a pinch (*shulei* 黍粟)⁸.

According to this statement, making devices and determining weights are both directly connected with mathematics in the sense of calculation. Thus, we should not expect to find a deductive system or qualitative discussion of practical machines based on principles of the circle and level in Chinese mathematical books⁹. Instead, what we do find are a number of quantitative results concerned with mechanical problems, and the way how ancient Chinese deal with mechanical problems using the established mathematical procedures.

Concerning the *Jiuzhang suanshu*, it is the earliest Chinese mathematical book coming down to us in writing. According to its commentator, Liu Hui 劉徽 (3rd century), there was a *Jiuzhang suanshu* compiled in Pre-Qin times (before 221 B.C.), and the main subjects of that book could be traced back to the *Jiushu* 九數 [nine branches of mathematics] system of the Zhou Dynasty (11th century B.C.). However, as a whole, the book was lost in the Qin Dynasty (221 B.C.-206 B.C.). In the first century B.C., two officials who were good at mathematics, Zhang Cang 張蒼 (?-152 B.C.) and Geng Shouchang 耿壽昌 (1st century B.C.), worked on a compilation of the *Jiuzhang suanshu* successively.¹⁰ In the year of 263, Liu Hui wrote his commentary on the *Jiuzhang suanshu*, which was the most ancient and important research on the book that has come down to us. Between 648 and 656, Li Chunfeng 李淳風 and his subordinates also made commentaries based on the text of the *Jiuzhang suanshu* and Liu Hui’s

Stake in Mathematical Proofs from Third Century China?”. *Science in Context*. 10 (1997). 227-251; Guo Shuchun, *Gudai Shijie Suxue Taidou Liu Hui* 古代世界數學泰斗劉徽 [An ancient world-wide leading Mathematician, Liu Hui]. Jinan: Shandong Kexue Jishu Chubanshe. 1992. For source materials and modern research concerning the *Jiuzhang suanshu*, see: Karine Chemla and Guo Shuchun, *Les neuf Chapitres*. Paris: Dunod, 2004. Pp. 1043-1090.

5 Pseudo-Aristotle, *Mechanical Problems* (translated into English by W. Hett). in *Aristotle* (W. S. Hett). Cambridge, Massachusetts: Harvard University Press. 1980. XIV, p. 331.

6 Ban Gu, *Hanshu* 前漢書 [Historical book of the Han Dynasty], in Wang Xianqian 王先謙, *Hanshu buzhu* 漢書補注 [supplementary commentary of Hanshu]. Xushoutang 虛受堂 edition, 1900, chapter 21. p. 2b-3a.

7 Ban Gu, *Hanshu*, Chapter 21, p. 2b-3a.

8 *Hao* 豪, *li* 厘 are the two last minimal units of length, *gui* 圭, *cuo* 撮 are the last minimal units of volume, and *shu* 黍 *lei* 粟 are the two last minimal units of weight. This sentence means that with mathematical method, one may have accurate measurement.

9 The “Mechanical problems” is arranged in such way. See: Pseudo-Aristotle, *Mechanical Problems*. 329-411. An analysis of Aristotelian mechanics, See: René Dugas, *A History of Mechanics*. Neuchatel: du Griffon. 1955. p. 19-24.

10 The date of the Compilation of the *Jiuzhang suanshu* is a well-studied subject. Based on Liu Hui’s preface of the *Jiuzhang suanshu*, which is the earliest document remained to us about the process of the forming of the book, Guo Shuchun 郭書春 points out that the book is completed in the first century B.C. at the latest. See: Guo Shuchun, preface, in *Jiuzhang suanshu huijiao* 《九章算術》彙校. [A Convergent Collation of *Jiuzhang suanshu*]. Shenyang: Liaoning jiaoyu chubanshe, 1990. Li Hui, preface, *Jiuzhang suanshu*, Wuying Dian Juzheng edition. French translation of Liu Hui’s preface, See: Karine Chemla. Guo Shuchun, *Les neuf chapitres le classique mathématique de la Chine ancienne et ses commentaires*. p. 126-129. For the Transmission of the text of the *Jiuzhang suanshu*, see: Guo Shuchun, preface, in *Jiuzhang suanshu huijiao*.

commentary. Afterwards, the *Jiuzhang suanshu* served as a model of Chinese mathematical work and was the main source of mathematical education in China.

The *Jiuzhang suanshu* is composed of nine chapters¹¹:

Chapter 1, *fangtian* 方田 [Square field]: The methods for calculating the boundaries and areas of fields.¹² In this chapter, the methods for calculating areas of rectangles, triangles, trapezoids, curvilinear figures (circles, segments, annuli and spherical crowns), and some basic algorithms involving computations with integers and fractions are given. In this chapter, Liu Hui stressed the significance of the *qitongshu* 齐同术 [art of harmonization and equalization]. The *qitongshu* was introduced in the method of addition of fractions. Equalization means to find a common denominator, and harmonization means to refigure the numerator in order to "harmonize" the numerator with the common denominator. This is a simple and basic method in arithmetic. However, Liu Hui pointed out that this *qitongshu* is a guiding principle of mathematics. He used the *qitong* principle to justify several different algorithms in the *Jiuzhang suanshu*¹³.

Chapter 2, *sumi* 粟米 [millet]: The main concern of this chapter is exchange and transformation of different goods¹⁴. The key mathematical method in this chapter is *jinyoushu* 今有術 [method of supposition]¹⁵, which is equivalent to the Rule of Three in European mathematics. The four terms of the proportional factors are as the following: *suoyou shu* 所有數 [the given number], *suoyou lü* 所有率 [the given *lü*], *suoqiu lü* 所求率 [the *lü* sought] and *suoqiu shu* 所求數 [the number sought].¹⁶ The general method is given as: "take the given number multiplied by the *lü* sought, [the result] is the dividend; take the given *lü* as the divisor. Dividing the dividend by the divisor, [one] gets the result."¹⁷ Liu Hui pointed out in his commentary on this method: "this is a great method 此都術也." He stresses that the *jinyou shu* is the method by which many complicated problems could be solved.¹⁸

Chapter 3, *cui fen* 衰分 [distribution by proportion]: The main mathematical method in this chapter is the *cui fen shu* 衰分術 [Procedure of distribution by proportion]: Add all of the proportional rates as the divisor, multiply the total number of things being distributed by each rate as the dividend. Dividing the

11 From the names of chapters and algorithms in the *Jiuzhang suanshu*, we may easily find their practical origins. These names are somehow chosen randomly from a topic of the first problem in the chapter or even the first term mentioned in the procedure of the algorithm. Though a detailed analysis, we could find that there is a system of algorithms existed in the text. For detail, See: Karine Chamela, Guo Shuchun, *Les neuf chapitres le classique mathématique de la Chine ancienne et ses commentaires*. p. 3-42.

12 Liu Hui, commentary on *fangtian*, in *Jiuzhang suanshu*, chapter 1. 1a.

13 For example, he provided a method for solving problems of linear equation system, which is equivalent to the method we use in modern time. Liu Hui stated, the procedure of multiplying the first coefficient of one equation with the one of the other equation respectively is of same significant with multiplying the divisor of one fraction with the one of the other fraction respectively when one add the two fractions, which was named as *tong* [homogenization] in the *qitong* method. And the procedure of multiplying the same number, the first coefficient of one equation, with each coefficients of other equation is of same significant with multiplying the dividend with the divisor of the other fraction, which was named as *qi* [harmonization]. Even the *qitong* method was not mentioned in such occasion, the *yi* [meaning] of *qitong* was existed there. A detailed discussion about the significant of *qitongshu*, see: Karine Chemla, *Esegesi e dimostrazione: I commentari ai 'Nove capitoli sui procedimenti matematici'* (Mathematics and exegesis: the commentaries on *The nine chapters on mathematical procedures*). in K. Chemla (gen.ed.), In collaboration with F. Bray, Fu Daiwie, Huang Yi-Long, G. Métailié (eds), *La scienza in Cina*, in: Sandro Petruccioli (gen.ed.), *Storia della scienza*, vol. II. Enciclopedia Italiana, Roma, 2001. p.142-149.

14 Liu Hui, Commentary on *Sumi*, in anonymous author, *Jiuzhang suanshu*. Beijing: *Wuying dian juzhen edition*, 1774. Chapter 2. 1a.

15 Liu Hui believed that the term *Jinyou* 今有 means the *jinyou lü* 今有率 [*lü* of having], which is the term of one proportional factor in the procedure. However, I believe that the term was just chosen in random. As most problems in the *Jiuzhang suanshu* begins with the term *jinyou*, which means suppose that. See: Liu Hui, commentary. *Jiuzhang suanshu*, chapter 2, 2a.

16 *Lü* is a general term used commonly in Chinese mathematical books. Liu Hui defined the term as: "For all the numbers with relation to other number, they are named as *lü*" [凡數相與者謂之率]. In the text, both numbers with relation and the relation between them are named as *lü*. Liu Hui, Commentary, *Jiuzhang suanshu*, chapter 1, p. 8b. For a detailed discussion about *lü*, see: Karine Chemla, Guo Shuchun, *Les neuf chapitres le classique mathématique de la Chine ancienne et ses commentaires*, pp. 956-959.

17 *Jiuzhang suanshu*, chapter 2. 2b-3a.

18 Liu Hui, commentary. *Jiuzhang suanshu*, chapter 2, 2a.

dividend by the divisor, the result is the share one should receive¹⁹. Some problems in this chapter use this method to calculate the sum of finite series of arithmetical and geometrical progressions.

Chapter 4, *shaoguang* 少广 [short width]: The topic of this chapter is calculations involving relations between areas, widths and lengths of rectangular fields, and the relations between diameters and areas of circular fields. Methods are given in this chapter for dividing by sum of integers and unit fractions as divisors, extracting square and cube roots, finding the circumferences of circles of known area, and finding the diameters of spheres of known volume.

Chapter 5, *shangong* 商功 [discussion of work]: the main part of this chapter is about methods concerning with calculation of work. These include algorithms for calculating volumes of different solids, including cylinders, cones and frusta of cones, rectangular parallelepipeds, prisms of constant trapezoidal cross-section, frustums of pyramid and pyramids, right triangular prisms, and some solids with curved surfaces and some other solids which have no corresponding names in Western mathematics, such as *yangma* 陽馬, *bienao* 鱉臚, *xianchu* 羨除, *chumeng* 芻甍, *chutong* 芻童, *panchi* 盤池, *minggu* 冥穀, *Quchi* 曲池, etc.

Chapter 6, *Junshu* 均輸 [fair transposition]: The main part of this chapter is concerned with levying taxes according to the distance between the taxpayer and the capital, and the number of taxpayers. In his commentary on the tenth problem of this chapter, Liu Hui named a method concerning two proportional relationships as *chong jinyoushu* 重今有術 [compound *jinyoushu*]. There are at least three pairs of quantities involved in the *chong jinyoushu*. Suppose that $a : b = c : d$, and at the same time $a : b = e : f$. Now, if both c, d, e, f are changed simultaneously, then, $a : b = ce : df$. Thus,

$$a = \frac{c \cdot e}{d \cdot f} \cdot b$$

In Liu Hui's words: "This is named *chong jinyou*. Even there are *lü* respectively. [We] do not look into the [processes] in the middle. So, let the former dividend multiply the latter dividend, and the former divisor multiply the latter divisor, and divide [them] in combination."²⁰ One problem concerning a weight in motion used this method.

Chapter 7, *yingbuzu* 盈不足 [excess and deficiency]: the topic of this chapter is how to deal with problems involving with excess and deficiency. The standard lay-out of such problems is: "suppose there are [persons] buy thing together. [Each] person pays 8 [*qian*], there are 3 [*qian*]’s surplus; [each] person pays seven [*qian*] there are 4 [*qian*]’s deficiency. Ask: how many is the number of person and the price of thing?"²¹ The *yingbuzu shu* is equivalent to the method of double false position in Western mathematics²². There are also some problems in this chapter that have no connection with excess and deficiency, but the compiler of the *Jiuzhang suanshu* constructed situations for which the method of

19 Liu Hui attributed this problem to the *jinyoushu*. Liu Hui, Commentary. *Jiuzhang suanshu*, chapter 3. 1a-b.

20 Liu Hui, commentary, *Jiuzhang suanshu*, chapter 6. 16b. The original text: 是謂重今有也。雖各有率，不問中間。故令後實乘前實，後法乘前法而並除也。

21 Anonymous author, *Jiuzhang suanshu*, chapter 7. 1a.

22 Qian Baochong argued that the *yingbuzu shu* was first invented in China, and it was transmitted into Arabic area, then to the Europe. At the beginning of 17th century, Jesuit missionary, Matteo Ricci (1552-1610) brought the European double false position method into China. At that time, parts of the knowledge in *Jiuzhang suanshu* could not be understood by Chinese scholars. Ricci's converted student Li Zhizao argued that the method of double false position is more advanced than traditional *yingbuzu* method. See: Qian Baochong 钱宝琮, *Jiuzhang Suanshu yingbuzushu liuchuan Ouzhou kao* 《九章算术》盈不足术流传欧洲考 [an investigation on the transmission of *yingbuzu shu* in the *Jiuzhang suanshu* to Europe]. in *Qian Baochong Kexue shi Lunwen xuanji* 钱宝琮科学史论文选集 [A Collection of Selected Qian Baochong's Papers on History of Science]. Beijing: Kexue Chubanshe, 1993. 83-96; Karine Chemla, Reflections on the world-wide history of the rule of false double position, or: how a loop was closed. *Centaurus*, 39. p. 97-120.

excess and deficiency could be used to solve them. We will discuss several such problems in detail in this paper.

Chapter 8, *fangcheng* 方程 [square]: The *fangcheng shu* 方程術 [*the art of fangcheng*] is equivalent to solving systems of linear equations. This method was invented to solve complicated problems.

Chapter 9, *gougu* 句股 [Right angled triangle]: this chapter is devoted to problems concerning right angled triangles.

There are a total of 246 problems in the *Jiuzhang suanshu*, among which, 22 problems relate to mechanical knowledge. Based upon their mechanical content, these problems belong to three groups: problems of weights, movement, and weights in motion. In the following, I will discuss each of these three groups of problems in turn. However, it should be kept in mind that this division is not in accordance with the original classification of the *Jiuzhang suanshu*, a point to which we will return.

2. The Calculation of Weight in the *Jiuzhang Suanshu*

There are six problems dealing with calculation of weights in the *Jiuzhang suanshu*. We first discuss a problem concerning specific gravity.

2.1. Knowledge Concerning Specific Gravity in the *Jiuzhang Suanshu*

The 12th problem in the *yingbuzu* chapter of the *Jiuzhang suanshu* shows how ancient Chinese mathematicians treated problems concerned with specific gravity.

今有玉方一寸重七兩，石方一寸重六。今有石立方三寸中有玉，並重十一斤，問玉石重各幾何。

Now given that [a cube of] jade with a side of one *cun* weighs 7 *liang*²³, [and a cube of] stone with a side of one *cun* weighs 6 *liang*, [suppose that] now there is a stone cube [with a side] of 3 *cun*, which contains jade. The whole [cube] weighs 11 *jin*. Question: How much do the jade and the stone [parts] each weigh?

答曰：玉一十四寸重六斤二兩。石一十三寸重四斤一十四兩。

術曰：假令皆玉，多十三兩，令之皆石，不足一十四兩。不足爲玉，多爲石，各以一寸之重乘之。得玉石之積重。

Answer: Jade, 14 [cubic] *cun* [and] weighs 6 *jin* and two *liang*; stone, 13 [cubic] *cun*, [and] weighs 4 *jin*, 14 *liang*.

Procedure: If the whole [piece] were [made of] jade, [then there would be] an excess of 13 *liang*; if the whole [piece] were [made of] stone, [there would be] a deficit of 14 *liang*. The deficit is equal to the [volume of] the jade [part], [and] the excess is equal to the [volume of] the stone part. Multiply each [volume] by the weight of a one *cun* [cube made of jade or stone respectively]²⁴. [Thus one] obtains the volume and weight of jade and stone.

In his commentary on this problem, Liu Hui gives a more detailed description of the reasoning behind this procedure:

立方三寸是一面之方，計積二十七寸，玉方一寸重七兩，石方一寸重六兩，是爲玉石重差一兩。假令皆玉，合有一百八十九兩，課於一十一斤，有餘一十三兩。玉重而石輕，故有此多，即二十七寸之中，有十三寸，寸損一兩，則以爲石重，故言多爲石。言多之數出於石以爲玉。假令皆石，合有一百六十二兩，課於一十一斤，少十四兩，故曰不足。此不足即以重爲輕，故令減少數于石重，即二十七寸之中有十四寸，寸增一兩也。²⁵

23 *Liang* 兩 is an unit of weight in China. In the *Luli zhi* 律曆志 (*records on temperament and astronomy*) of *Hanshu*, there is a description about the units of weight: “24 *zhu* 銖 is [one] *liang*, 16 *liang* is [one] *jin* 斤, 30 *jin* is [one] *jun* 鈞, 4 *jun* is one *dan*.” See: Ban Gu, *Qian Hanshu*, chapter 21, 20a.

24 In other words, multiply the deficit by the weight of a one *cun* jade cube and the excess by the weight of one *cun* stone cube.

25 Anonymous author, *Jiuzhang suanshu*, chapter 7, 11a-b.

A cube of 3 *cun* [means] each face has a side of [3 *cun*]. Calculating the volume, [the result] is 27 [cubic] *cun*. A jade cube with a side of one *cun* weighs seven *liang*, [and] a stone cube with a side one *cun* weighs 6 *liang*. This means that the difference in weight between [a one *cun*] jade [cube] and [a one *cun*] stone [cube] is one *liang*. If [the whole piece] is jade, [its weight] should be 189 *liang*. Comparing [this weight] with 11 *jin*, there is an excess of 13 *liang*. Jade is heavy and stone is light. That is why there is such an excess. That means, within the 27 [cubic] *cun* [of stone], there is 13 [cubic] *cun* [that is not jade]; if [this 13 cubic *cun* were] decreased by one *liang* per [cubic] *cun*, [this] would change [the weight of jade part] to the weight of stone. Therefore, it is said that the excess is equal to [the volume of] stone. This [means] that the amount of excess is because the stone [contained within the cube] is taken for jade. If the whole piece were stone, [its weight] would be 162 *liang*. Comparing [this] to 11 *jin*, it is 14 *liang* less. That is why it is said [that there is] a deficit. This deficit is [because] a heavy [thing] is taken for a light [thing]. Therefore, now subtract the difference [of 14 *liang*] from the weight of the stone²⁶. This means, in 27 [cubic] *cun*, there are 14 [cubic] *cun*, [which should be] increased by one *liang* [per cubic] *cun*.

Liu Hui's commentary provides a clear explanation of the reasoning behind the procedure given in the main text of the *Jiuzhang suanshu*.

In the above problem, the weights of pieces of stone and jade of standard shape and standard size were given at the beginning. The way in which these were expressed and the roles these quantities played in calculations indicate that they are the same as what is meant by specific gravity in modern sense. But was this a matter of chance, or did the compiler of the *Jiuzhang suanshu* consciously try to determine such quantities? That is, did the ancient Chinese at the time when the *Jiuzhang suanshu* was compiled try to find out the weights of some materials of unit size, and then use them to calculate the weights of things made of these materials in the same way that we use specific gravity today? I believe that the ancient Chinese may have had this same concept of specific gravity in mind based upon their own experience.

The above problem in the *Jiuzhang suanshu* is by no means an isolated example of the use of specific gravity in ancient Chinese texts. In the *Shihuo zhi* 食貨志 [record of food and economy] of the *Hanshu* 漢書, "gold with the side of [one] *cun* weighs one *jin*" is given²⁷. Qiu Guangming believes that this shows that the weight of gold of unit size was used as a standard unit of weight²⁸. In a mathematical book more than half a millennium later than the *Jiuzhang suanshu*, the *Sunzi suanjing* 孫子算經 [Mathematical Classic of Master Sun, about 400 A.D.]²⁹, there is the following table of weights of materials of unit size:

黃金方寸重一斤
 白銀方寸重一十四兩
 玉方寸重一十兩
 銅方寸重七兩半
 鉛方寸重九兩半
 鐵方寸六兩
 石方寸重三兩³⁰

26 Here, the stone means the whole piece containing both stone and jade.

27 The original text in the *Hanshu*: 黃金方寸重一斤. Ban Gu, *Hanshu*, chapter 24, second part, p. 1a-b.

28 Qiu Guangming 丘光明, *Zhongguo gudai duliangheng* 中國古代度量衡 [Weights and measures in ancient China]. Beijing: Shangwu Yinshu Guan, 1996. 103.

29 See: Qian Baocong, *Sunzi Suanjing tiyao* 孫子算經提要 [introduction to *Mathematical Canon of Master Su*], in *Suanjing Shishu* 算經十書 [Ten Classics of Mathematics] (Qian Baocong critique ed.), in *Li Yan Qian Baocong kexue shi quanji* 李儼錢寶琮科學史全集 [A Complete Collected Edition of Li Yan and Qian Baocong's Publication on History of Science]. Dalian: Liaoning jiaoyu chubanshe, 1998. [Reprinted] Vol. IV. P. 218.

30 Sun Zi 孫子, *Sunzi suanjing* 孫子算經. Beijing: Wuyingdian juzhen edition, 1774, chapter 1, p. 2a-b. There was not a standard system of measures and weights in the time when the *Sunzi suanjing* was written. We know that the system of the Xinchao 新朝 [New Dynasty] (Wang Mang 王莽 Reign, 9 B.C.-24 A.D.) was still in used. Liu Fu made a detailed study on the system of the Xinchao, according to his result, in that system, one *jin* is equal to 226.7

A golden cube with side of [one] *cun* weighs 1 *jin*
 A silver cube with side of [one] *cun* weighs 14 *liang*
 A jade cube with side of [one] *cun* weighs 10 *liang*
 A copper cube with side of [one] *cun* weighs 7 and a half *liang*
 A lead cube with side of [one] *cun* weighs 9 and a half *liang*
 A iron cube with side of [one] *cun* weighs 6 *liang*
 A stone with side of [one] *cun* weighs 3 *liang*

The way of expressing relative weights in this table is in accordance with the one given in the problem in the *Jiuzhang suanshu*, and was followed by later mathematicians³¹. Relying on the above materials, we may conclude that the ancient Chinese consciously measured the specific gravity of different materials, and they used a formal way to express specific gravity which is identical to the one given in the *Jiuzhang suanshu*.

From this method used in the *Jiuzhang suanshu*, it is clear that ancient Chinese mathematicians were already familiar with the method of calculating weights of objects from their volumes, and knowing the specific gravities of the materials of which they were made. Taking their ability to calculate the volumes of solid objects and vessels or various shapes into consideration, it is likely that they could also calculate the weights of objects of different shapes. It is well known that the calculation of the weights of objects is of great importance in official affairs, however, in the *Jiuzhang suanshu*, there is not a chapter or even a section devoted to this special subject. Instead, there is only one problem concerning specific gravity, for which a very complicated situation is given, and the method of solution is truly ingenious. This is crucial to understanding the character of the development of mathematics in ancient China. We will come back to this point later.

2.2. Problems Concerning the Balance

There are no problems about the function of the balance or steelyard in the *Jiuzhang suanshu*, but two problems concerning the balance both focus on ways of calculating weights.

The ninth problem of the Fancheng chapter:

今有五雀六燕集稱之衡。雀俱重，燕俱輕。一雀一燕交而處，衡適平。並雀燕重一斤，問雀燕一枚各重幾何？

答曰：雀重一兩一十九分兩之一十三，燕重一兩一十九分兩之五。

術曰：如方程，交易質之，各重八兩。

Now [a group of] five sparrows and [a group of] six swallows gather on [opposite ends of] the beam of a balance. The [group of] sparrows are heavy, and the [group of] swallows are light. One swallow changes place with one sparrow; the beam is just level. Altogether, the sparrows and swallows weigh one *jin*. Question: how much do one sparrow and one swallow each weigh?

Answer: [One] sparrow weighs 1 and 13/19 *liang*; [one] swallow weighs 1 and 5/19 *liang*.

gram; one chi is equal to 23.1 centimeters. See: Liu Fu 刘复, *Xin jialiang zhi jiaoliang yu tuisuan* 新嘉量之校量与推算 [Examination and calculation of the system of measure and weights of the *Xinchao*], in *Gongye Biaozhun yu dulianghen* 工業標準與度量衡 [Industry Standards and Weights and Measures]. Vol. 1. Num. 4. 1934. Relying on this result, we transfer the quantities in the above table into modern units: Gold, 18.39 g/cm³; Silver, 16.09 g/cm³; Jade, 11.49 g/cm³; Copper, 8.62 g/cm³; lead, 12.01 g/cm³; iron, 8.05g/cm³; stone, 3.45 g/cm³. Except silver, other quantities are close to modern measure results. Li Di provides us his study on the results in his paper, which are only slightly different from the result above. See: Li Di 李迪, *Woguo gudai de bizhong ceding yu yingyong* 我國古代的比重測定與應用 [the measurement and application of specific gravity in ancient China]. in *Keji shi wenji* 科技史文集 [Collected papers of history of science and technology]. Num. 12. Shanghai Kexue jishu chubanshe, 1984. 122-126.

31 There is table of special gravity with same structure contained in the *Suanxue Baojian*. Wang Wensu 王文素, *Suanxue Baojian* 算學寶鑒 [Precious Mirror of Learning of Mathematics]. Manuscript. Preserved in Beijing Library. 1524, chapter 20, p. 56a.

Procedure: Follow [the procedure of] *fangcheng*. Exchange and weigh them; each [group] weighs eight *liang*.

The description of the method shows us that by finding the weights of each group of birds on the beam when it is level, one may use the Fangcheng method to solve such problems. As there is already a detailed description about the method of *fangcheng* in the text directly preceding this problem, the procedure is simply stated as: “use the procedure of *fangcheng*.”³² Liu Hui gave his own detailed explanation about the equations and the method of elimination of unknowns needed to solve this problem in his commentary:

此四雀一燕與一雀五燕衡適平，並重一斤，故各八兩。列兩行程數，左頭位其數有一者，令右行偏除。亦可令于左行，而取其法實于左。左行數多，以右行取其數。左頭位減盡，中下行算當燕與實。右行不動。左上空，中法下實，即每枚當重宜可知也。³³

[When there are] four sparrows and one swallow [on one end] and one sparrow and five swallows [on other end], the beam is just level. Altogether [the sparrows and swallows] weigh 1 *jin*. So, each [end weighs] 8 *liang*. Place [counting rods] in two columns [to represent] the value of the number [of each kind of bird and the total weight on each side]. [If] the first number of the left column is 1, subtract [each number in the left column] from [the corresponding one] in the right column. Or [one] can also subtract [the numbers in the right column] from [corresponding ones in] the left column, and find out the dividend and the divisor from the left column. The [first] number of the left column is larger. Find out the numbers [of the dividend and divisor] from the right column. When the [number at] left top place is subtracted to zero,³⁴ the counting rods at the middle and the bottom row represent [the number of] swallows and the dividend. Do not touch [the rods of] the right column. The top place of the left column is empty. [Taking] the middle [value] as the divisor [and] the bottom [value] as the dividend, the weight of each [swallow] can be known.

The *fangcheng* method is equivalent to the method of solving a system of linear equations. The procedure of the problem above may be presented as follows:

$$\begin{pmatrix} 4 & 1 \\ 1 & 5 \\ 8 & 8 \end{pmatrix} \quad \begin{pmatrix} 3 & 1 \\ -4 & 5 \\ 0 & 8 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 \\ -9 & 5 \\ -8 & 8 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 \\ -14 & 5 \\ -16 & 8 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ -19 & 5 \\ -24 & 8 \end{pmatrix}$$

This results in:

$$\text{Number of swallows} = 24/19$$

Thus the weight of 1 swallow is 1 *liang* and 5/19 *liang*.

The 18th problem in the *Yingbuzu* chapter is quite similar to the problem above. The problem reads:

今有黃金九枚，白銀十一枚，稱之重適等。交易其一，金輕十三兩，問金銀一枚各重幾何？
答曰：金重二斤三兩一十八銖，銀重一斤一十三兩六銖。

Now there are 9 pieces of gold, [and] 11 pieces of silver. Weigh them, the weights are exactly equal [to each other]. Exchange one piece [of gold for one piece of silver]; then the [side with] gold is 13 *liang* lighter [than the side with silver]. Question: How much are the weights of one piece of gold and one piece of silver respectively?

Answer: [one piece] of gold weighs 2 *jin*, 3 *liang* and 18 *zhu*. [One piece] of silver weighs 1 *jin* 13 *liang* and 6 *zhu*.

32 A general introduction about the *fangcheng* procedure and the procedure of solving system of linear equation were given in the first two problems of the 8th chapter of *Jiuzhang suanshu* and Liu Hui's commentary attached to this problem. See: *Jiuzhang suanshu*, chapter 8, 1a-8a.

33 *Jiuzhang suanshu*, chapter 8, 11b-12a.

34 In other words, continue subtracting the values in the right column from the left column until the top value in the left column reaches zero.

術曰：假令黃金三斤，白銀二斤一十一分斤之五，不足四十九，于右行。令之黃金二斤，白銀一十一分斤之七，多一十五。于左行。以分母各乘其行內之數，以盈不足維乘所出率，並以爲實，並盈不足爲法，實如法得黃金重。分母乘法，以除，得銀重。約之得分也。

Procedure: If [one piece of] gold [weighs] three *jin*, [and one piece] of silver [weighs] two and five-elevenths *jin*, [there should be] a deficit 49 [*liang*]. [Lay down counting rods representing the value 49] at the right column. If [one piece of] gold weighs two *jin*, [and one piece of] silver weighs 1 and seven-elevenths *jin*, [there should be] an excess of 15 *liang*. [Lay down counting rods representing the value 15] at left column. Multiply the denominators with the number in its column respectively; cross-multiply the deficit and the excess with the ratios of pay out; add [the results] as the dividend; add the excess and deficit as the divisor; divide the dividend by the divisor. [Thus one] obtains the weight of [one piece of] gold. Multiply the denominators with the divisor, and take the result to divide [the dividend]. [Thus one] obtains the weight of [one piece of] silver. Reduce to obtain the fraction.³⁵

The above two problems are essentially the same in their mathematical and mechanical meanings. The annotator, Liu Hui, pointed out in his commentary on the problem “swallow and sparrow” in chapter 9 that “the problem of ‘gold and silver’ is equivalent to this”³⁶. If so, then why should the compiler of the *Jiuzhang suanshu* have put them in two different chapters, and solved them with two different methods? Another matter deserving our attention is that both problems concern a balance with equal arms. However, none of these problems gives an explanation of the principle of the balance or how to use it. And neither the compiler nor Liu Hui gives any details concerning the instrument used for weighing in these problems. In fact, there is no need to weigh swallows or sparrows in such a way. In a word, practically, this problem is of no use. All of these problems are connected with the structure of the book mathematically, and serve to reveal the character of mathematics in ancient China.

3. Problems Concerned with Movement

There are several problems concerning numeral relations among velocity, time, and distance of travel in the *Jiuzhang suanshu*.

3.1. Simple Problems of Catching-up

There are several simple catching-up problems. We give one example here to show how such problems are dealt with in the *Jiuzhang suanshu*:

今有客馬日行三百里，客去忘持衣。日已三分之一，主人乃覺，持衣追及與之，而還至家，視日四分之三。問主人馬不休，日行幾何。答曰：七百八十裏。

術曰：置四分日之三除三分日之一。半其餘以爲法，副置法，增三分日之一，以三百里乘之爲實，實如法得主人馬一日行。³⁷

Now it is given that a guest’s horse travels 300 *li* each day. The guest leaves, [but] forgets to take his cloth. When the sun is at one third, the host discovers [this]. Taking the cloth, [he] catches up to [the guest] and gives him [the cloth back]. [When] he arrives back home, [he] looks at the sun; [it] is at three-fourths. Question: [If] the host’s horse does not stop, how much does [it] travel each day?

Answer: 780 *li*.

Procedure: Lay down [counting rods representing] $3/4$ of a day; subtract $1/3$ of a day [from it]. Halve the remainder, [and] take [the result] as the divisor. Then, lay down the [counting rods representing] the divisor and add $1/3$ of a day [to it]; multiply the result by 300 and take [the result] as the dividend. Divide the dividend by the divisor. [Thus, one] obtains the [distance] the host’s horse travels.

In his commentary on the procedure, Liu Hui makes clear the meaning of every step. The result of “[from] $3/4$ day subtract $1/4$ day” is the time the host needs for catching up with the guest and returning

³⁵ *Jiuzhang suanshu*, chapter 7, 12b-14a.

³⁶ *Jiuzhang suanshu*, chapter 8, 11a-b.

³⁷ *Li* is a unit of distance in China.

home. “Halve the remainder” gives the result of the average time for one trip. Adding $1/3$ to the result above gives the amount of time the guest travels before the host catches up with him. Multiplying this by 300, the distance the guest travels in one day, the result is the distance the guest travels before the host catches up with him, which is also the distance the host travels to catch up with the guest. Dividing this distance by the time the host needs to cover the distance, the result is the distance the host travels in one day³⁸.

We are interested in the expression of quantities involving movement in this problem. It is clear that the day was taken as a standard unit of time. Thus, the distance a horse travels in one day may be regarded as velocity. Practically, the movement of horses can not be considered as a uniform motion. However, it is evident that the method given in the problem assumes a constant velocity over the course of the entire day. And in the problem, the compiler of the *Jiuzhang suanshu* states the conditions of the problem clearly, that is: the horse should not stop. This means that he did not define the distance the horse travels in one day as the sum of the distances it travels, but as the distance it continually travels in one day. It seems that the compiler takes it for granted that the speed of the horse is uniform. Based on this analysis, we may say that this problem is equivalent to uniform motion; but not even this concept of uniform motion is mentioned in the text.

Besides several catching-up problems similar to the above, there is also a problem concerning movement in different directions.

Next we consider the more complicated problems which are concerned with regular variable motion.

3.2. Problems Concerned with Variable Motion

Problems concerned with variable motion may be sorted into two groups. One group consists of problems of uniform accelerated motion; the other group includes several problems in which the velocity of motion changes according to geometrical progressions. We begin with the first group.

3.2.1. The Problem of a Good Horse and a Bad Horse

Problem 19 of the Yingbuzu Chapter is the only problem related to uniform accelerated motion:

今有良馬與駑馬發長安至齊。齊去長安三千里，良馬初日行一百九十三里，日增一十三里。駑馬初日行九十七里，日減半里。良馬先至齊，複還迎駑馬。問幾何日相逢及各行幾何。

答曰：一十五日一百九十一分日之一百三十五而相逢。良馬行四千五百三十四里一百九十一分里之四十六；駑馬行一千四百六十五里一百九十一分里之一百四十五。

術曰：假令十五日不足三百三十七里半，令之十六日多一百四十里。以盈不足維乘假令之數，並而為實。並盈不足為法，實如法而一得日數。不盡者，以等數除之而命分。

Now given that a good horse and a bad horse leave Changan for Qi³⁹. Qi is 3000 *li* from Changan. The good horse travels 193 *li* the first day, and [it] increases [its traveling distance] 13 *li* [each following] day. The bad horse travels 97 *li* the first day, and [it] decreases [its traveling distance] half a *li* [each following] day. The good horse arrives at Qi earlier, and [it goes] back to meet the bad horse. Question: how many days will it take for them to meet and how far will each horse have traveled?

Answer: they meet each other after 15 and $135/191$ days, The good horse travels 4534 and $46/191$ *li*, [and] the bad horse travels 1465 and $145/191$ *li*.

術曰：假令十五日不足三百三十七里半，令之十六日多一百四十里。以盈不足維乘假令之數，並而為實。並盈不足為法，實如法而一得日數。不盡者，以等數除之而命分。

Procedure: Suppose that [the answer is] 15 days; then there is a deficit of 337 and a half *li*. Suppose that [the answer is] 16 days; then, there is an excess of 140 *li*. Cross-multiply the deficit and the excess with the supposed numbers (i.e. 15 days or 16 days); add [the results] and [take the result] as the dividend. Add the excess and the deficit [and take the result] as the divisor; divide the dividend

38 Anonymous author, *Jiuzhang suanshu*, chapter 6, p. 20b-21b.

39 Qi and Changan are the names of two cities.

by the divisor. [Thus one obtains] the number of days. [For] the indivisible part, divide [both the dividend and divisor] by [their] common divisor and get the fraction.

求良馬行者，十四乘益疾里數而半之，加良馬初日行里數，以乘日分子，如日分母而一，所得前良馬凡行里數。以乘十五日，得良馬十五日之凡行。又以十五日乘益疾里數，加良馬初日之行，以乘日分子，如日分母而一。所得，加前良馬凡行里數，即得。其不盡而命分。求駑馬行者，以十四乘半里，又半之，以減駑馬初日之行里數，以乘十五日，得駑馬十五日之凡行，又以十五日乘半里，以減駑馬初日之行，餘以乘日分子，如日分母而一。所得加前里，即駑馬定行里數。

To find the distance the good horse travels⁴⁰, multiply 14 by the increasing number of *li* [for each day], and halve [the result]; add [the result] to the distance the good horse travels the first day and multiply the result by 15 days. [Thus one obtains] the total distance the good horse travels in 15 days. Then [for the remaining 135/191 of a day], multiply 15 by the increasing number of *li* [for each day] and add the distance the good horse travels the first day⁴¹. Multiply [the result] by the dividend (i.e, 135) of the fraction of a day [the good horse travels] and divide [it] by the divisor (i.e. 191). Add the result to the number of *li* the good horse travels in 15 days. [Thus, one] obtains the distance the good horse travels. To find the distance the bad horse travels, multiply 14 by half a *li*; halve the result again; subtract it from the distance the bad horse travels the first day; multiply by 15 days. [Thus, one] obtains the distance the bad horse travels in fifteen days. Then, [for the remaining 135/191 of a day] multiply 15 days by half a *li*; subtract it from the distance the bad horse travels the first day; multiply the remainder by the dividend and divide it by the divisor. Add the result to the distance [obtained] above. [Thus, one] obtains the distance the bad horse travels.

Now, let us examine this problem more closely. Literally, the problem may be interpreted as follows: the horses travel day and night every day; in the first 24 hours, the good one covers 193 *li*, and thereafter, the distance it travels increases by 13 *li* per day. We know that it is impossible for a horse to run for 15 days, day and night, without stopping. Nevertheless, it is reasonable for the purposes of this problem to take the *day* as a unit of time. If then, the distance a horse travels in the first day is taken as its initial speed, in modern terms this would be expressed as v_0 . Now, we have another problem, because the good horse increases the distance it can cover by 13 *li* per day; but how should we express this increase in the speed?

There are two alternatives. One is that every day, the speed changes, in which case the change in speed from day to day forms a discrete arithmetic progression. The other alternative is that the horse does not change its speed staggered, but we appeal to some sort of “mean speed theorem” whereby the total distance covered may be accounted for in terms of a uniform acceleration.⁴² For both of these possibilities, if the horse travels for an integral number of days, we may compute the distance the good horse travels from day to day by the method given in the problem, which is equivalent to the following formula:

$$d = 193 + \sum_{i=1}^{t-1} (193 + 13i)$$

The general method for finding the sum of a finite arithmetic progression is given in *Zhang Qiujian suanjing* (Mathematical Canon of Zhang Qiujian, fifth century, before 484).⁴³ However, we believe that this method was already known at the time of the compilation of the *Jiuzhang suanshu*. In chapter 3, chapter 6, and chapter 7 of the *Jiuzhang suanshu*, there are problems concerning finite arithmetic

40 Dai Zhen thought that this part is the commentary made by Liu Hui. See: *Jiuzhang suanshu*, *Siku quanshu* edition. But some other scholars, including Wang Lai and Li Huang pointed out that this part is the original text of the *Jiuzhang suanshu*. From the style of the arrangement of the text, Guo Shuchun believes that the opinion of Wang Lai and Li Huang is more reasonable. See: Li Huang, 39b; Guo Shuchun (commentary and translation into modern Chinese), *Jiuzhang suanshu*.

41 The original text is lost. The translation in the brackets are based on the commentary made by Dai Zhen in 1774.

42 There is another possibility that is the speed of the horse changed rulelessly. As it is not consistent with the algorithm of the problem, we will not discuss this possibility in this paper.

43 Qiang Baochong, *Zhongguo shuxue shi* 中国数学史 [History of Mathematics in China], Beijing: Kexue Chubanshe, 1964, p. 80-81.

progressions. Although the procedures there are based on special cases, it seems that general methods for handling arithmetic progressions were already known.⁴⁴

From the quotation, it seems that the distances the horses traveled may be found through the following steps: find the average distance the horses travel over 15 days; multiplying the results by 15, one gets the distance the horses travel in 15 days. In modern terms, the method for calculating the distance the good horse travels may be expressed as follows:

$$\left(\frac{14 \times \text{incrli}}{2} + \text{dis}_1\right) \times 15$$

Now, let us consider the method of calculating the distance the horse travels in a fractional day as is the case in some problems. If we accept that the velocity of the horses changes from day to day, then it is possible to determine this in terms of horses traveling at a uniform speed every day. In the fractional day, the distance the horse travels is just the fractional multiple of the distance the horse traveled in the whole day. Take the good horse as an example. The total distance traveled is:

$$(15 \times \text{incrli} + \text{dis}_1) \frac{135}{191}$$

Adding the two parts together, we have the distance the good horse travels:

$$\left(\frac{14 \times \text{incrli}}{2} + \text{dis}_1\right) \times 15 + (15 \times \text{incrli} + \text{dis}_1) \frac{135}{191}$$

In the above formulas, the *incrli* means increasing *li*, that is the increment of the distance the good horse travels per day, and the *dis*₁ the distance the good or inferior horse travels for the first day. In the same way, the case of the inferior horse may be expressed as:

$$\left(\text{dis}_1 - \frac{14 \times \text{decrli}}{2}\right) \times 15 + (\text{dis}_1 - 15 \times \text{decrli}) \frac{135}{191}$$

Each of the two formulas may be regarded as a word-for-word translation of the “procedure” given in the *Jiuzhang suanshu*. Therefore, we may safely say that when the speed of the horses changes from day to day, the procedure in the *Jiuzhang suanshu* gives exact results for the distances as well as the times the horses travel.

However, if the speeds of the horses changes by uniform accelerations, the situation would be a little different. As the velocity changes continuously, in any fraction of the day, the average speed the horses travel during this part of the day will not have reached the average speed they will traveled in the whole day. So, the second part of both of the formulas given above cannot give the exact distances the horses travel.⁴⁵

Even if the method for calculating the distances the horses travel is an approximate one, the answer for the time the horses travel is an exact result, even under the condition of uniform acceleration. This is because the *yingbuzu* method, which is equivalent to the method of double false position method, is valid for linear problems.

44 The 17th problem of chapter 6, *Junshu* chapter is: “Supposing, the length of a golden cone-shaped whip is 5 *chi*. The base cut at 1 *chi* long weights 4 *jin*, whereas the tip cut at 1 *chi* long weights 2 *jin*. Ask: what is the weight of each segment of 1 *chi* long? Answer: the tip 1 *chi* long weights 2 *jin*. The next 1 *chi* weights 2 *jin* 8 *liang*, the third *chi* weighs 3 *jin*. The fourth 1 *chi* weighs 3 *jin* 8 *liang*. The base 1 *chi* weighs 4 *jin*.” The weight of every *chi* formed an arithmetic progression. In the procedure, the common difference of arithmetic progression was given. Anonymous author, *Jiuzhang suanshu*, chapter 6, 21b-22b.

45 Shen Kangshen and J. N. Crossley and A. W.-c Lun pointed out that: “some modern mathematicians complain that the answer given in *The Nine Chapters* is only an approximate one. Such a remark is not justified, for the sum formula for an arithmetic progression is valid only if *x* is a natural number”. (Shen Kangshen, J. N. Crossley and A. W.-C. Lun, P. 282. They are reasonable, but if we believe that the speed of the horse changed in uniform acceleration, we can not deny that the answer the *Jiuzhang suanshu* is not an exact result.

3.2.2. The Problems in Which the Velocity Changes in Geometric Progression.

In the *Jiuzhang suanshu*, there are several problems concerned with non-uniform acceleration. In two of them, the velocity changes in terms of a geometric progression.

今有垣厚五尺，兩鼠對穿，大鼠日一尺，小鼠亦日一尺，大鼠日自倍，小鼠日自半，問幾何日相逢，各穿幾何。答曰：二日一十七分日之二，大鼠穿三尺四寸十七分寸之一十二，小鼠穿一尺五寸十七分寸之五。

術曰：假令二日，不足五寸，令之三日，有餘三尺七寸半。⁴⁶

Now given that the thickness of a wall is 5 *chi*, two rats are gnawing [it] from opposite directions. The big rat [gnaws] 1 *chi* [in the first day], [and] the small rat [gnaws] 1 *chi* [in the first day] too. [The rate at which] the big rat [gnaws] doubles each day, and [the rate at which] the small rat [gnaws] is halved each day. Question: how many days will take [them] to meet? How far will each rat gnaw?

Answer: Two and 2/17 days. The big rat gnaws 3 *chi*, 4 and 12/17 *cun*; the small rat gnaws 1 *chi*, 5 and 5/17 *cun*.

Procedure: Suppose that [the answer] is two days, there is a deficit of five *cun*; Suppose [the answer] is three days, there is an excess of 3 *chi*, 7 and a half *cun*.

Liu Hui offers the following commentary on the above procedure:

大鼠日倍，二日合穿三尺，小鼠日自半，合穿一尺五寸，並大鼠所穿，合四尺五寸，課於垣厚五尺，是為不足五寸，令之三日大鼠穿得七尺小鼠穿得一尺七寸半，並之以減垣厚五尺，有餘三尺七寸半。以盈不足術求之即得。以後一日所穿乘日分子如日分母而一，即各得日分子之中所穿，故各增二日定穿，即合所問也。⁴⁷

[The rate at which] the big rat [gnaws] doubles each day; so it should gnaw three *chi* in two days. [The rate at which] the small rat gnaws halves each day; so it should gnaw 1 *chi* and five *cun* [in two days]. Add [the distance the small rat gnaws] to [distance] the big rat gnaws; [the result] should be 4 *chi* and five *cun*. Subtract the result from the thickness of the wall, [which is] 5 *chi*. This is the 5 *cun* deficit. If [the answer is] 3 days, the big rat should gnaw 7 *chi* [and] the small rat should gnaw 1 *chi*, 7 and a half *cun*. Add them, [and] subtract [the total] from the thickness of the wall, [which is] 5 *chi*. There is an excess of 3 *chi*, 7 and a half *cun*. Using the *yingbuzu* method to find [the value of the time], [the result] is obtained. Multiply the [distance each] rat gnaws during the last day by the numerator of [the fraction of] the day, and divide it by the denominator of [the fraction of] the day. Thus, the [distance] each rat gnaws in the fractional day is obtained. Then, add [the results to the distances] they gnaw in two days. [The result] answers the question.

In this problem, the speed at which the rats gnaw through the wall changes according to a geometric progression. There are several problems also involving geometric progressions in *Jiuzhang suanshu*, and the *Yingbuzu* method is always used in solving them. However, the *Yingbuzu* method is only valid when the problem can be interpreted in terms of a linear equation, and yet the problem with the two rats involves an exponential equation. Therefore, the solution given in the *Jiuzhang suanshu* can only be an approximate one. Not only that, like the problem discussed earlier of the good horse and inferior horse, the problem involving the rats also deals with fractional amounts, i.e. the speed at which the rats gnaw through the wall changes continuously according to an exponential equation, and the distances are expressed in fractions of a day, and since the speed changes continuously, then final solution computed by the method in the *Jiuzhang suanshu* can only be approximate. In the nineteenth century, after the European methods were transmitted into China, several mathematicians pointed out the errors of this procedure, and introduced new methods for solving the problem.

In the above three problems, speed is expressed in a uniform way, always in terms of distances traveled in one day. From this it seems clear that the ancient Chinese use the day as a standard unit of time. Thus, distances per day represent speed or velocity. It should be noted that all three of these problems

⁴⁶ Anonymous author, *Jiuzhang suanshu*, chapter 7, 18a.

⁴⁷ Anonymous author, *Jiuzhang suanshu*, chapter 7, 18 a-b.

involving variable motion are, in practical terms, impossible. We can not imagine a horse increasing its speed steadily for 3000 *li*, or a rat doubling the distance it can gnaw through a wall every day. In fact, the specific terms of these problems were mathematical inventions, perhaps devised by mathematicians with astronomical calculations or other problems of pursuit in mind. I believe that the ancient Chinese had already noticed that some times and speeds could increase or decrease steadily, or that the change in speed could be greater (or less) than in the case of uniform acceleration motion. By considering the numerical quantities in such problems under the ideal conditions of uniform change, then such problems could be solved by established mathematical algorithms. We shall return to this point in our final discussion of the connections between mechanics and mathematics below.

4. Problems Concerning Weights in Motion

Most of the problems concerning weights in motion in the *Jiuzhang suanshu* are in the *junshu* Chapter. *Junshu* (fair levies) means the fair distribution of taxes according to the number of taxpayers, the distances between the taxpayers and the place where the tax is to be paid, the cost of transportation, and sometimes various additional factors.

The following problem exhibits the standard way in which such problems were presented in the mathematical text:

今有均賦粟，甲縣四萬二千算，粟一斛二十，傭價一日一錢，自輸其縣。乙縣三萬四千二百七十二算，粟一斛一十八，傭價一日一十錢，到輸所七十裏。丙縣一萬九千三百二十八算，粟一斛一十六，傭價一日五錢，到輸所一百四十裏。丁縣一萬七千七百算，粟一斛一十四，傭價一日五錢，到輸所一百七十五裏。戊縣二萬三千四十算，粟一斛一十二，傭價一日五錢，到輸所二百一十裏。己縣一萬九千一百三十六算，粟一斛一十，傭價一日五錢，到輸所二百八十裏。凡六縣，賦粟六萬斛，皆輸甲縣。六人共車，車載二十五斛，重車日行五十裏，空車日行七十裏，載輸之間，各一日，粟有貴賤，傭各別價，以算出錢，令費勞等。問縣各粟幾何。⁴⁸

Now there is fair levy of millet. County A [has] a count 算 of forty-two thousand [tax paying units]. [The price of] one *hu*⁴⁹ of millet [is] twenty [*qian*]⁵⁰. The price of hiring [labor] is one *qian* per day. [The levy] is transported to its own county. County B [has] a count of thirty-four thousand two hundred and seventy-two [tax paying units]. [The price of] one *hu* of millet [is] eighteen [*qian*]. The price of hiring [labor] is ten *qian* per day. [The distance] to the levy [collection] place is seventy *li*. County C [has] a count of nineteen thousand three hundred and twenty-eight [tax paying units]. [The price of] one *hu* of millet [is] 16 [*qian*]. The price of hiring [labor] is 5 *qian* per day. [The distance] to the levy [collection] place is 140 *li*. County D [has] a count of seventy thousand seven hundred [tax paying units]. [The price of] one *hu* of millet [is] 14 [*qian*]. The price of hiring [labor] is 5 *qian* per day. [The distance] to the levy [collection] place is 175 *li*. County E [has] a count of twenty-three thousand forty [tax paying units]. [The price of] one *hu* of millet [is] 12 [*qian*]. The price of hiring [labor] is 5 *qian* per day. [The distance] to the levy [collection] place is two hundred and ten *li*. County F [has] a count of nineteen thousand one hundred and thirty-six [tax paying units]. [The price of] one *hu* of millet [is] 10 [*qian*]. The price of hiring [labor] is 5 *qian* per day. [The distance] to the levy [collection] place is 280 *li*.

[In total], the six counties should pay a levy of sixty thousand *hu*. All of the levy should be transported to County A. There are six people per vehicle and each vehicle carries twenty-five *hu*. A loaded vehicle travels fifty *li* per day, and an empty vehicle travels 70 *li*. Loading and paying [the levy] each takes one day. The prices of millet are all different. And the prices of labor are different. [Each county] should pay money (i.e. millet) according to the count of [tax paying units]. Let the expense of millet and labor be equal [for all the counties]. Question: How much millet should each county pay?

It is clear that in ancient China, when the tax laws were issued, it was understood that the speed of a heavily burdened vehicle is slower than that of a lighter vehicle, or one carrying no weight at all. This

48 Anonymous author, *Jiuzhang suanshu*, chapter 6, 8a-10b.

49 *Hu* is a unit of measure.

50 *Qian* is a unit of money in ancient China.

knowledge was surely a result of direct experiential knowledge. Moreover, experience would also have led to considering the sizes of different taxpaying populations, their distances from the taxpaying center, and then, with the added factor of transporting different amounts (weights) of grain as tax over the given distances in mind, finding a proportional means of deciding a standard by which to determine a fair distribution of taxes. Among the problems in this chapter, several emphasize the factor of weights in motion, and we examine two of these here as particular examples of such problems.

今有程傳委輸，空車日行七十里，重車日行五十里，今載太倉粟輸上林，五日三返。問太倉去上林幾何。答曰：四十八里分十八分里之一十一。

術曰：並空重里數，以三返乘之爲法，令空重相乘，又以五日乘之爲實，實如法得一。

Suppose that in the process of transportation, an empty vehicle travels 70 *li* per day, and a vehicle carrying a load travels 50 *li* per day. Now, the millet from the imperial depository is transported to the imperial garden within five days over three round trips. Question: How far is the imperial depository from the imperial garden? Answer: Forty-eight and eleven eighteenths *li*.

Procedure: add the number of *li* [the vehicle travels] when empty and when loaded; multiply this by three round trips as the divisor; multiply [the number of *li* the vehicle travels] when empty and when loaded; And multiply [the result] by five day as the dividend; and then, divide the dividend by the divisor to get the solution.

Liu Hui comments as follows on this procedure:

此術，重往空還，一輸再還道。置空行一里七十分日之一，重行一里用五十分日之一，齊而同之，空重行一里之路往返用一百七十五分日之六。定言之者，一百七十五里之路往返用六日。故并空重者并齊也。空重相乘者，同其母也。于今有術，五日爲所有數，一百七十五爲所求率，以此所得則三返之路。今求一返者，當以三約之，故令乘法而并除，亦當約之也。⁵¹

In this procedure, [the vehicle] carries a load [to the imperial garden], and returns empty. After one trip, [the vehicle] returns and repeats the circuit. Lay down [counting rods representing] one seventieth of a day in which an empty vehicle travels one *li*; [and the counting rods representing] one fiftieth of a day in which a loaded vehicle travels one *li*. Harmonize [the numerators] and homogenize the denominators⁵². For [the vehicle] to travel empty [one way] and loaded [the other] round trip over a road of one *li* takes 6/175 of a day. [Given this] one can definitely say that [the vehicle] takes six days [to travel] round trip over a road of 175 *li*. Thus, to add [the number of *li* the vehicle travels] empty and loaded is to add the harmonized [numerators]; to multiply [the distance the vehicle travels] empty and loaded is to equalize the denominators. According to the method of Supposition (*Jinyou shu* 今有術) five days is the given number (*suoyou shu* 所有數); 175 is the sought after *lǜ* (*suoqiu lǜ* 所求率). The result from this is the distance of three round [trips]. Now, seeking [the distance] of one round [trip], divide it by three.

The basic idea for solving this problem is first to find the time of a round trip in terms of a standard unit of distance, in this case one *li*. Thus, a heavy wagon will cover one *li* in 1/50 of a day, while an empty one will complete a round trip in 1/70 of a day, i.e. it will take 6/175 of one day to make one round trip over a distance of one *li*. Therefore, it follows that in 6 days, 175 round trips can be made. The solution to the problem now proceeds in terms of applying the method of *Supposition* (Rule of Three). Given that the problem assumes that it takes 5 days to complete 3 round trips, it is then possible to determine the distance from the imperial garden. This method is the one generally used to solve problems involving the transport of different weights over various distances, by first reducing the elements of a given problem to standard units of distance and time.

The following problem is another example that proceeds using the same procedure.

今有負籠重一石行百步五十返。今負籠重一石一十七斤行七十六步，問反幾何。答曰：五十七返二千六百三分返之一千六百二十九。⁵³

51 Anonymus author, *Jiuzhang suanshu*, chapter 6. p. 15a-16a.

52 For the method of Qitong, see the first section of this paper.

53 The original text of *Jiuzhang suanshu* is: "Suppose bearing with a basket-load of 1 *dan* 17 *jin* and traveling 76 *bu*, [one] can make 50 roundtrips. Now, [he] travels 100 *bu* bearing a basket-load of 1 *dan*." (*Jiuzhang suanshu*,

術曰：以故所行步數乘故籠重斤數爲法。今籠重斤數乘今步，又以返數乘之爲實，實如法得一返。⁵⁴

Now given that [a person] carrying a basket weighing one *dan* traveling a distance of 100 *bu* makes 50 roundtrips, suppose [he] carries a basket weighing 1 *dan* 17 *jin* to travel a distance of 76 *bu*. Question: how many roundtrips does [he make]? Answer: 57 and 1629/2603 roundtrips.

Procedure: Take the given number of *bu* traveled and multiply by the number of *jin* the basket weighs as the divisor; Multiply the supposed number of *jin* the basket weighs by the supposed number of *bu* traveled; then, multiply it (i.e. the product) by the number of roundtrips. [The result] is the dividend. Divide the dividend by the divisor to get the number of round trips.

Liu Hui comments as follows:

此法負一斤一返所行之積步，此實者一斤一日所行之積步，故以一返之課除終日之程，即是返數也。

This divisor is the product in *bu* [one] travels round trip carrying a load of one *jin*. The dividend is the product in *bu* [one] travels in one day carrying a load of one *jin*. Thus, take [the distance of] one round trip to divide the distance [one] travels in one day; [the result] is the number of roundtrips.

The Tang Dynasty commentator Li Chunfeng added the further explanation:

此負籠又有輕重，於是爲術者因令重者得返少，輕者得返多，故又因其率以乘法實者。重今有之義也。然此意非也。按：此籠雖輕而行有限，籠過重則人力遺。力有遺而術無窮。人行有限而籠輕重不等。使其有限之力隨彼無窮之變，故知此術率乖理也。⁵⁵

These basket-loads are different in weight; thus because of this, the creator of this method let the one carrying the heavy [load] make fewer roundtrips, [and] the one carrying the light [load] make more roundtrips. So, [he] multiplies the divisors and the dividends according to the *lii*⁵⁶. That is the meaning of the compound method of supposition. But this reasoning is flawed. Commentary: Although the basket is light, the distance traveled is short. If the basket is too heavy, then a person's strength [*li*] is insufficient. [A person's] strength [may be] insufficient but the method [assumes it is] limitless. [And] the [distance] a person travels is limited but the weight of the basket varies. [The method] allows [a person's] limited strength to vary along with the infinite change [in weight]. Thus, [one] knows that the *lii* in this procedure violates reason.

The above material is the only part of the *Jiuzhang suanshu* which provides any indication of how ancient Chinese mathematicians thought about the problem of weights in motion.

From these problems in the *Jiuzhang suanshu*, it is clear that the inventor of the method understood the relation between weight and the time needed to move a given weight over a given distance; the heavier the weight, the longer the time (assuming the effort or force applied is equal). Alternatively, experience would have shown that the heavier the weight to be moved, the shorter the distance the carrier would cover in a standard unit of time. By assuming the weights and distances are in inverse proportion, for a given distance, the weights to be moved and their respective times of travel are also in inverse proportion. Thus for a given weight to be moved, the corresponding time and distance will also be in proportion. By employing these proportional relationships, the author of the method reduced the practical problem to one that could be solved with established mathematical methods, in particular, the *jinyou* method or familiar Rule of Three. The method could be regarded as a theoretical application of the inventor's analysis of the problem of weights in motion. In the former section, we saw how the author of the *Jiuzhang suanshu* reduced the problem of movement to a mathematical problem. However, Li Chunfeng further pointed out that the method is not in accord with reason. His argument

chapter 6, 13b) Guo Shuchun corrects the text according to the style of lay out of the *Jiuzhang suanshu* and the result given in the book. See: Karine Chemla, Guo Shuchun, *Les Neuf Chapitres*, 508-510.

54 Anonymous author, *Jiuzhang suanshu*, chapter 6, p. 13b-14b.

55 Li Chunfeng, commentary, in Anonymous author, *Jiuzhang suanshu*, chapter 6, p. 13b-14b.

56 Here *lii* could be treated as ratios. In this problem, the weights of burdens and the numbers of round trips one could make in one day are in inversed proportion; the distances of travel and the numbers of round trips are also in inversed proportion. Therefore, the numbers of round trips are in a compound proportion with weights of burdens and the distances of travel. This is the meaning of compound *jinyoushu* in the next sentence.

was doubtless based on an every-day observation: a light burden has little or no effect on motion, especially if the distance of travel is short, but if the burden is too heavy, it would be impossible to carry at all. So the method given in the *Jiuzhang suanshu* cannot be correct, as it violates common sense. Consequently, Li Chunfeng provided his own method for solving such problems:

假令空行一日六十里，負重一斛行四十里，減重一斗進二里半；負重三斗以下與空行同。今負籠重六斗，往返行一百步，問返幾何。答曰：一百五十返。術曰：置重行率加十里，以里法通之爲實，以一返之步爲法，實如法而一，即得也。⁵⁷

Suppose that without a load, a [person can] travel 60 *li* in one day. Carrying a load of one *hu*, [a person] can travel 40 *li*. Reducing the load by one *dou*, [a person] can travel two and a half *li* more. [If a person] carries a load of less than three *dou*, [he can travel] the same [distance] as without a load. Now, [suppose a person] carries a basket weighing six *dou* and travels 100 *bu*. Question: how many roundtrips can [he] make. Answer: 150 roundtrips. Procedure: Lay down [counting rods representing] the *liü* [of the distance] traveled with a load⁵⁸; add ten *li* [to it]. Use the method of conversion to change [all measures of distance in] *li* into *bu*⁵⁹, [and take the result] as the dividend. Take the distance of one roundtrip as the divisor. Divide the dividend by the divisor to get [the result].

Here Li Chunfeng gives more detailed conditions according to his own understanding of the practical problem of weights in motion.

As 1 *hu* = 10 *dou*, the person who carries a burden of only 6 *dou*, according to Li Chunfeng's rule (since for each *dou* less in weight, the distance traveled increases by 2.4 *li*), then with 6 *dou* (4 *dou* less than 10 *dou*) means that the distance covered should increase by 10 *li* (i.e. 4 times 2.5 *li*). Therefore, the person can travel 50 *li* every day carrying a burden of 6 *dou*. Given that one round-trip covers 100 *bu*, then the number of round trips that can be made is:

$$50 \times 300 \div 100 = 150 \quad \text{round trips.}$$

In this problem, the person carries the same weight in both directions.

Our last example to be considered here concerning weights in motion in the *Jiuzhang suanshu* involves weights moving on a slope:

今有武馬一匹，中馬二匹，下馬三匹，皆載四十石至阪，皆不能上。武馬借中馬一匹，中馬借下馬一匹，下馬借武馬一匹，乃皆上，問武、中、下馬一匹各力引幾何。答曰：武馬一匹力引二十二石七分石之六，中馬一匹力引一十七石七分石之一，下馬一匹力引五石七分石之五。術曰：如方程，各置所借，以正負術入之。⁶⁰

Now given that there is one strong horse, [a team of] two average horses and [a team of] three weak horses. Each [of these three teams] carts 40 *dan*⁶¹, to a slope, which none is able to ascend. If the strong horse is teamed with one average horse, or the [two] average horses are teamed with one weak horse; or the [three] weak horses are teamed with the strong horse, then all [the teams] would be able to ascend [the slope]. Question: How much [weight] can the strength of a strong horse, or an average horse, or a weak horse draw?

Answer: the strength of a strong horse can draw [a weight of] 22 and 6/7 *dan*; the strength of an average horse can draw [a weight of] 17 and 1/7 *dan*; and the strength of a weak horse can draw [a weight of] 5 and 5/7 *dan*.

Procedure: According to the *fangcheng* method, lay down [counting rods representing] the teams, and use the method of positive/negative to solve [the problem].

⁵⁷ Anonymous author, *Jiuzhang suanshu*. Chap 6, pp. 13b-14b.

⁵⁸ *liü* means the number of *li* the person travels with a burden of 1 *hu*.

⁵⁹ *Li fa* 里法 [method of *li*] means the method of transforming units of *li* to units of *bu*. Namely, 1 *li* is equal to 300 *bu*.

⁶⁰ Anonymous author, *Jiuzhang suanshu*, chapter 8, p. 13 a-b.

⁶¹ *Dan* is a unit of weight in ancient China. One *dan* is equal to 100 *jin*.

In terms of modern mechanical knowledge, problems involving motion on inclined planes must consider among other things friction, the force of gravity, angular momentum, and composition of forces. With all of these factors in mind, some scholars find the solution to this problem totally wrong, and argue that if the answers were correct, the horses would be able to lift the burden directly rather than draw it up a slope⁶². But such critics have not noticed that, in the problem, what is asked for is the drawing force, the force one horse needs to pull its burden up the slope, rather than the force needed to lift a given weight or draw it up an inclined plane. But in fact, this problem is meant to illustrate application of the *fangcheng* method, which would have appeared as follows on the counting board:

1	0	1
0	2	1
3	1	0
40	40	40

Applying the plus/minus method is equivalent to diagonalizing the matrix of numbers:

0	0	1
0	2	1
7	1	0
40	40	40

from which it follows that the weak horse can only pull $5 \frac{5}{7}$ *dan* up the slope.

Although force is mentioned in this problem, the function of force is not considered, and the relationship between force and movement is not discussed either. And there is no analysis of the relationship between the angle of the slope and the force needed to pull up a given weight, even though the problem makes it clear that the compiler knew that drawing a weight up an inclined plane requires more force than to move it along a flat, horizontal surface. It would be extremely interesting if the case of the inclined plane had been analyzed, but the ancient Chinese mathematicians did not do so.

5. The Relation Between Mathematics and Mechanical Knowledge in Ancient China

From the above information, it would appear that in ancient China there was no separate category for or systematic study of mechanics. Although Chinese mathematicians provided numerical solutions for a wide variety of practical or pseudo-practical problems relating to mechanical knowledge, they were not interested in any theoretical analysis of the particular circumstances or causes of mechanical phenomena—like balances or motion on inclined places. Furthermore, knowledge related to mechanics was not carefully arranged in the *Jiuzhang suanshu*. They were not arranged in any particular order, but problems with the same or similar mechanical significance appear in different chapters, and the compiler provided different numerical methods for their solutions. I have mentioned at the beginning of this paper that mechanical problems or mechanics in general as a special subject did not exist in ancient China. What Chinese mathematicians focused on were methods providing numerical solutions for all kinds of problems concerned with calculation. At the same time, they were interested in deepening their research on algorithms. More theoretical discussions of the reason and reasoning behind mechanical problems may be found in philosophical texts (like the *Mojing*). As for the *Jiuzhang suanshu*, the problems relating to mechanical knowledge occur in the following chapters, as may be seen from the following table:

Chapter <i>Junshu</i> (6):	moving weight 4 movement 7 weight 1
Chapter <i>yingbuzu</i> (7):	weight 2 movement 4
Chapter <i>Fangcheng</i> (8):	weight 1 moving weight 1
<i>Gougu</i> chapter (9):	movement 1

62 See: Dai Nianzu, Wei Zhong, Wang Dakai.

What must be kept in mind is that this is a mathematical book, and problems were distributed from chapter to chapter depending on the concerns of the compiler. What, then, were the mathematical concerns of the compiler? On this question there is a relevant passage in the *Zhoubi Suanjing* 周髀算經 [*Mathematical Canon of Zhoubi*, 1st century B.C.]. The following is crucial for understanding the character of mathematics in ancient China:

昔者，榮方問于陳子曰：今者聞夫子之道。知日之高大，光之所照，一日所行，遠近之數，人所望見，四極之窮，列星之宿，天地之廣袤，夫子之道皆能知之，其信有之乎？

陳子曰：然

榮方曰：方雖不省，願夫子幸而說之。今若方者可教此道耶？

陳子曰：然。此皆算術之所及。子之於算，足以知此矣。若誠累思之。

In the past, Rong Fang asked Master Chen “I have recently heard of your Way. Comprehending the height and size of the sun; the [area] illuminated by [its] light, the amount of its daily motion; the figures for its greatest and least distances; the extent of human vision; the ends of the four [corners of the earth]; the lodges of stars, [and] the length and breadth of heaven and earth - Your Way can know all of these [things], is it really so?”

Master Chen replied: “Yes.”

Rong Fang said: “Although I am not intelligent, I hope that Master will favor me with an explanation. Can someone like me be taught of this Way?”

Master Chen replied: “Yes. All these things can be attained through mathematics. Your knowledge of mathematics is sufficient to understand this. If you are sincere and earnest in thinking about it [you can understand this].”

於是，榮方歸而思之，數日不能得。

複見陳子曰：方思之不能得，敢請問之。陳子曰：思之未熟。此亦望遠起高之術，而子不能得，則子之於數未能通類。是知有所不及，而神有所窮。夫道術言約而用博者，智類之明；問一類而以萬事達者，謂之知道。今子所學，算數之術。是用智矣，而尚有所難，是子之智類單。夫道術所以難通者，既學矣，患其不博；既博矣，患其不習；既習矣，患其不能知。故同術相學，同事相觀，此列士之愚智、賢不肖之所分。是故能類以合類，此賢者業精習智之質也。夫學同業而不能入神者，此不肖無智而業不能精習。是故算不能精習。吾豈以道隱子哉？⁶³

At this, Rong Fang returned and thought about it. After several days, [he] was still unable to understand.

[He] again went to see Master Chen and said: “I have thought about it and [I am still] unable to understand. May I venture to inquire about it?”

Master Chen said: ‘You thought about it, but [your understanding] is not well developed. This is also the method of surveying distances and heights. But you did not understand it, since you are unable to apply your knowledge of mathematics to different categories of problems. This is because your understanding has still not reached [a deep level] and your resolve has been limited. The art of Way is expressed in few words but its application is broad, [this is called] elucidating the wisdom of categories; inquiring into one category [one] can obtain [knowledge of] myriad of things; this is called knowing the Way. Now, what you are studying is the art of mathematics. This requires wisdom. Nevertheless, [you] still have difficulty, this is [because] your wisdom of categories is narrow. The reason why the art of Way is difficult to understand is because even after [one] has studied, one still suffers from lacking breadth of knowledge; even after attaining broad knowledge, [one] still suffers from lacking practice; even after having practice, [one still] suffers from not being able to know. Therefore [one] studies similar arts [in comparison] with each other, and [one] examines similar affairs [in comparison] with each other. This is what distinguishes foolish from

63 Anonymus author, *Zhoubi suanjing* 周髀算經 [*Mathematical Canon of the Zhoubi*]. Beijing: Wuying Dian Juzheng edition, 1774, chapter 1, Section 2, pp. 1a-5b. The translation from *Zhoubi suanjing*, with some variations, is from Christopher Cullen, *Astronomy and Mathematics in Ancient China: the Zhou bi suan jing*. Cambridge: Cambridge University Press, 1996, pp. 176-178.

intelligent [scholars] and worthy from unworthy scholars. Therefore, this ability to distinguish categories to combine them is the quality of a worthy person whose endeavors are profound and practices are intelligent. Those who study the same endeavors but are unable to plumb the profundities [of these things] are unworthy and lack wisdom; thus, [they] are unable to profoundly practice their endeavors. This is why [you] are unable to practice mathematics profoundly. Why should I conceal the Way from you?"

From what Master Chen expresses here, it is clear that a mathematician must know how to solve problems belonging to different categories with established mathematical methods, and how to combine the methods derived from solving problems belonging to different categories. Thus, the *qitongshu*, *jinyoushu*, *cweifenshu*, and the *yingbuzushu* are different mathematical methods that could be applied to different categories of problems⁶⁴. Chinese mathematicians often said that solving practical problems concerned with calculation was the aim of their research, but not all of them were solely interested in finding numerical solutions to concrete practical problems. This may explain why problems concerned with weight are not to be found grouped together in one chapter. Instead, the compiler of the *Jiuzhang suanshu* sought to provide different methods for solving problems in the same category of mathematical significance, and this is why there is no section devoted specifically to calculating the weight of things knowing their volumes and specific gravities. In fact, problems relating to mechanical knowledge were simply taken as examples wherever they were needed to demonstrate how to transform a given problem into one that could be solved with the algorithms given in the *Jiuzhang suanshu*.

Based on the above discussion, we may conclude that ancient Chinese mathematicians applied themselves to developing new algorithms, and using their algorithms to solve a wide variety of problems relating to calculation. Some problems concerned with mechanical knowledge were among those they considered, because they sought to provide numerical solutions to these problems and show their readers how to use established mathematical methods in different situations. Thus, they did not attribute to these problems concerned with weight, motion, or moving weights any special significance as a special category of physical problems, any more than they did to the significance of problems about calculating the area of a rectangular field. Above all, neither the authors nor later commentators tried to provide any theoretical discussion about the function or special status of mechanical problems.

64 As mentioned at the beginning of this paper, Chinese mathematicians focused on providing numerical solutions to problems concerned with calculation, and inventing new algorithms. The algorithms are arranged in a progression from easy to difficult, and the methods given in earlier parts of the text are commonly used in later parts of the book.

Wang Zheng and the Transmission of Western Mechanical Knowledge to China

ZHANG Baichun and TIAN Miao

In general, when the issue of the transmission of European scientific knowledge and technology in China is raised, the implication is almost unconsciously accepted that the transmission happened in a unilateral way - imparted by Europe and received by China¹. This underlying assumption has had a significant influence on research on the history of science and technology in 17th- to 19th-century China². It leads this research inevitably toward issues such as the content of the European knowledge transmitted into China, the Western sources of this knowledge, the impact of European knowledge upon Chinese tradition, the standard of the Chinese understanding and adoption of the knowledge concerned, etc.³. At the same time, any content inconsistent with European thought found in a document concerned with European science and technology serves only to provide evidence for misunderstanding of this knowledge by Chinese scholars, or their incapacity to catch up with European standards. However, if we wish to develop a comprehensive understanding of the process of transmission, we have to take account of all aspects of this process. Thus, examples of knowledge inconsistent with European thought contained in a translation of a European text or one designed to introduce European knowledge are of the greatest importance to us, as they provide us with precious information about the way foreign knowledge was assimilated by Chinese scholars, and how it was imparted by European missionaries within a foreign cultural tradition⁴.

During the end of 16th century and the first half of 17th century, thought from the European and Chinese traditions met directly. Most of the Chinese scholars who participated in the transmission of European scientific and technological knowledge were thoroughly versed in Chinese traditional learning and were interested in science and technology. It would be quite normal if they used the knowledge they were already equipped with to understand and interpret the knowledge from abroad, and it is certain that in doing so misunderstandings and misinterpretations would result. Such evidence is very precious for an understanding of the process of transmission of knowledge and scientific thought between two such great cultural traditions. Until recently, little effort had been made to conduct research on the knowledge and thought brought into the corpus of the Chinese version of European knowledge by Chinese transmitters, and whether traditional Chinese knowledge and thought could or could not be consistent with those of Europe. In this paper, we endeavor to inquire into the above mentioned problem through a case study of the compilation of *Yuanxi qiqi tushuo lu zui* 遠西奇器圖說錄最 [A Record of the Best Illustrations and Descriptions of Extraordinary Devices of the Far West], more commonly known as *Qiqi tushuo*. We will pay special attention to the experience of Wang Zheng 王徵 (1571-1644) in its compilation, and the part played by traditional Chinese thought and learning in his contribution.

1 We will not be dealing here with research on the influence of Chinese culture and tradition upon European civilization.

2 Studies have shown that Chinese scholars played a far more significant role than that of mere passive acceptors in the transmission of European knowledge. They participated directly in the translation of European scientific works, and in the selection and reshaping of the knowledge transmitted in China.

3 We are in no way trying to deny the importance of research into these areas. Such research provides a solid grounding for the study of the history of the transmission of European knowledge.

4 A comprehensive understanding of the transmission of European knowledge in China requires research on the following aspects: the personalities who did or did not participate in the transmission, and their motivations for doing so; the cultural and social context of the transmission; and the content of the transmission. Existing studies already provides us abundant information concerning participants and opponents of the transmission of European knowledge. Research on the social and cultural context of the transmission as well as the content of European knowledge transmitted into China has also been productive. Nicolas Standaert has given a full account of these aspects in his *Handbook of Christianity in China*, Volume One (Leiden: Brill, 2001), pp. 635-1800.

1. A Short Introduction to Wang Zheng and His Time

Wang Zheng was born in 1571 in Jingyang county, Shanxi province, and lived through the reigns of the last two emperors of the Ming dynasty, the Wanli 万历 and Jiajing 嘉靖 reigns. His father was a teacher of Confucian classics and arithmetic. Wang passed the imperial exams at the provincial level in 1594, when he was 24. From the following year, he attempted the highest imperial examination ten times, finally succeeding in 1622. Soon after that, he was appointed an official in Guangping county and then Yangzhou city successfully. In 1631, he was promoted to be a supervisor of military affairs. However, he was soon removed from his post due to a rebellion by low ranking military officers, and returned to his home town. In 1643, his home town was overrun by one of the rebelling armies. Refusing to serve them, he committed suicide the following year, as the Ming dynasty collapsed. Such an act of loyalty was not unusual at times of dynastic transition.

However, what concerns us is his involvement in the transmission of European technology. The reason for his interest in European technology is connected with his traditional ideals of serving society and his interest in knowledge of diverse origins.

According to Wang's own account, he did not concentrate solely on the learning directly required for examinations, the interpretation of the Confucian Canons made by Zhu Xi 朱熹 (1130-1200). He was interested in a wide variety of knowledge. From a young age, he was interested in device-making, and, for a period of time, he indulged in Daoist theories of longevity and immortality⁵.

In 1616, he met the Spanish Jesuit missionary, Diego de Pantoja (1571-1618). On 24th January, 1601, Pantoja had accompanied Matteo Ricci (1552-1610), a pioneer of the transmission of Catholicism and European knowledge into China, on his arrival at Beijing. He went on to compile two books, among them, there is *Qike* 七克 (*the Seven Victories*), in which he explained how one should overcome the seven deadly sins of pride, envy, greed, anger, gluttony, lust, and sloth⁶. The Catholic ethics revealed in *Qike* do not violate the orthodox ones of Confucianism. After reading *Qike*, an important Chinese Scholar Xiong Mingyu 熊明遇 (1579-1649) commented: "Unexpectedly, Western scholars could also render outstanding service to Confucius"⁷. During 1625 and 1626, Wang helped the Jesuit missionary Nicolas Trigault (1577-1628) to compile and publish a Chinese-Latin phonetic dictionary *Xiru Ermu zi* 西儒耳目資 (*An Aid to the Ear and the Eye of Western Scholars*). However, in his preface to this book, he stressed that, through the study of Western writing, he understood why the ancient Chinese classic the *Yi Jing* was the origin of all written characters. He divided Latin letters into the mother of characters (zimu 字母) and the father of characters (zifu 字父), and connected them with Chinese Yinyang theory and the 64 divinatory symbols.

From his experience, we may see that Wang was originally open to knowledge of different traditions. This open attitude toward non-Confucian knowledge was not so unusual among Chinese scholars of the 17th century. The most influential scholar of that time, Wang Yangming, studied Buddhism and Daoism before establishing his system of neo-Confucian doctrines. Lu Jiuyuan 陆九渊, a scholar of the Song dynasty, said: "There is a Sage who appears in the Eastern Sea. The heart is the same, and the reason is the same. There is a Sage who appears in the Western Sea. The heart is the same, and the reason is the same. There is Sage who appears in the Southern and Northern Seas. The heart is the same and the reason is the same."⁸ Lu was generally regarded as the pioneer of Wang Yangming's doctrine, and this

5 In 1594, several months after he succeeded in the provincial examination, his mother died. By chance, he read a sentence in a Daoist text: "if one person achieves immortality, his forebears for nine generations are raised to the heavens". Wishing to pay back the benevolence of his ancestors, he decided to study Daoist arts. See his preface to *Liangli Lüe* 两理略 (An outline of two administrations), in Wang Zheng Yizhu 王徵遗著 (*Wang Zheng's Posthumous writings*), Li Zhiqing 李之勤 ed. (Shanxi Renmin Chubanshe 陕西人民出版社, 1984) p.12. In addition, Wang styled himself Liaoyi daoren (了一道人), which means "Daoist Knowing The One". *One* is a crucial term in Daoism, which means the beginning of the whole universe. This shows us that even after he converted to Catholicism, he still retained a profound interest in Daoist theories.

6 See Waltner, Ann, "Demerits and Deadly Sins: Jesuit Moral tracts in Late Ming China", in Stuart B. Schwartz (ed.), *Implicit Understandings: Observing, Reporting and Reflecting on the Encounters between Europeans and Other Peoples in the Early Modern Area* (Cambridge: Cambridge Univ. Press, 1994), pp. 422-448.

7 Xiong Mingyu, *Qike Yin* (*Introduction to Qike*), in *Qike, Tianxue Chuhan* (*First collectanea of heavenly studies*) edition, 1628. p.2a-b.

8 Lu Jiuyuan, *Lu Xiangshan Ji* 陆象山集 (*Collection of Lu Xiangshan*), pp. 482-483.

saying was taken as a slogan by the scholars who converted to Catholicism. In fact, numerous scholars of the Ming dynasty, including such famous converts to Catholicism as Li Zhizao 李之藻 (1565-1630), studied Buddhism, Daoism or Catholicism. Wang Zheng claimed: “(As for) learning, (I) originally don’t mind whether (it is) fine or rough, (but) always expect (it) to be of benefit for the world. (As for) a person, I also don’t ask whether (he is) a Chinese or a westerner, (but) always expect that he does not disobey Heaven.”⁹ In his various works, Wang tried to incorporate Catholic ideas into Confucianism. We believe that this was not only an effort to legitimize Western knowledge and Catholicism in China, but also that his satisfaction that Western knowledge and religion did not violate Confucian tradition was the basis for his adoption of Western knowledge and his conversion to Catholicism¹⁰.

There is no solid evidence to tell us exactly when Wang converted to Catholicism. Even though he was interested in Daoism, and converted to Catholicism, he remained a Confucian scholar. In his *Liangli Lüe*, he mentions that whenever “I arrived at a county, I would first pay homage at the temple of Confucius. I would examine the sacrificial utensils and books. Obeying the Sage, esteeming (his) “Way” (Dao) and respecting (his) learning are the main acts of righteousness of a country.”¹¹

Wang believed that both European knowledge and thought were in accordance with Confucianism. His remarks show us his attitude toward Western missionaries and their knowledge and the reasons behind it:¹²

夫西儒在茲多年，士大夫與之游者靡不心醉神怡。彼且不驕不吝，奈何當吾世而覲面失之？古之好學者裹糧負笈，不遠數千里往訪。今諸賢從絕徼數萬裏外齎此圖書以傳我輩，我輩反忍拒而不納歟？諸賢寥寥數輩，胥皆有道之儒。來賓來王，視昔越裳肅慎，不啻遠之遠矣。正可昭我明聖德，來遠千古罕儷之盛。邇來餘省新從地中掘出一碑，額題“景教流行中國碑頌”，乃唐郭子儀時所鑄，千載如新，與今日諸賢所傳崇敬天之教一一若合符節。所載自唐太宗以後凡六帝，遞相崇敬甚篤也。在昔已然，今又何嫌忌之與¹³？

The western scholars have lived here for many years. (Among) the scholar-bureaucrats who have associated with them, all have been fascinated and delighted. They are neither arrogant nor stingy. How can it be appropriate in our age to meet (them) and let them go. Those of old who loved learning wrapped food and carried books, and went to the trouble of traveling several thousand li to visit (learned people). Now, various worthy persons have brought these books from the farthest frontiers a myriad li away in order to transmit them to us. How can we have the heart to reject and not to accept (them)? These worthy persons are only a few people and all are true scholars¹⁴. Vassal states pay tribute (to the emperor), and princes have an audience (with him). Compared with the Yuechang¹⁵ and Sushen¹⁶ of former times, (the western scholars are) no less distant than they! (This) truly makes manifest that the flourishing of the sagely virtue of our Ming (dynasty) attracts from afar rarities to rival any age. Recently, a stele has been newly unearthed in our province, at the head of which is inscribed “the stele eulogy of the popularity of the Jing religion [i.e. Nestorianism] in China.” (It) was carved during the time of Guo Ziyi in the Tang Dynasty. After a thousand years (it still looks) like new. (The eulogy) matches in every particular with the heaven-revering religion (Catholicism) that is transmitted by the present worthy scholars like (the two halves of) a tally¹⁷. (It is) recorded (that) all six emperors from the time of Tang Taizong [the second emperor of the Tang

9 Wang Zheng’s preface in *Qiqi tushuo*.

10 Catherine Jami has analyzed the efforts Chinese converts and scholars who admired Western learning made in legitimizing Western knowledge and Catholicism in China in her paper, “‘European Science in China’ or ‘Western Learning’?—Representations of Cross-Cultural Transmission, 1600-1800”, *Science in Context*, 12:3 (1999), pp. 413-434.

11 Wang Zheng, *Liangli Lüe*, p. 27.

12 Wang Zheng’s preface in *Qiqi tushuo*.

13 Wang Zheng’s preface in *Qiqi tushuo*.

14 The phrase used here, *you dao zhi ru*, literally means « Confucian scholars who possess the Way ».

15 Yuechang was the name of a state in the Southern Sea, and is said to have given a special kind of bird in tribute to the Duke of Zhou at the start of the Zhou Dynasty (ca. 11th century B.C.).

16 Sushen was a name of a people far to the Northeast of ancient China who sent such tributes as Hu-arrows and stone arrowheads to Kings Wu and Cheng at the start of the Zhou Dynasty.

17 A *fujie* or tally had characters written or engraved on it., and was then divided into two parts. These could subsequently be matched up as a method of authentication.

Dynasty], successively revered (it) very sincerely. It was already thus in the past, why object to it now?

Wang's suicide in 1644, by which he displayed his loyalty to the Ming emperor, is proof of his life-long commitment to Confucianism¹⁸. Loyalty toward the emperor was one of the prime principles of Confucianism, while suicide was forbidden by Catholicism.

Now, let us provide an outline of the context of the transmission of European knowledge in China. In the 16th century, direct contacts between China and Europe were established¹⁹. From then on, Jesuit missionaries acted as the main transmitters of European scientific and technological knowledge into China. Nicolas Standaert lists four commonly accepted characteristics of Jesuit corporate culture in China:

1. Accommodation or adaptation to Chinese culture;
2. Propagation and evangelization 'from the top down';
3. Indirect propagation of the faith by using European science and technology in order to attract the attention of the educated Chinese and convince them of the high level of European civilization;
4. Openness to and tolerance of Chinese values.

He argues that these characteristics were formed in the context of contemporary Chinese culture and social conditions²⁰. The role Jesuit missionaries played in the transmission of European mechanical knowledge was directly connected with the third characteristic in this list, which was in fact caused by the second, both of them tools for pursuing the great aim of the Jesuits, as Standaert argues, the Christianization of China.

The reason why Jesuit missionaries decided to transmit European scientific and technological knowledge is two-fold. First, scientific and technological knowledge could be quoted to prove the validity of Christian theories²¹. Second, scientific and technological knowledge could help in attracting the attention of Chinese scholars. The latter played a more important role in the transmission of device-making methods in China.

Not long after arriving in China, Matteo Ricci committed himself to "building up throughout the empire a network of friendly contacts and developing ever broadening Christianizing masses". He hoped that, after having converted "a goodly number of Christians, then perhaps it will not be impossible to present some memorial to the emperor asking at least that the right of Christians to practice their religion be accorded"²². In order to attract their attention, Matteo Ricci "sought to make himself all things to all men, in order to win them all to Christ"²³. Even though there was persistent resistance to this strategy from missionaries of other schools and even among Jesuit missionaries, the transmission of scientific and technological knowledge was one of the long-term enterprises of the Jesuit missionaries in China.

As for Chinese scholars, mechanical knowledge, especially applied mechanical knowledge was especially significant to them. At that time, there was an urgent need for devices used for irrigation and flood defense, while internal rebellions and external threats meant that there was a dire need for military devices.

So, *Qiqi tushuo* was compiled in the context of a combination of the needs of Chinese scholars and the strategies of Jesuit missionaries.

18 See Zhang Pengfen's preface in *Qiqi tushuo*.

19 On the relationship between Europe and China, see Fang Hao, *Zhongxi jiaotong shi* 中西交通史 (*The History of Communication between China and the West*) (Changsha: Yuelu shushe, 1987), p. 732.

20 N. Standaert, "Jesuit Corporate Culture as Shaped by the Chinese", In *Jesuits - Cultures, Sciences, and the Arts* (1540-1773), John W. O'Malley, Gauvin Alexander Bailey, Steven J. Harris, T. Frank Kennedy, eds. (Toronto: University of Toronto Press, 2000), pp. 352-363.

21 In his *Tianzhu Shiyi*, Matteo Ricci tried to prove the existence of God by introducing Aristotle's mechanical theories concerning motion. Li Madou 利玛窦, *Tianzhu Shiyi* 天主实义 (*Tianxue Chuhan* Edition, 1630), p.3b.

22 George H. Dunne. *Generation of Giants*, Paris: Univ. of Notre Dame Press, 1962. pp. 85-88.

23 Matthew Ricci. Translated by Louis J. Gallagher, *China in the 16th century—the Journals of Matthew Ricci 1558-1610*. New York: Random House, 1953. P. 277.

2. The Relationship between the Compilation of *Qiqi tushuo* and Confucian Tradition

Qiqi tushuo was compiled by the German Jesuit Johann Terrenz (1576-1630) and Wang Zheng between November 1626 and February 1627. By analyzing the sources relating to the compilation of *Qiqi tushuo* and Wang's part in the process, we find that Confucian tradition and the trend of Confucian thought in Wang's time are significant factors in its production.

According to Song Boyin's 宋伯胤 chronological research on Wang Zheng, Wang took an interest in devices from his youth. In 1577, when he was seven, he began studies with his uncle Zheng Jian 張鑑, a scholar-official. Influenced by him, Wang became interested in making military devices. Between 1623 and 1626, he used his devices for various purposes, such as dredging rivers, flood defense and military defense. His choice was not random. Wang introduced his achievements in device-making to his readers:

考工指南而後，代不乏宗工哲匠，然自化人奇肱之外，巧絕弗傳。而木牛流馬遂擅千古絕響，餘甚慕之愛之。間嘗不揣固陋，妄制虹吸、鶴飲、輪壺、代耕及自轉磨、自行車諸器，見之者亦頗稱奇²⁴。

After the (ancient text) *Kao Gong (Ji)* and the south-pointing (carriage), there was no lack of great craftsmen and wise artisans in (every) generation. However, except for the Huaren²⁵ and Qigong²⁶, ingenious and consummate (skills) were not transmitted. The *muniu liuma*²⁷ then enjoyed unrivaled fame, and I admired and loved it very much. Occasionally, without heeding my ill-informed (situation) and ignorance, I rashly made various machines, such as a *hongxi* (lit. "rainbow-sucking")²⁸, *heyin* (lit. "crane-drinking"), *lunhu* (wheeled clepsydra), *daigeng* (plough substitute), *zizhuanmo* (self-rotating mill) and *zixingche* (self-moving quadricycle). The people who saw these said (they were) quite extraordinary.

All these machines constructed by Wang were concerned directly with farming and daily life. Besides the influence of his uncle, his admiration for a former famous Confucian Fan Zhongyan 范仲淹 may also have played a role in shaping of his interest in device-making. In 1587, he expressed his admiration for Fan, a model of an official and scholar in Confucian tradition²⁹. Fan was renowned for his sense of responsibility towards the nation and its people, organizing flood-control work and attending to military affairs³⁰. That Wang should follow Fan's example was normal in the framework of Confucianism.

We should admit, however, that Wang's interest was not widely shared by his contemporaries. Most Chinese scholars concentrated on personal moral perfection and debates about problems concerning the interpretation of ancient Confucian classics. Activities relating to practical matters were commonly regarded as demeaning. A conversation recorded (or invented) by Wang in his preface of *Qiqi tushuo* shows us the contradictory ideas that prevailed about device-making among Chinese scholars. Wang begins with a question presented by someone:

今茲所錄，特工匠技藝流耳。君子不器，子何敝敝焉於斯？

Now, what is recorded here is merely such things as the arts and crafts of artisans. A Junzi [a man of honor in the Confucian system] does not like utensils³¹. Why do you exhaust yourself at this?

24 Wang Zheng's preface in *Qiqi tushuo*.

25 化人 (Huaren) were legendary ingenious people during the reign of the Zhou king Mu (ca. 10th century B.C.). See *Liezi*, ch. 3 (*Zhuzi Jicheng* edition. Shanghai: Shanghai shudian, 1986), pp. 31-32.

26 奇肱 (Qigong) was the name of a legendary country where many marvelous machines were made. See *Shanhai Jing*, ch. 7 (Shanghai: Shanghai guji chubanshe, 1980), p. 213.

27 *Muniu liuma* means "wooden ox and gliding horse", devices reputedly used to transport grain across mountain invented by Zhuge Liang in the Three Kingdoms period (early 3rd century A.D.).

28 *Hongxi* is a single-cylinder force-pump, *Heyin* a flume-beamed swape, *Lunhu* a combined clock, *Daigeng* a winch-driven cable plough, *Zizhuanmo* a weight-driven geared mill and *Zixingche* a weight-driven geared quadricycle.

29 See Song Boyin, *Wang Zheng Nianpu* 王徵年譜 (*Chronicle of Wang Zheng's life*), (Shanxi shifan daxue chubanshe, 1990), p. 15.

30 See Huang Zongyi 黃宗義, *Gaoping Xuean* 高平學案 (*Scholarly Biography of Gaoping*), in *Songyuan Xuean* 宋元學案 (*Biography of scholars in the Song and Yuan Dynasties*, first printed in 1846), ch. 3, (*Zhejiang guji chubanshe*, 1994), pp. 180-183.

31 The Analects of Confucius (*Lun Yu* 論語, 為政) says, "*jun zi bu qi* (君子不器)." The Chinese term "*bu qi*" should really be translated as "being unlike a utensil", and means that their use is not confined to one aspect.

Wang answered:

茲所錄者，雖屬技藝末務，而實有益於民生日用、國家興作甚急也。”儻執不器之說而鄙之，則尼父系《易》，胡以又雲：“備物制用，立成器以為天下利，莫大乎聖人”³²。

Although what is recorded here belongs to such trivial affairs as arts and crafts, nevertheless, they are actually beneficial to people's livelihood (and) daily use, extremely essential for the prosperity of the nation. If we should hold the view of disliking utensils and despising them, then why did Confucius say in his commentary to the Book of Changes (*Yi*), “(As for) preparing things for use, and establishing finished utensils of benefit to all under heaven, no one is greater than the Sage.”

In fact, from the beginning of the 17th century, there was a rapid growth of interest in such practical matters among scholars, linked to a rejection of certain more esoteric elements of neo-Confucianism and the revival of concrete learning. This trend was pushed to its zenith by the economic and social crises that accompanied the transfer of dynasties³³. Quite a few scholars called for attention to be paid to practical fields under the slogan of recovering ancient learning and eliminating the influence of Buddhism and Taoism. Some of these scholars were attracted by the Jesuits and their learning, and some of them converted to Catholicism³⁴. Wang was one among them. As mentioned above, Wang admired devices mentioned in ancient sources and legends. In his preface to *Qiqi tushuo* he again says:

余不敏，竊嘗仰窺制器尚象之旨，而深有味乎璿璣玉衡之作一器也，規天條地，七政咸在，萬祀不磨，奇哉！蔑以尚已³⁵。

I am not smart, (but) I have admired and pried out the principles of device-making and illustration in private, and had a deep interest in (the fact that) by making one machine, the *xuanji yuheng* 璿璣玉衡³⁶, (one) models the heavens and orders the earth. The sun, moon and five planets were all on (it), and it is perpetuated after a myriad generations. How extraordinary! Nothing can surpass (it).

His admiration for ancient devices did not disappear after he had met the European missionaries. His recollection provides us with precious information about how he used European knowledge to reconstruct ancient devices:

憶余少年時，妄意武侯木牛流馬，必欲仿而行之。輒准杜氏《通典》尺寸作法，再四為之摩擬，迄無能成，然弗肯中止。往往考古證今，旁咨遠訪，窮索苦思。忘食寢，廢應酬，一似癡人。乃癡想之極，會得西儒自鳴鐘法，遂頓生一機巧，私儀必可成也。如法作之，果遂成。不敢妄擬木牛流馬，爰名之為自行車焉³⁷。

I can still remember that when I was young, I fantasized about the *muniu liuma* (wooden ox and gliding horse) of Wuhou 武侯 (Zhuge Liang 諸葛亮, 181-234 A.D.) and was determined to imitate it. At once (I) followed the measurements and method of construction in Du's *Tong Dian* 通典, trying to model (it) again and again. Although I was never successful, nevertheless, I was not willing to stop. (I) was constantly examining what was ancient to verify my current (labors), consulting with those both near and far. Deep in thought, I forgot to eat and sleep, and neglecting social intercourse, I was just like a crazy man. Just when (my) obsession reached its limit, (I) learned the method of the self-sounding bells [clocks] of the Western Scholars. Then, I suddenly had an ingenious idea, privately conjecturing that (the device) could be completed. Following this method to make it, it was actually completed. I dared not presume to compare it with the *muniu liuma*, so I named it as a self-moving vehicle.

32 Wang Zheng's preface in *Qiqi tushuo*. Wang actually changes the quotation from the Appendix to the Book of Changes 易·繫辭 slightly. In the opening phrase 備物致用, he uses the character 制 to replace the character 致. The two characters are pronounced the same, and it is also possible that this is simply a printing error.

33 The new dynasty, the Qing Dynasty began in 1644.

34 The most famous convert of that time is Xu Guanqi 徐光啟. Xu hoped that Catholicism would act as an assistant to Confucian replacing Buddhism.

35 Wang Zheng's preface to *Qiqi tushuo*.

36 *Xuan ji* 璇璣 means an instrument concerning the first four stars in the bowl of the Big Dipper; *yu heng* 玉衡 means an instrument which simulates the running of the Little Bear. Here *xuan ji yu heng* 璇璣玉衡 means a kind of astronomical instrument.

37 Wang Zheng. *Liangli Lüe*, in Wang Zheng *Yishu*, pp. 81-82.

Thus, inspired by the mechanisms of European clocks, Wang invented a device in imitation of an ancient one. We can find several such instances of the use of European knowledge and technology to reconstruct supposedly lost ancient devices through the 17th to 19th centuries in China. The reason why Wang's statement interests us is that it shows that for him the process of using European methods to reconstruct ancient devices, or putting it another way, the process of putting the foreign method into a traditional framework, was relatively smooth. There is another thing that draws our attention in the above quotation. Wang appeared quite indifferent to the clock-making technology itself. We know that European clocks were generally welcomed by Chinese officials and even emperors. However, it seems that what interested Wang most about the clock-making technology is that it allowed him to reach an understanding of the mechanisms of an ancient device, one that was useful in transportation. Let us return to the material we quoted at the beginning of this section. In it, the devices Wang admired were water-lifting devices, transport devices and astronomical instruments. From the above discussion, we know that he was interested in such devices before he had contact with European knowledge, and furthermore, all these devices are useful for daily life. We know that in traditional Chinese culture extraordinary devices were regarded as the product of uncanny techniques, and unless they were beneficial to people they were generally disdained. Clocks could be of use in daily life, but it is possible that for Wang they were not of urgent need in comparison with the devices he mentions in his preface to *Qiqi tushuo*.

To sum up, based on an analysis of available sources and Wang's experience, we argue that traditional Chinese modes of thought were central to the motivation behind the compilation of *Qiqi tushuo*. In the following section, we will argue that traditional reasoning and thought also had their influence on the content of the book.

3. The Influence of Ancient Chinese Thought on the Content and Structure of *Qiqi tushuo*

We have argued in the above discussion that it was Wang Zheng who proposed the idea of compiling a book about European device-making, and that his motivation was situated within the framework of the Confucian ethical system. In this part, we will focus on the content and the structure of *Qiqi tushuo*, and will inquire into the problem of the ways and extent to which Chinese culture and thought influenced the shaping of the book.

In 1615, Wang acquired a book entitled *Qi ke (the Seven Victories)*, the first he read about Catholicism. In 1616, he met with Diego de Pantoja (1571-1618) the book's author.³⁸

In 1622, Wang composed a memorial on resisting enemy attack, in which he introduced some military devices invented by him and his friends. According to research by Song Boyin³⁹, Wang made the following devices before 1623, that is before he met Jesuit missionaries: *hongxi* (lit. "rainbow-pumping") 虹吸, *heyin* 鶴飲 (lit. "crane-drinking"), *longwei* 龍尾 (screw) and *hengsheng* 恒升 (piston-pump). *Longwei* and *hengsheng* were first introduced by Sabatino de Ursis and Xu Guangqi in *Taixi shuifa* 泰西水法 (*Hydraulic Methods of the Great West*, 1612). Wang must, therefore, have read the *Taixi shuifa* before 1623.

The *Lunhu* 輪壺 (wheeled clepsydra), *daigeng* 代耕 (plough substitute), *zizhuanmo* 自轉磨 (self-rotating mill) and *zixingche* 自行車 (self-moving quadricycle), which are described in *Zhuqi tushuo* (1626) in detail, are based on the principles of the mechanisms of European clocks⁴⁰. Therefore, Wang must have had a detailed understanding of the European clocks that had been introduced into China by Jesuits after 1582⁴¹ prior to 1626. Wang makes the following remarks in his preface to *Qiqi tushuo*:

偶讀《職方外紀》，所載奇人、奇事未易更僕數，其中一二奇器絕非此中見聞所及。如云多勒多城，在山巔取山下之水，以供山上，運之甚艱。近百年內有巧者製一水器，能盤水直至山城，絕不賴人力，其器自能晝夜轉運也。又云，亞而幾墨得者，天文師也，承國王命，造一航海極大之船。船成，將下

38 Song Boyin. *Wang Zheng Nianpu*. pp.40-42.

39 Song Boyin. *Wang Zheng Nianpu*. pp.59, 69.

40 Wang Zheng's preface in *Qiqi tushuo*. For these devices, see note 28 above.

41 Zhang Baichun 張柏春. Ming-Qing shiqi ouzhou jixie zhongbiao jishu de chuanru ji youguan wenti 明清時期歐洲機械鐘錶技術的傳入及有關問題, *Ziran bianzhengfa tongxun* 自然辯證法通訊, 17:2 (1995), pp.38-46.

之海。計雖傾一國之力，用牛馬駱駝千萬，莫能運也。幾墨得營作巧法，第令王一舉手引之，舶如山岳轉動，須臾即下海矣。又造一自動渾天儀，其七政各有本動，凡列宿運行之遲疾，一一與天無二。其儀以玻璃為之，悉可透視，真希世珍也。《職方外紀》，西儒艾先生所作，其言當不得妄。余蓋爽然自失而私竊嚮往，曰：“嗟乎！此等奇器何緣得當吾世而一睹之哉”？丙寅冬，余補銓如都，會龍精華、鄧函璞、湯道未三先生以候旨修歷，寓舊邸中。余得朝夕晤請教益甚謹也。暇日，因述《外紀》所載質之，三先生笑而唯唯。且曰：“諸器甚多，悉著圖說，見在可覽也，奚敢妄？”余亟索觀。……於是，亟請譯以中字⁴²。

(I) read *Zhifang Waiji* 職方外紀 (*Record of the Places Outside the Jurisdiction of the Office of Geography*) by chance. The extraordinary persons and extraordinary things recorded (in it) are not easily listed, and among them are one or two extraordinary machines that can certainly not be compared with those of which I have knowledge of here. For example, it is said that (in) the city of Duoleduo (Toledo), (people) fetched water from the foot of a mountain to the mountaintop in order to supply their needs, and that the transportation of it was extremely difficult. Within the last hundred years, there was an ingenious person who made a water-lifting device. (It) could “spiral” water up to the city on the mountain without relying on any manpower, and it could run day and night by itself. It is also said that Archimedes, who was an astronomer, received the king’s order to construct a huge ship. The ship was completed, and was to be put out to sea, yet it could not be moved by a nation’s workforce nor a myriad beasts of burden. Archimedes invented an ingenious method. The king simply raised a hand and pulled it, the ship turning like a mountain and descending into the sea in a very short time. (Archimedes) also constructed an automatic armillary sphere, the sun, moon and five planets keeping to their original motions, and the speed of the motion of all the constellations not differing from those in the heavens. This instrument was made of glass, and was transparent, truly a rare treasure in the world. *Zhifang Waiji* 職方外紀 is written by the western scholar Aleni, and his words in it should not be fanciful. It suddenly dawned upon me that I was missing the mark and yearning (for it) in private, said: "Alas! Such extraordinary machines, how can I get an appropriate opportunity to have a look at them in my lifetime?" In the winter of the *bingyin* 丙寅 year (1626), I went to the capital waiting for my appointment. At that time, while waiting for the imperial edict to compile the calendar, three gentlemen, Long Jinghua 龍精華 (N. Longobardi, 1559-1654), Deng Hanpu 鄧函璞 (Johann Terrenz) and Tang Daowei 湯道未 (Jean Adam Schall von Bell, 1592-1666) were living in their old residence. (Therefore), I was able to meet them and learn from them day and night, and was very happy. One day when not busy, I talked about what *Waiji* recorded and question them about it. The three gentlemen politely smiled and affirmed it. Moreover, they added that “there were many such devices, and they are recorded (in the form of) illustrations and descriptions. (The illustrations and descriptions) are now here and can be read. How dare (we talk) preposterously?” I immediately ask to see them. ... Thereupon, (I) earnestly requested to translate (them) into Chinese.

It is clear that Wang’s admiration for Western devices led to the compilation of *Qiqi tushuo*, and that his admiration was grounded in the statecraft of Confucianism.

The contents of the *Qiqi tushuo* were drawn from many European works. In 1618, the Jesuit missionary Nicolas Trigault (1577-1628) set sail from Lisbon to China, bringing with him 22 missionaries and about 7000 volumes of European books. According to Fang Hao’s study, only a portion of these books arrived in China⁴³. Among these, less than one hundred were concerned with mechanical knowledge, the *Qiqi tushuo* being compiled on the basis of these⁴⁴. Compared with a straight-forward Chinese translation of a European book, authors can put more subjective ideas into a compiled work. This has made analysis more complicated for us, and at the same time, more interesting. Before we get into the

42 Wang Zheng. Preface, in *Qiqi tushuo* (1627). Chapter 1.

43 Fang Hao 方豪, *Mingji xishu qiqian bu Liuru zhongguo kao* 明季西書七千部流入中國考. *Wenshi zazhi* 文史雜誌, 3: 1&2 (1944), pp. 47-51.

44 At the beginning of his preface of *Qiqi tushuo*, Wang mentions: “Illustrations and descriptions of extraordinary devices are books brought by Western scholars from their countries. These books are one branch of more than 7,000 books (they brought into China). (And) this (the *Qiqi tushuo*) only represents one tenth of the books in this branch”. Wang Zheng, preface. p. 1a.

content and structure of the *Qiqi tushuo*, let us first have a look at the method and the process of its complication. Only after we have made the method of compilation clear can we explain the role of the two authors in the selection of the book's content.

We know that only a very small number of Chinese scholars could read Latin in the 17th century. Wang Zheng is something of an exception. He assisted Trigault in the compilation of a Latin-Chinese dictionary, *Xiru Ermu Zi* 西儒耳目資 (*An Aid to the Ear and the Eye of Western Scholars*), through which he became acquainted with Latin. Nevertheless, his standard of Latin was not sufficient to carry out the task of translating a technological work in Latin into Chinese. He introduces the situation thus:

有物有像，猶可覽而想像之。乃其說則屬西文西字。雖余嚮在里中，得金四表先生為余指授西文字母字父二十五號，刻有《西儒耳目資》一書，亦略知其音響乎。顧全文全義則茫然其莫測也⁴⁵。

There are objects and illustrations. (We) can imagine (the objects) by looking at (their illustrations), but their descriptions are in a Western script. However, when I was in my hometown, (I) received instruction from the gentleman Jin Sibiao 金四表 (Nicolas Trigault) on the 25 letters of the Western script, and (I) printed a book, *Xiru Ermu Zi* (*An Aid to the Ear and the Eye of Western Scholars*). (I) also knew a little about the pronunciation of these letters. Nevertheless, I was at a loss as to the meaning of a complete text and could not infer (it).

Clearly, it was beyond Wang's ability to understand the content of a Latin text. So, the actual method of compilation was as follows:

取諸器圖說全帙，分類而口授焉。余輒信筆疾書，不次不文，總期簡明易曉，以便人人覽閱⁴⁶。

(Terrenz) took out all the books with the illustrations and descriptions of devices, (and he) communicated it orally by categories. In great haste I wrote it down, not putting the text in sequence or embellishing it. (I) intended that it would be concise and easy to understand so that everyone could read (it) conveniently.

This way of translating a foreign text into Chinese had been used as a standard method for a long period of time in China. In the Tang Dynasty, it had been used for the translation of Buddhist Sutra. In 1382, the first emperor of the Ming Dynasty ordered Arabic astronomers working in the Imperial Astronomical Bureau to translate Arabic astronomical works into Chinese. He gave detailed rules for the method of translation:

尔西域人素习本音，兼通华语，其口以授儒，尔儒译其义辑成文焉。惟直述，毋藻绘、毋忽⁴⁷。

The person from the Western Region⁴⁸ should be a native speaker, and also familiar with Chinese. They communicate (the contents) orally to Confucian scholars, who then translate their meaning and compile it into texts. Narrate directly, without embellishing or leaving anything out.

This rule leaves little space for the Chinese translator to add his own ideas to the text. Therefore, we may conclude that the Western participant, Johann Terrenz, is the key person in the interpretation of the knowledge communicated. However, as Wang states, when shown illustrations of European devices, he could easily understand their function, and hence could ask Terrenz to interpret the descriptions of the devices he was interested in. On the other hand, Terrenz could decide whether he would like to fully fulfill Wang's requests. A few lines of introduction about Johann Terrenz's background may give us some clues as to when he might not accede to Wang's requests.

45 Wang Zheng, Preface to *Qiqi tushuo*.

46 Wang Zheng, Preface to *Qiqi tushuo*.

47 Zhu Yuanzhang 朱元璋, rescript, quote from Ruan Yuan 阮元, *Chouren Zhuan* 畴人传 (*Biographies of Mathematicians*), Guanwo Shengshi Huigao edition, 1843. Chapter 29, p.2b.

48 Chinese generally named Central and West Asia Xiyu 西域 (Western Region).

Johann Terrenz was the most competent scientist among the Jesuit missionaries in China⁴⁹. He studied medicine at Padua, and was acquainted with several languages. He became the 7th academician of the Academy of Lincei in 1611. He was, therefore, a scholar of some renown, and deeply imbued with European learning.

We can well imagine that there would be some unavoidable differences and even contradictions in the outlook of these two scholars concerning technology, brought up as they were in two such different cultural traditions. As a result of his background, Wang was interested primarily in the following content of European sources:

第專屬奇器之圖之說者不下千百餘種。其器多用小力轉大重，或使升高，或令行遠，或資修築，或運芻餉，或便泄注，或上下舫船，或預防災祲，或潛禦物害，或自春自解，或生響生風，諸奇妙器無不備具。有用人力物力者，有用風力水力者，有用輪盤，有用關捩，有用空虛，有即用重為力者，種種妙用，令人心花開爽⁵⁰。

There are very many different kinds of books that specifically deal with the illustration and description of extraordinary devices. These devices usually use a small force to turn big weights, making them rise, travel a long distance, aid construction, transport fodder and food, facilitate drainage and flow (of water), raise and lower boats and ships, guard against disasters, resist damage, automatically pound and saw, produce sound (and) wind, etc. Thus are prepared all kinds of wonderful devices. Some use manpower or the force of objects; some use wind power and water power, others use rotiform plates, rotatable mechanisms, vacuity, or even a weight as force. All these wonderful usages realize all ones expectations.

Wang directly relates the devices he has read about in Western sources to statecraft and the needs of daily life. Furthermore, he sets out his rule for the selection of the contents of the *Qiqi tushuo*:

然圖說之中巧器極多，第或不甚關切民生日用，如飛鳶、水琴等類，又或非國家工作之所急需，則不錄。特錄其最切要者，器誠切矣。乃其作法或難，如一器而螺絲轉太多，工匠不能如法；又或器之工費甚鉅，則不錄。特錄其最簡便者，器俱切俱便矣。而一法多種，一種多器，如水法一器有百十多類，或重或繁，則不錄。特錄其最精妙者⁵¹。

However, there are very many ingenious devices among the illustrations and descriptions. If (some) are not really related to needs of everyday life, such as the “flying kite” and water-driven organ, or if (some) are not in urgent need for the nation’s purposes then (I) do not recorded (them here). (I) have only recorded the most crucial and necessary ones. A machine may certainly be crucial, but if the method of making it is difficult, for example, if a device has too many screws, (so that) craftsmen can not follow the method (to make it), or if the amount of work and cost required for a device is very great, then (I) have not recorded (them). (I) have only recorded the simplest and most convenient ones. A device may be crucial and convenient, however, if a method produces many types (of devices), or one type has many (individual) examples, for instance one type for a hydraulic method may have over a hundred examples of devices, some repeated and some complicated, then I have not recorded them (all). I have only recorded the finest and most ingenious ones.

So, Wang did not really need a theoretical work on technology, and he did not intend to transmit an entire system of European theoretical knowledge about device-making into China. His intention is clearly stated - the introduction of some useful devices from the West, ones of use to people and the nation that are relatively straight-forward to construct. His design of the content of the *Qiqi tushuo* is strictly confined to the illustration and description of the devices he was interested in. After the compilation of *Qiqi tushuo*, he used the knowledge in the book in his statecraft, making some devices in

49 According to M. D’Elia’s study, besides Johann Terrenz, there were two other competent scientists who set off for China, Wenceslaus Kirwitzer and John Alberich. John Alberich died during the voyage, and Kirwitzer died soon after his arrival in China. Thus, neither contributed to the transmission of European scientific knowledge into China. See: M. D’Elia, *Galileo in China—Relations through the Roman College between Galileo and the Jesuit Scientist-Missionaries (1610-1640)*. Cambridge: Massachusetts: Harvard University Press, 1960, pp. 25-28.

50 Wang Zheng, Preface to *Qiqi tushuo*.

51 Wang Zheng, Preface to *Qiqi tushuo*.

order to improve conditions in the counties where he worked as an official⁵². As we argued above, his conduct is completely in accordance with Confucian ethics, and his thought regarding technology and device-making was no different to that of an ordinary Confucian scholar such as Fan Zongyan, who Wang had admired from a young age.

However, the content of *Qiqi tushuo*, especially the original design of the content, is different from that as originally planned by Wang. In front of the main text of the work, there is a schema for the content of the whole book:

第一卷重解--此學總為運重而設。儻無重，何必運，且將何運？故重之解列為一卷。

第二卷器解--重不得起，須用器而起，器不一而足也。器之中又求最巧之器。故器之解列為一卷。

第三卷力解--巧器用以起重、引重、轉重，固矣。然器必借力而運，或人力馬力，或風力水力，或即借重物之力，故力之解列為一卷。

第四卷動解--有重於此，或欲升之高，或欲致之遠，或欲令其轉旋往來而不已。此皆運動法也。或薦，或揭，或推，或曳，或手轉足躡，種種不同。故動之解列為一卷。⁵³

Chapter one: Explanations of weights — This (branch of) learning is generally devised for transporting weights. If there were no weights, why transport and what would be transported? Therefore, explanations of weights are listed as one chapter.

Chapter two: Explanations of devices — When a weight can not be hoisted, it has to be hoisted by a device. (Such) devices are too many to be enumerated. We have sought out the most ingenious among them. Therefore, explanations of devices are listed as one chapter.

Chapter three: Explanations of force — Ingenious devices are obviously used to hoist, pull and turn weights. However, the device must rely on force to transport (the weight), and may variously use the force (provided by) people, horses, wind, water or a heavy object. Therefore, explanations of force are listed as one chapter.

Chapter four: Explanations of motion — Here is a heavy object. (We) may want to raise it up high, move it far away, or make it turn ceaselessly or move back and forth. These all are methods of motion. There are many ways to do this, by supporting, prizing, pushing, towing, or turning by hand (and) treading by foot. Therefore, explanations of motion are listed as one chapter.

This logical, sequential schema reveals the system of European mechanics step by step. Taking the first chapter as an example, let us analyze the details of its contents.

The first chapter contained 61 sections, including the definition of the center of gravity and the way of finding the center of gravity of different geometric shapes, the definition of specific gravity, the numerical relation of the weight, specific gravity and volume of a thing, as well as the proportional relation among weights and volumes of things with different specific gravity, etc., and some theory concerned with fluids and buoyancy. This chapter also provides some general discussions about cosmology. We will not go any further into the details of the content of this chapter⁵⁴. Here, we only want to point out that all the content contributes to theoretical discussions about basic concepts and knowledge of European mechanics, and that European mathematical methods were widely used in this chapter. The character of this chapter, and, furthermore, the structure of the content of the whole of the book, is in accordance with that of a European scholarly mechanical work.

We believe that this content was designed by the European compiler, Terrenz. In 1629, Terrenz became one of the main compilers of a collection of astronomical works, the *Chongzhen lishu* 崇禎曆書

52 See Wang Zheng, *Liang Liliue*, in Wang Zheng Yizhu, p.83.

53 Wang Zheng. Preface to *Qiqi tushuo*.

54 The content also shows characteristics of the interaction between thought and knowledge developed in different culture traditions. We are completing a paper that provides a detail discussion about the content of *Qiqi tushuo*. Deng Yuhan, Wang Zheng. *Qiqi tushuo*, chapter 1.

(*Chongzhen reign-period manual of mathematical astronomy*). The works in this collection were classified into five fundamental categories (基本五目): derivation (法原), numbers (法數), calculation (法算), devices (法器), and integration (法器). It is generally accepted among historians of astronomy and mathematics that Terrenz exerted a major influence on the design of the structure of the collection. We find that this structure is essentially the same as the schema of the content of *Qiqi tushuo*.

However, the actual structure of *Qiqi tushuo* is somewhat different from the schema at the start of the book. The *Qiqi tushuo* consists of three chapters. The contents of the first two chapters are just as they are given in the schema, one contributing to a theoretical discussion of weights, the other to a theoretical discussion of basic devices. The third is composed of illustrations and descriptions of a whole set of machines. Just as Wang remarked, all the machines introduced in the third chapter are concerned with everyday life and the nation's requirements. Hence, the last two chapters as originally planned by Terrenz are actually replaced by illustrations and descriptions of machines.

We think that the reason for this change is simple. Wang and Terrenz actually worked together for less than three months. In the winter of the sixth year of the Tianqi reign, Wang went to Beijing and met with Terrenz⁵⁵. In the 12th month of the same year, he was appointed as an official at Yangzhou. Before he left the capital, Beijing, he wrote a preface to *Qiqi tushuo*⁵⁶. Terrenz, meanwhile, remained in Beijing until his death. In the traditional Chinese calendar, the winter starts with the beginning of the tenth month and ends at the end of twelfth month. In the sixth year of Tianqi reign, the first day of the tenth month was 19th November, 1626 of the Gregorian calendar, the last day of the twelfth month was 15th February, 1627 of the Gregorian calendar.

Without Terrenz's interpretations, it would be impossible for Wang to write the two chapters of theoretical discussion on European mechanics. One thing we need to point out here is that before the separation of the two compilers, Terrenz was not able to realize his design for the last two chapters of *Qiqi tushuo*, while Wang accomplished one chapter he was most interested in, namely selected translations of such European books on machines as Agostino Ramelli's *Le Diverse e Artificiose Machine del Capitano* (1588), Jacques Besson's *Théâtre de Instruments Mathématiques et Mécaniques* (1578), Faustus Verantius' *Machinae Novae Fausti Verantii Siceni, cum Declaratione Latina, Italica, Hispanica, Gallica et Germanica* (written c. 1595), Vittorio Zonca's *Novo Teatro di Machini e Edificii* (1607), and Heinrich Zeising's *Theatrum Machinarum* (1613). From 1627 to his death, Wang produced some works on the basis of chapter three of *Qiqi tushuo*. There is no evidence that he spent time researching theoretical knowledge on the basis of chapter one and chapter two.

Concerning the transmission of technology in China, Bray has argued that there was active transmission and passive transmission of knowledge. Applied to our discussion, we can say that Wang Zheng passively accepted European theoretical mechanical knowledge from Terrenz. It is possible that he was also interested in aspects of this knowledge, but it was not one of his main concerns. On the other hand, he very actively took part in the transmission of the machines he was very interested in. When the situation did not allow him to finish the compilation of both branches of mechanical knowledge as designed by Terrenz, Wang turned to the one he actively intended to transmit.

Traditional Chinese thought not only influenced the selection of the contents of the *Qiqi tushuo*, but also on the model of reasoning employed in the book.

Wang mentions that Terrenz had emphasized the significance of mathematics in device-making:

譯是不難，第此道雖屬力藝之小技，然必先考度數之學而後可。蓋凡器用之微，須先有度有數。因度而生測量，因數而生計算，因測量、計算而有比例，因比例而後可以窮物之理，理得而後法可立也。不曉測量、計算，則必不得比例；不得比例，則此器圖說必不能通曉⁵⁷。

Translating these (is) not difficult, however, although this knowledge belongs to trivial techniques of the art of force, nevertheless, (one) must at first investigate the study of measures and numbers [i.e. mathematics] before (the translation) can be done. To understand the subtleties of the devices, first there must be measures and numbers. It is from measures that measurement is produced, and from

55 Song Boyin. *Wang Zheng Nianpu*. pp.79-80.

56 Song Boyin. *Wang Zheng Nianpu*, p.93.

57 Wang Zheng, preface.

numbers that calculation is produced. From measurement and calculation comes proportion, and from proportion one is then able to make a thorough inquiry into the reason of things. After obtaining the reason the method can be established. Without knowing measurement and calculation, (one) certainly cannot obtain proportions, and without proportions, (one) certainly cannot understand the illustrations and explanations of these machines thoroughly.

At that time in Europe, mechanics was commonly regarded as one branch of mathematics. Therefore, it is natural that Terrenz would suggest to Wang to learn mathematics before attempting the translation of mechanical works. However, if we consider this the only reason why Terrenz might suggest that Wang learn mathematics we will miss some information provided by the above material. It is quite possible that Terrenz's statement is drawn mainly from Simon Stevin's first definition in the first book of the *Art of Weighing* and Guido Ubaldi's *Mechanicorum liber*. "The art of weighing is the art which teaches the ratios, proportions, and properties of weights or gravities of solids"⁵⁸. And we know that Simon Stevin's *Art of Weighing* was already identified as one of the main European sources of *Qiqi tushuo*. E. J. Dijksterhuis has pointed out that Archimedes treated mechanics as a branch of mathematics, and that he developed his system of statics following the model of Euclid's geometry. Simon Stevin studied and supplemented the work of Archimedes. In the *Art of Weighing*, following Archimedes' model, Stevin set out definitions and Postulates.⁵⁹ And for most propositions in the book, Stevin not only gave an explanation, but also provided a mathematical proof. For Terrenz, mathematical proof could not only guarantee the correctness of a method, but also demonstrate mathematics as a way of reasoning. The method could only be established after the reason behind it had been sorted out. In the *Chongzhen lishu*, Terrenz also provided proofs to astronomical theories contained in the book. However, it seems that Wang did not accept this model of reasoning. The *Qiqi tushuo* does not contain any mathematical proofs. Of those propositions taken from Stevin's book, only the main text of the proposition, the method and the explanation, are selected, and sometimes they are even reshaped. The mathematical proofs are all deleted. We surmise that Wang thought that since the explanation already makes clear the correctness of the proposition, so a mathematical proof as given by Stevin was not needed. Here, Wang followed the traditional Chinese model of reasoning. In ancient Chinese mathematical books, Chinese mathematicians also established the correctness of their algorithms using this model.

Finally, we want to point out that the design of the text of *Qiqi tushuo*, especially the third chapter, which was Wang's favorite part and in which he had more liberty in deciding the lay out, is identical with the design of illustrations and descriptions of devices in ancient Chinese books⁶⁰. Wang had compiled his *Zhuqi tushuo* 诸器图说 (*Illustration and description of devices*, 1626) using this type of design before the compilation of *Qiqi tushuo*. Furthermore, when he was around 70, he compiled a book *Ela jiya youzhuo zhuqi tushuo ziji* 额辣济亚扁造诸器图说自记 (*Record of the illustration and description of devices made with the blessing of the Gratla*), in which, he introduced 24 categories of devices, the text arranged in a form similar to the third chapter of *Qiqi tushuo*⁶¹.

4. Conclusions

Through the above analysis of Wang Zheng's reasoning and modes of thinking used in the compilation of *Qiqi tushuo*, we may conclude that Chinese concepts and knowledge were incorporated into the Chinese publication of European knowledge in the following ways:

1. Making use of their liberty to choose the subjects for transmission, Chinese scholars greatly affected the content of European knowledge transmitted into China. In the case of Wang Zheng, we find that his choice was made relying on Confucian concepts of statecraft.

58 Simon Stevin, *The Art of Weighing*, in: *Principal works of Simon Stevin*. Amsterdam: C. V. Swets & Zeitlinger, 1955. P. 97.

59 E. J. Dijksterhuis, Introduction to 'The Art of Weighing', in: *The Principal Works of Simon Stevin*, (Amsterdam: C. V. Swets & Zeitlinger, 1955), vol. 1, p. 37.

60 See: Zhang Baichun and Tian Miao, "Zhongguo gudai Jixie yu Qiwu de Tuxiang Biaoda 中国古代机械与器物的图像表达 [Pictorial Representation of machines and utensils in ancient China]," *Gugong Bobu Yuan Yuankan* 故宫博物院院刊 [Palace Museum Journal], 2006, No. 3. pp. 81-97.

61 See Song Boyin and Ming Jingyang, *Wang Zheng Xianshe Nianpu*, pp. 162-168.

2. In the process of translation and compilation, Chinese scholars played an important role in the translation of European terms, consciously or unconsciously transmitting their understanding of these terms.
3. In the translations, Chinese participants added their interpretation of European knowledge based on their own knowledge, which might come from traditional Chinese sources or from previous translations of European knowledge. In either case, Chinese knowledge was inserted into the body of the newly introduced knowledge.
4. Where a Chinese participant had considerable liberty in the transmission of a branch of knowledge, such as in the case of Wang Zheng and the compilation of *Qiqi tushuo*, they might also modify the European system of knowledge.

From this case study, we try to unveil one model of the transmission of European knowledge in 17th-century China. It is true that the transmission mainly relied upon the European transmitter, as there were hardly any Chinese scholars who could understand European languages at that time. However, the position of these European transmitters and the social and cultural context of 17th-century China determined that the transmission of European knowledge in China would not be simply unilateral. As we argued in the first section of this paper, the main transmitters of European scientific and technological knowledge, the Jesuit missionaries, did not aim at transmitting contemporary European knowledge in its entirety. Their strategy was to use European knowledge about science and technology as a bait to attract Chinese scholars toward Catholicism. Therefore, the knowledge they introduced had to chime with the interests of Chinese scholars. In the case of the compilation of *Qiqi tushuo*, we find that the book was compiled because a Chinese scholar was interested in the arts of device-making. During this period it was very difficult for Chinese scholars, even for converts to Catholicism, to fully appreciate the advantages of systemic and theoretical European scientific knowledge. In the case of *Qiqi tushuo*, Wang Zheng selected the parts of the knowledge that specially interested him according to the perceived need of Confucian statecraft. Though he respected Jesuit missionaries and admired the knowledge they introduced into China, he welcomed the newly introduced knowledge as a useful tool for Confucians and interpreted it within the system of Confucianism. This was normal in the social and cultural context of 17th-century China. Converts such as Wang, attracted as they were by western knowledge, still held fast to Confucianism as the core of their ideology, and thus they could only accept the knowledge in accordance with or as interpreted by the Confucianism of their time. As they participated in the transmission of European knowledge, so the system and the content of the knowledge were modified or reshaped.

Western Sources of the *Qiqi Tushuo*

Peter DAMEROW and Urs SCHOEPLIN

The Roman Catholic religious order of the Jesuits, the *Societas Jesu*, was founded in 1534. Its main goal was to spread and defend Catholicism. Shortly after the foundation of the order, the Jesuits started missionary activities in the Far East. But only at the end of the sixteenth century, that is in the late Ming and early Qing period, when the Jesuits adopted a new strategy, did they become able to influence for about 50 years the policy of the Chinese upper class.¹

This new strategy was based on adapting to the Chinese culture, on the one hand, and on impressing Chinese intellectuals by the competence of Western scholars and organizing the transfer of Western scientific and technological knowledge to China, on the other hand, before attempting to convert the Chinese people to Christianity. The French Jesuit Nicolas Trigault (1577-1628) and the German Jesuit Johannes Schreck (1576-1630, latinisiert Terrentius) traveled in 1614 through Europe collecting scientific books and instruments, among them a telescope from Galileo. The Jesuit Beitang library in Peking contained about 7000 European books brought to China by such activities. Several were translated into Chinese or were used to compile Chinese books describing Western science and technology.²

The book *Yuanxi qiqi tushuo luzui*, or for short *Qiqi tushuo*, *The Record of the Best Illustrations and Descriptions of Extraordinary Devices of the Far West*, provides an outstanding example of this transmission of knowledge from Europe to China at the beginning of the 17th century. It was written by Johannes Schreck in cooperation with the Christianized Chinese administrator Wang Zheng (1571-1644, baptized under the name Philip). The book was first published in 1627 and was republished later several times.

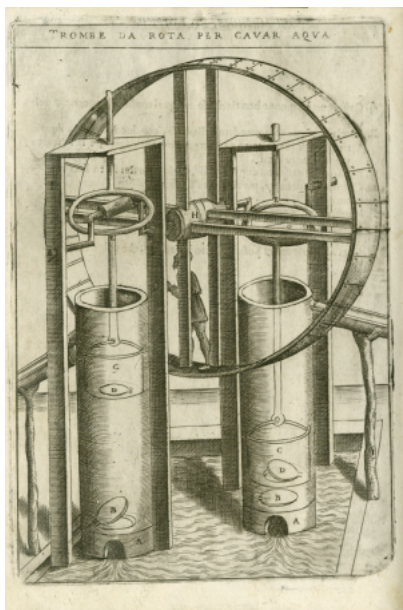


Figure 1: Zonca 1607³



Figure 2: *Qiqi tushuo* 1627

1 The influence was at times so strong that, as Matteo Ricci reports, Christianized Chinese scholars purged their libraries and burned numerous manuscripts and books that were considered to deal with subjects prohibited by the Catholic church; see Needham, J. *Science and Civilisation in China*, Vol. 4.1, Cambridge 1962 (Cambridge University Press), p. 244.

2 For the history and content of the library see: Verhaeren, H. (ed.) *Catalogue of the Pei-t'ang Library*, Beijing 1944-1948 (Imprimerie des Lazaristes).

3 Zonca, V. *Novo teatro di machine et edificii*, Padova 1607.

With the *Qiqi tushuo* Schreck intended to transmit European knowledge to China. He based his work on the widespread European technical literature of his time (see figures 1 and 2). It comes therefore as no surprise that the identification of the Western sources for details of the text has frequently been the focus of investigations by historians of science. Already Needham started to compare the machines depicted and described in the *Qiqi tushuo* with machines depicted in the European literature. In particular, he tried to determine which of these machines were new to the Chinese.⁴

Such identifications of direct connections of details in the *Qiqi tushuo* with specific contents in European books covers, however, only one aspect of the transfer of knowledge from Europe to China. This is indicated already by the names of Western authors explicitly mentioned in the text. The text frequently refers to Archimedes. In the preface, not only are the names of Agricola and Ramelli mentioned as sources for illustrations of machines, but also Vitruvius and Stevin are cited as persons who possessed the background knowledge needed to understand the functioning of machines. We know furthermore that Schreck himself had such background knowledge. He studied in Padua where he also got to know Galileo. Later his scientific scholarship was so highly esteemed that he became a member of the Accademia dei Lincei shortly after Galileo.

Considering this broader background knowledge of Western technology, we are concerned with the question of what role it played in the transmission of knowledge to China. Was the successful transmission of specific technical knowledge to China dependent on a good fit of such background knowledge between Europe and China? Or more generally: How can knowledge of this kind be transmitted? Do social and cultural conditions determine such backgrounds and make the transmission of knowledge possible or impossible?

Let us go a little more into details concerning these questions before we return to the role of the *Qiqi tushuo* in the specific transmission process with which we are dealing here.

Any transmission of knowledge is based on direct or indirect communication between individuals. This is true for the historical transmission between generations within a culture as well as for the exchange between cultures. As a result of such communication processes between individuals the basic form of knowledge transmission is a kind of slow diffusion process. This process spreads knowledge, once it is acquired, continuously over time and space.

But this basic transmission process is complemented by discontinuities resulting in a variety of knowledge communities and knowledge traditions. To give an example, interruptions of knowledge traditions may result from the rise and decline of cultures such as the rise and decline of the ancient Babylonian or Greek cultures. Discontinuities in knowledge traditions may also result from the confrontation of traditions of different cultures such as the confrontation of the Chinese culture with early modern Western science and technology.

What kind of knowledge could be transmitted and how it has been transformed in the process of transmission was widely determined by systemic conditions in Europe and China. Three sets of such conditions can easily be identified:

1. the compatibilities and incompatibilities of knowledge systems in Europe with those in China,
2. the important role of the knowledge of practitioners for the rapid development of European science and technology in the Renaissance and in the Early Modern era,
3. the internal incoherence of the body of Western scientific knowledge in the Early Modern era, which resulted from difficulties in integrating the various developments of technical and scientific knowledge into one coherent system.

Let us briefly review how these three conditions influenced the transfer of knowledge from the West to China as it is represented by the *Qiqi tushuo*.

As far as the compatibility between systems of knowledge in Europe and China is concerned, it has to be noticed that between any cultures there exist cross-cultural similarities in cognitive constructions

4 Needham, J. *Science and Civilisation in China*, Vol. 4.2, Cambridge 1965 (Cambridge University Press), pp. 215-218.

which are acquired in the process of growing up and in the socialization of a child into the adult community. Similarities and parallelisms concerning, for instance, notions such as space, time, body, motion, force etc., form a solid base for the transmission of knowledge. Certain correspondences between concepts in Western sources and representations of transmitted knowledge such as the *Qiqi tushuo* are thus not the result of the transmission process but rather its precondition. The investigation of the transmission process of Western mechanical knowledge to China thus necessarily includes the identification of such common notions and the determination of how they are embedded in the different conceptual systems of both cultures and how they are modified by different connotations within these systems.

Another source on which the transmission of knowledge is based is the shared intuitive knowledge of practitioners built up in the process of professionalization. This type of shared knowledge results from the use of tools and devices in handiwork and craftsmanship. Insofar as similar devices of this kind, for instance, balances or pulleys, exist in both Europe and China, their professional use provides similar experiences which may give rise to the construction of mental models based on similar notions and expressed by similar terminologies.

These commonalities and discrepancies of European and Chinese conceptualizations of professional mechanical experiences set conditions for the transmission of knowledge from Europe to China. It is thus another requirement an investigation of this transmission process has to meet to identify common mechanical technologies and to investigate the conceptualization of practical experiences emerging in the context of their use in both cultures. Since basic mechanical technologies, in particular the use of so-called simple machines such as the lever and the pulley, were the same in both cultures already before the Jesuits started their activities, the Jesuits could count on a common understanding of mechanical phenomena which had only to be activated by an adequate translation of networks of mechanical terms.

This explains one of the most striking differences between the *Qiqi tushuo* and its Western sources: European scholars of the Renaissance and the Early Modern era show a strong tendency to justify scientific knowledge by derivations from general principles. In contrast, the *Qiqi tushuo* uses techniques of classification to link bits of knowledge to general principles. Since in both cases the generalizations result from reflections on mechanical technologies, both representation techniques could serve the same goal, that is, to encode the mechanical knowledge to be transmitted.

Let us have a closer look at the knowledge that the Jesuits tried to transmit. It is well known that at the beginning of the seventeenth century the development of scientific knowledge was widely influenced by the conflicts between the rapidly developing early modern science and the church. As a consequence, the knowledge represented by western science and technology was a patchwork of partly controversial theories, often affiliated with certain groups of the conflicting political parties, rather than in coherent theoretical systems.

Let us look in some detail at the sources that represent this body of knowledge.

The first source to be mentioned is the *scholastic Aristotelianism* which developed after Saint Thomas Aquinas introduced the ancient Aristotelian philosophy into the dogmas of the Roman Catholic Church. According to Aristotle three types of motion have to be distinguished: natural motion, violent motion, and the circular motion of heavenly bodies. Based on such notions Aristotle developed his cosmology encompassing the idea of the Earth resting at the center of the cosmos surrounded by the spheres of celestial bodies. This world view was popularized by Sacrobosco's *Treatise on the Sphere*. Scholastic activities brought the Aristotelian cosmology in accordance with the biblical account of the Genesis.

Another ancient tradition, the tradition of *theoretical mechanics*, developed independently of Aristotelianism although it also had its roots in Aristotle's work, as represented by the *Problemata Mechanica*. Archimedes contributed to this tradition deductive theories such as his theory of equilibrium or his theory of floating bodies. Heron decomposed mechanical devices into elements such as lever, pulley, wedge and screw and showed how to derive theoretical consequences concerning the functions of these devices from the theoretical interpretation of these elements. He thus bridged the gap between theoretical and practical mechanics. Vitruvius in his ten books on architecture showed how

such machines had to be applied in construction work. A specific outcome of this tradition is the medieval science of weight, represented for instance by the work of Jordanus Nemorarius. It is characteristic of this tradition that, following Archimedes, mechanical knowledge was predominantly represented in deductive form.

A third ancient tradition in the body of technological knowledge relevant at the time of the Jesuit mission in China is the mathematics tradition. This tradition comprised in particular the tradition of Euclidean geometry and the theory of proportions, which up to the time of Galileo was the main theoretical tool for representing functions. An additional, more powerful tool for representing functions was only provided by the geometrical interpretation of the Aristotelian doctrine of the alteration of qualities in his work *On Generation and Corruption*.

These widely independent ancient traditions were complemented in the Renaissance and in the Early Modern era in two ways. On the one hand, numerous well-illustrated books on technology were written which tried to represent the professional knowledge of practitioners, printed publications such as those of Agricola, Besson, Ramelli, Taccola, or Zonca, to give some examples. On the other hand, scientists with a critical attitude towards scholastic Aristotelianism such as Galileo, Kepler, Descartes, and Stevin⁵ tried to create “New Sciences” with conceptual foundations deviating from the concepts of Aristotle.

Jesuit scholars were well acquainted with all these traditions. In spite of the major conflict between scientists and the Roman Catholic Church about Copernicanism, Jesuits contributed actively with editions, commentaries, and their own treatises to the development of the new sciences, without touching upon this conflict.

Let us finally return to the question posed at the beginning, the question of what are the sources of the *Qiqi Tushuo* with respect to the broader background knowledge of the Western culture as we briefly outlined it here. A first glance on the structure of the *Qiqi tushuo* shows already that all the traditions we mentioned are substantially represented in the *Qiqi tushuo*.

Following a substantial preface of the Chinese coauthor Wang Zheng, the treatise begins with lists of technical terms used in the book, comprising

1. scientific disciplines,
2. relevant Chinese literature,
3. terms for the construction of machines,
4. Latin characters,
5. terms for the elements of machines,
6. terms for the use of prime movers, mechanical devices, and complex machinery,
7. terms for capabilities of mechanical devices,
8. terms for the advantages of mechanical devices, and
9. terms which classify representations of machines by illustrations and descriptions.

After the prefaces and term lists follow two very abstract chapters about techniques to use forces as they are represented by what is called the science of weights. These chapters are arranged in opposition to each other.

The *first* deals more or less with the internal conceptual structure of the science of weights and its methodological consequences.

The *second* deals with the external use and application of the science of weights.

The style of these chapters and also the terminology show an influence of scholastic Aristotelianism. They contain references to Archimedes, but also to the Bible. Categories of quantity, measure and weight are said to be universally established by the Creator; and Adam together with his wife Eve are

⁵ In particular Simon Stevin’s work *The Elements of the Art of Weighing* (in Dijksterhuis (ed.) *The Principal Works of Simon Stevin*, Vol. 1, pp 35-285) was a source Schreck draw on when he was compiling the *Qiqi tushuo*.

the first to construct mechanical devices and machines after they were expelled from paradise. After these general sections follows the main content of the book, arranged in three chapters.

The *first* chapter contains a detailed elaboration of everything known about weight at that time. It starts with Aristotelian cosmology and the inherent concept of gravity. These cosmological considerations are then related to Archimedes' theory of the center of gravity and to his theory of floating bodies and of specific weight as it is expounded in contemporary Western treatises.

The *second* chapter contains the theory of mechanical devices, that is, mechanics in the narrow sense of the Western mechanical tradition (see figure 3). It first elaborates general characteristics of machines. This is followed by sections on simple machines corresponding to Heron's classification as it was revived in the sixteenth century by Guidobaldo del Monte, sections concerning the balance, the steelyard, the lever, the pulley, the wheel, and the screw.

Finally the third chapter follows the Western Renaissance and early modern tradition of illustrated books on machines and mechanical devices representing the professional knowledge of practitioners. Each machine is represented by a full page illustration (see figure 4) complemented by an extensive description (see figure 5).



Figure 3
Theory of simple machines

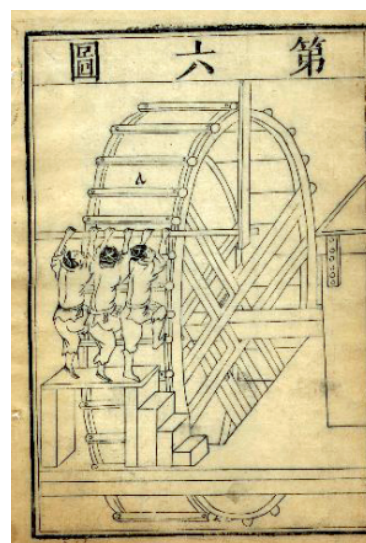


Figure 4
Illustration of a compound machine

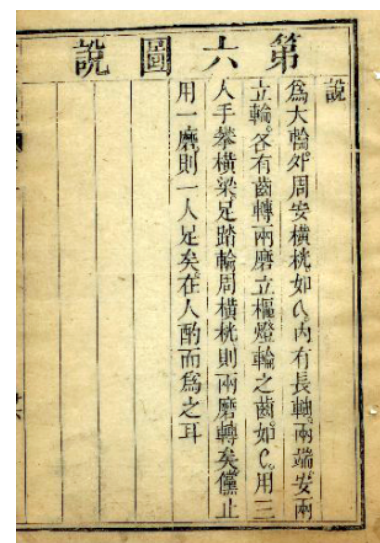


Figure 5
Description of a compound machine

This brief overview of the structure and content of the *Qiqi tushuo* shows not only its close relation to Western sources, but also a remarkable difference. We do not know of any contemporary Western treatise which outlines essential contents of all the Western traditions enumerated above. Given the combination of abstract theory and practitioners' knowledge as it is displayed in the *Qiqi tushuo* this book turns out to represent a process of knowledge integration that is unparalleled in the Western sources.

This observation immediately raises a further question: Is this integration process based on theoretical means of the Western tradition, or is it based on the translation of Western concepts into a Chinese theoretical framework, or is it based on merging Western and Chinese sources?

Unfortunately, this question turns out to be difficult to answer and we are unable to give a satisfying answer here. Chinese terms used for the theoretical integration cannot be translated one by one into corresponding Latin terms of the Western traditions. The same is possibly true for Chinese traditions. The theoretical integration has to be studied as an approach in its own right. It is a major challenge for our ongoing research to reconstruct then the indirect references to Chinese and European background knowledge in order to get a deeper understanding of the knowledge transmission process between Europe and China represented by the *Qiqi tushuo*.

The Context of Jesuit Mechanics

Rivka FELDHAY

As is well known, the *Collected diagrams and explanations of wonderful machines from the far west* (*Yuanxi Qiqi Tushuo Luzui*), published in 1627, was the fruit of a fortunate cooperation between the Jesuit missionary, physician, naturalist and mathematician Johann Terrenz Schreck (1576-1630) and the Chinese scholar Wang Zheng. My aim, in this short essay, is to situate Schreck in his historical context as a Jesuit scientist in order to portray, in very general terms, the background knowledge and cultural attitudes that Schreck was very likely to bring into this cooperation.

Johann Schreck, born in Constance, was the son of a lawyer who taught jurisprudence in Nuremberg. Versed in many languages both antique and modern, among them Latin, Greek and Chaldean as well as French, English and Portuguese – he studied medicine and traveled widely in Europe until he enrolled at the University of Padua in 1603, where he became an acquaintance of Galileo Galilei. It was probably in 1603 that he adopted a Latin name – Terentius. In 1611 Schreck was nominated member of the Academia dei Lincei of Prince Federico Cesi in Rome, for whom he prepared a Latin edition of the Natural History of Mexico. The year 1611, however, became crucial for the subsequent biography of Schreck because in that same year he decided to join the Jesuits and dedicate himself to the religious life. Thus, he left the Academy and began studying theology. Schreck remained in Rome until 1618, though most of the last two years (since 1616) he traveled all over Europe as the adviser to Nikolaus Trigault. Together they were preparing the mission to China, collecting funds, equipment, books and scientific instruments, among them a telescope donated by Cardinal Frederico Borromeo of Milan which was the first telescope to reach China.

What was the position of Jesuit mathematicians in the field of mechanics during the second decade of the 17th century, and particularly during the years 1611-1618 spent by Terrenz Schreck in Rome? Who were the prominent speakers among them, and what were the problems at the center of their attention?

Traditional histories of mechanics have usually described the field, at the beginning of the seventeenth century, as divided between two schools.¹ On the one hand, historians used to refer to the School of Urbino, initiated in the work of Commandino² which relied on and perfected the 15th century purification of the Archimedean tradition from the remnants of scholastics, including non-mathematical components and purely “physical” concepts. As against this purified reading of mechanics as a mathematical science originating in Archimedes, an alternative tradition stemming from the pseudo Aristotelian *Mechanical Questions*³ was posited, especially by those who chose to follow in the footsteps of Pierre Duhem’s monumental work on Leonardo Da Vinci and the origins of statics⁴. In this historiographical tradition, Galileo’s early work on mechanics⁵ was considered as a point of conversion of the two traditions. The story of the two traditions has already been severely criticized.⁶ A look at the environment of the Collegio Romano in the second decade of the 17th century indicates that a third center of writing, studying and debating mechanical problems was crystallizing as a space of interesting encounters between the two traditions, opening new possibilities for the development of a new science of mechanics but also creating new constraints.

During the second half of the sixteenth century Balthazar Torres and Christopher Clavius lay the foundations - both intellectual and institutional – for an encounter between the Archimedean and

1 See S. Drake & I. E. Drabkin, *Mechanics in Sixteenth-Century Italy*, Madison 1969, pp. 13ff; O. L. Rose, *The Italian Renaissance of Mathematics*, Genève 1976.

2 F. Commandino, *Archimedis opera non nulla a Federico Commandino Urbinate nuper in latinum conversa, et comentariis illustrata*, Venetiis 1558.

3 Aristotle, *Mechanica*, trans. E. s. Foster in Works of Aristotle (W. D. Ross, ed.), Vol. 6, Oxford 1913.

4 P. Duhem, *Les origines de la statique*, 2 vol. Paris, 1905-6; idem, *Études sur Leonard de Vinci*, 3 vol., Paris 1906.

5 G. Galilei, *Le Meccaniche* (ca. 1600) Translated with Introduction and Notes by S. Drake, Madison 1960.

6 J. Renn (ed.), *Galileo in Context*, Cambridge 2001, see especially: L. Renn & P. Damerow, & S. Rieger, & Camerota, M., “Hunting the White Elephant: When and How did Galileo Discover the Law of Fall?”

Aristotelian traditions. Clavius maintained an elaborate correspondence with Commandino (that is lost) and was in close contact with Maurolico, thus encompassing Archimedean contents (rectifications, quadratures and cubatures etc) and methods (exhaustion and centers of gravity) in his own work,⁷ and introducing it to his environment. His correspondence testifies to early quadratures by his students already in the 16th century (for example that of the Jesuit Bernardino Salino). Luca Valerio, a Jesuit between 1570-1580, got his education at the Collegio Romano and followed Maurolico in using Archimedean heuristics – i.e. methods of hanging ideal geometrical figures and indivisibles.⁸ Around 1585 Galileo sent his early work on centers of gravity [*Theoremata circa centrum gravitates solidorum*] to Clavius who sent back his remarks and criticisms. By the end of the sixteenth century Clavius and his follower Grienberger already trained young mathematicians in their Mathematical Academy – an informal seminar for the most talented Jesuits interested in the mathematical sciences that served as a meeting place for non Jesuit mathematicians as well.⁹ Thus Paulus Guldin who studied theology around 1609 and stayed in Rome until 1618 was member of the academy and already deeply involved in his work on centers of gravity to be published later in his *Centrobaryca*.¹⁰ By 1603-4 some of the most important works in the field of Archimedean mathematics were published in Rome by people who were all closely connected to the Jesuits: Marino Ghetaldi, who was not Jesuit but nevertheless frequented the Academy published his *Promotus Archimedes* in 1603. Luca Valerio and Villalpando published their own books: *De centro gravitatis* and *Tomi III apparatus urbis ac temple Hierosolymitani* respectively in 1604.

However, the encounter between the Archimedean and Aristotelian traditions mainly took place within the framework of discussion of the more “physical” works of Archimedes, namely those concerned with weights in equilibrium [*On the Equilibrium of Planes*] and bodies in fluids [*On Floating Bodies*]. The main difference between a pure Archimedean approach and a pseudo-Aristotelian approach stemmed from the different terms used in the definition of equilibrium, the first focusing on the inverse proportion between the mathematical quantities of weight and distance from fulcrum, the second on the difference in the velocities of two equal weights positioned in unequal distances from fulcrum. This difference was at the heart of the image of the Archimedean and pseudo-Aristotelian approaches constructed by historians who tended to picture the Archimedean approach as limited to a static, geometric description and the pseudo-Aristotelian approach as dynamic. Most scholars today, however, reject this schematic historical representation, being aware of the dynamical elements that infiltrated even the work of the most orthodox early modern Archimedean, probably via the medieval tradition of mechanical discussion that already mixed Archimedean and Aristotelian concepts and methods, and through the interest of Renaissance engineers and practical mathematicians in machines. Even Commandino used the term “momentum” in his definition of equilibrium, thus testifying to a dynamical understanding of weight, and the same is true of Guidobaldo del Monte who was concerned to emphasize the historical continuity between the older pseudo-Aristotelian tradition and the Archimedean tradition.¹¹ This does not mean, however, that sixteenth century writers on mechanical were completely unaware of the tension that existed between the two traditions. The first among them was probably Maurolico¹² who, in his *Problemata Mechanica* from 1613 already pointed out the tension between the pseudo Aristotelian *Mechanical Questions* and Archimedes’ *On the Equilibrium of Planes*. Still, in that work he tried to incorporate the *Questions* within an Archimedean context by defining “momentum” as the product of weight and distance from fulcrum – a quantity that Archimedes did not explicitly define – and consequently declared that equilibrium means equality of momenti. In addition

7 M. Scaduto, “Il matematico Francesco Maurolico e I gesuiti”, in *Archivium historicum Societatis Iesu*, XVIII, 1949, pp. 126-141; P. D. Napolitani, “Maurolico e Commandino”, in P. Nastasi (a cura di), *Atti del convegno Il Meridione e le scienze (secoli XVI-XIX)*, Palermo 1988, pp. 2281-316; C. Clavius, *Corrispondenza*, U. Baldini and P. D. Napolitani (eds.), Pisa 1992.

8 P. D. Napolitani, “Metodo e statica in Valerio con edizione di due opere giovanili”, *Bolletino di storia delle scienze matematiche*, II, 1, 1982, pp. 3-173.

9 U. Baldini, “L’accademia di matematica del collegio romano (1553-1612)”, in *Saggi sulla cultura della Compagnia di Gesu* (secoli XVI-XVIII), (Padova 2000), pp. 49-98.

10 P. Guldin, *Centrobaryca*, Vienna 1635-1641.

11 G. del Monte, *Mechanicorum liber*, Pesaro 1577.

12 See U. Baldini, “Archimede nel seicento italiano”, in C. Dollo (ed.), *Archimede: Mito Tradizione Scienza*, Firenze 1992, pp. 237-289.

to the Archimedean quantities of body, weight and momentum, Maurolico also posited a fourth non-Archimedean quantity of “vis” or “impetus” in his work on mechanics.

In the second decade of the 17th century overlapping with Terrenz Schreck presence in Rome, many Jesuit mathematicians became aware of the tension between the Archimedean and the pseudo-Aristotelian approaches to mechanics, and the errors sometimes brought about by the use of the Aristotelian terminology. A good example can be found in Giuseppe Biancani’s chapter dedicated to the *Questions* in his *Aristotelis loca mathematica* from 1615. Nevertheless, the official policy of the Society of Jesus as stated in its *Ratio studiorum* from 1599 demanded commitment to an Aristotelian cosmological and metaphysical framework of thought. That was probably the basis for accepting the Galileo’s early project stated in his *Mechanics* that resonated in later Jesuit syntheses of mechanics, such as P. Casati’s *Mechanicorum libri*.

To conclude: Jesuit texts on mechanics exhibit an interesting intersection between two different aspects essential for understanding the development of mechanics in the 17th century. First, the mathematical treatment of weights in equilibrium, associated with weights in motion through the concepts of “momentum”, “virtus”, “potentia” touches upon the most fundamental issue of the connection between statics and dynamics within an Aristotelian-Archimedean framework of thought. Second, Jesuit texts involve intense use of such concepts as “impetus” and “velocitas” – traditionally associated with the science of motion in natural philosophy, much more than with a conversation on mechanics. Such use testifies to the appropriation of growing parts of traditional Aristotelian physics into a new framework by a new type of Jesuit “scientists”, namely Jesuit physico-mathematicians. The study of mechanics by Jesuits reveals that the science of mechanics was not the exclusive domain of the “moderns”.

Mechanical Knowledge in the Context of Pre-Modern Chinese Salt Industry

Hans Ulrich VOGEL

When dealing with mechanical practice and knowledge in the pre-modern Chinese production sector, we have to be aware that there existed quite a huge gap between practice and theory. This simply means that mechanical devices have been used intuitively in practice and have been described or depicted in books or on illustrations, but that almost no attempt has been made to provide some kind of theoretical explanation. Only in rare instances can we find statements that may be qualified as expressions of mechanical knowledge. Both these phenomena, the sheer existence of mechanical devices in practical use and their description in books or illustrations without their being accompanied by theoretical explanations on the one hand and – more rarely – the situation that they were provided with some kind of theorizing on the other have to be taken into account, but at the same time have to be given justice in their own way.

1. Primary Sources and Secondary Literature

The Chinese primary sources which are at our disposal for tracing mechanical practice and knowledge in the traditional Chinese salt industry are first of all a number of works that deal specifically with the salt production sector:

1. *Aopo tu* 熬波圖 (Illustrated Boiling of Sea Water), by Chen Chun 陳椿, 1334. Ed. *Siku quanshu zhenben* (Taipei: Taiwan shangwu yinshuguan, 1981). For a translation and a study of this work, see Yoshida Tora, *Salt Production Techniques in Ancient China: The Aobo Tu*. Translated and revised by Hans Ulrich Vogel. Leiden: Brill, 1993.
2. *Yanjing tushuo* 鹽井圖說 (Illustrated Explanations of the Salt Wells [of Shehong 射洪 in Northern Sichuan]), by Ma Chi 馬季 or Ma Ming-Heng 馬明衡 (responsible for the text) and Yue Yufang 岳諭方 (illustrator), ca. 1587. The illustrations are lost. The text is contained in *Shuzhong guangji* 蜀中廣記 (Extensive Records from Shu [Sichuan]), by Cao Xuequan 曹學佺 (1574-1647). Late Ming Period. Ed. *Siku quanshu zhenben* (Taipei: Taiwan shangwu yinshuguan, 1981). For an investigation and translation of this text, see Hans Ulrich Vogel, „Ma Jis ‘Yanjing tushuo’ (Illustrierte Abhandlung über die Salzbrunnen [Sichuans]) aus der späten Ming-Zeit und seine technikgeschichtliche Bedeutung“, *Zeitschrift der Deutschen Morgenländischen Gesellschaft*, 155.1:253-294 (2005).
3. [*Qingdai jingyan huajuan*] [清代井鹽畫卷] [Scroll on Well Salt (Production in Sichuan) during the Qing Period], ca. 1740. For an investigation of this scroll and a translation of its inscriptions, see Hans Ulrich Vogel, „Die Darstellung der Salzproduktion in Sichuan – Eine chinesische Bildrolle aus der Mitte des 18. Jahrhunderts“, *Thesis: Wissenschaftliche Zeitschrift der Bauhaus-Universität Weimar*, 4/5, Heft 2002, S. 328-345 (Internationale Tagung am Lehrstuhl für Bauaufnahme und Baudenkmalpflege: Investitionen im Salinenwesen und Salzbergbau: Globale Rahmenbedingungen, regionale Auswirkungen, verbliebene Monumente – Gewidmet Rudolf Palme (1942-2002).
4. *Diannan yanfa tu* 滇南鹽法圖 (Illustrations of the Salt [Production] Methods of Yunnan), by Li Bi 李苾, 1707. This scroll is stored in the China National Museum. For a recent study of its content see the article of Zhu Xia 朱霞, „Cong ‘Diannan yanfa tu’ kan gudai Yunnan shaoshu minzu de jingyan shengchan” 從滇南鹽法圖看古代雲南少數民族的井鹽生產 (Production of Well Salt in the Ancient Period by the Minorities of Yunnan as seen from the “Illustrations of the Salt [Production] Methods of Yunnan), *Ziran kexueshi yanjiu* 自然科學史研究 (Studies in the History of Natural Sciences), 23.2: 132-147 (2004).
5. *Ziliujing fengwu mingshi shuo* 自流井風物名實說 (Explanations of Designations and Objects of Customs and Notable Things [of the Salt Works] of Ziliujing), by Wu Dingli 吳鼎立, 1871. In

Luo Tingquan 羅廷權, Lü Shangzhen 呂上珍 et al., *Fushun xian zhi 撫順縣志* (Gazetteer of Fushun District [in Sichuan]), 1872, chap. 30, pp. 12b-29a. The text is also reprinted *Jingyanshi tongxun 井鹽史通訊* (Well Salt History Newsletter), 1984, no. 1 (12), pp. 71-76.

6. *Ziliujing ji 自流井記* (A Record [of the Salt Industry] of Ziliujing [in Sichuan]), by Li Rong 李榕, 1876. The text is in *Shisanfeng shuwu quanji 十三峰書屋全記* (Complete Collection of the Thirteen-Peaks-Studio), in *Li Shenfu xiansheng quanji 李申夫先生全集* (The Complete Works of Mr. Li Shenfu [Li Rong]), comp. by Jiang Dejun 蔣德均. Ed. Taipei: Wenhai chubanshe. The *Ziliujing ji* was translated by Lien-Che Tu fang, “An Account of the Salt Industry at Tzu-liu-ching [Szechuan], *Tzu-liu-ching chi*, by Li Jung [1890], (with introductory note)”, *ISIS*, 39:228-234 (1948).
7. *Sichuan yanfa zhi 四川鹽法志* (The Salt Laws Gazetteer of Sichuan), compiled officially at the request of Ding Baozhen 丁寶楨, the Governor-General of the Province, by Luo Wenbin 羅文彬 et al., 1882.
8. *Sichuan Nanlang yanwu tushuo 四川南閬鹽物圖說* (Illustrated Explanations of the Salt Affairs of the Nanlang [Saline] in Sichuan), 1910.

A more general primary source of information is Song Yingxing's *Tiangong kaiwu* (Heaven's Crafts in the Instigation of Things) of 1637. An English translation is available from Sun E-Tu Zen and Sun Shiou-Chuan, *T'ien-Kung K'ai-wu: Chinese Technology in the Seventeenth Century*, by Sung Ying-Hsing, University Park, Penn., and London 1966. Recently, this book was also translated into German by Konrad Herrmann, *Erschließung der himmlischen Schätze, von Song Yingxing*, Bremerhaven: Wirtschaftsverlag Nw, 2004. For a critical edition of the Chinese text, see Pan Jixing, *Tiangong kaiwu jiaozhu ji yanjiu* (An Investigation and Annotated Edition of the *Tiangong kaiwu*), Chengdu: Ba Shu shushe, 1989.

When we come to the secondary sources we should mention the interesting and instrumental study written by Zhang Xuejun 張學君 on “Achievements in Physics in the Production of Well Salt in Ancient Sichuan” (Gudai Sichuan jingyan shengchan zhong de wulixue chengjiu 古代四川井鹽生產中的物理學成就), published in 1986 in *Yanyeshi yanjiu 鹽業史研究* (Salt Industry History Research), 1986, no. 1, pp. 108-116. An important work providing a convenient introduction into the history of mechanics and mechanical thought in China is Dai Nianzu 戴念祖 and Lao Liang's 老亮 *Zhongguo wulixueshi daxi: Lixueshi 中國物理學史大系：力學史* (Series of Books on the History of Physics in China: A History of Mechanics), Changsha: Hunan jiaoyu chubanshe, 2001. By taking this book as a first basis, we can immediately see that a number of items exist which readily refer to mechanical devices and, possibly, mechanical thinking in the areas of salt production. These are especially those chapters Dai and Lao's book that deal with levers, pulleys, winches, gears and cog-wheels, the concept of work and energy, the use of the specific gravity of liquids and methods for its measurements, as well as the notion of crystals.

2. Selected Modern Western Definitions of Mechanics

In order to distinguish between Western traditions of mechanics on the one hand and Chinese conceptions that may prove suitable for comparisons on the other, it may be helpful to introduce some Western definitions of mechanics. Thus, the electronic version of the *Encyclopaedia Britannica*, 1994-2001, gives the following definition:

Mechanics:

science concerned with the motion of bodies under the action of forces, including the special case in which a body remains at rest. Of first concern in the problem of motion are the forces that bodies exert on one another. This leads to the study of such topics as gravitation, electricity, and magnetism, according to the nature of the forces involved. Given the forces, one can seek the manner in which bodies move under the action of forces; this is the subject matter of mechanics proper.

Historically, mechanics was among the first of the exact sciences to be developed. Its internal beauty as a mathematical discipline and its early remarkable success in accounting in quantitative detail for

the motions of the Moon, the Earth, and other planetary bodies had enormous influence on philosophical thought and provided impetus for the systematic development of science into the 20th century.

Mechanics may be divided into three branches: statics, which deals with forces acting on and in a body at rest; kinematics, which describes the possible motions of a body or system of bodies; and kinetics, which attempts to explain or predict the motion that will occur in a given situation. Alternatively, mechanics may be divided according to the kind of system studied. The simplest mechanical system is the particle, defined as a body so small that its shape and internal structure are of no consequence in the given problem. More complicated is the motion of a system of two or more particles that exert forces on one another and possibly undergo forces exerted by bodies outside of the system. ...

The *Brockhaus Naturwissenschaften und Technik*, Wiesbaden: Brockhaus, 1983, vol. 3, p. 236 (transl. by HUV) writes as follows:

... History. In classical antiquity and the middle ages mechanics was the 'art' to move bodies contrary to their natural behaviour through human interventions in the natural processes and to construct and build the devices necessary for this. In the fourth century B.C. the first moves toward a theoretical understanding of mechanics were made by Archytas of Tarent. The contrast between forced and natural motions was also retained during the middle ages (Jordanus Nemorarius). It was only by Galileo Galilei towards the end of the sixteenth century that this concept was overcome. ...

For avoiding the complexities of the respective cultural and scientific traditions and environments we may agree to use a simple workable definition of mechanics as the art to move bodies or the motion of bodies under the action of forces as well as the construction of devices necessary for this and, as a further step, a theoretical understanding of mechanics. Consequently, the use of the specific gravity of liquids and methods for its measurements as well as the notion of crystals which are also discussed in Dai Nianzu and Lao Liang's book belong rather to the field of "physics" and will therefore not be treated here. To the art of moving bodies we, however, can certainly count the adoption of such devices as pulleys, wheels, winches, capstans, and levers.

3. The Salt Industry

In traditional China, four major kinds of salt were produced: sea salt, salt from salt lakes, well salt, and earth salt. While in medicine and pharmacy dozens of other salts were used, it were these four kinds of salts which covered the salt demand for ordinary human consumption. Most important was sea salt which was harvested all along the Chinese coast and which constituted about 80 percent of the empire-wide output, followed by salt produced at the salt lake of Xiezhou with about 10 percent and the salt boiled at the salt wells in Sichuan and Yunnan with about 6 percent. Salt produced by leaching earth rich in salt particles and by boiling down the resulting brine was of rather minor importance.

4. The Well Salt Industry

It was especially in the well salt industry of Sichuan where salt producers made use of numerous mechanical devices. In Sichuan, from about the third century B.C. at the latest, salt producers started to exploit subterranean brine with the help of shaft wells. Around the middle of the eleventh century A.D. deep-drilling was invented in this province which resulted in a process of a gradual replacement of the exploitation of natural salt springs and shaft wells by deep-drilling for brine. The technical devices used in drilling, hoisting and brine transportation show that at least an intuitive and empirical grasp of physical and mechanical principles was at work. We can readily see this on a picture from the *Sichuan yanfa shi* (Ill. 1) which shows the pushing down of wooden pipes into the well with the help of a scissor-shaped tool.

5. Pulleys

Four brick relieves which survived in tombs of the Eastern Han period show how salt production was carried out in ancient Sichuan.¹ On these tomb brick relieves the brine hoisting device occupies quite a prominent place. It consists of a structure with four legs and two storeys. At the top of the roofed hoisting tower a pulley is fixed which guides the rope. On Type A brick the pulley is mounted on a kind of triangular frame, while on Type B brick a long axle is shown. Each end of the rope is equipped with a bucket or, more probable, a bag made of leather. Two pairs of men standing on two different levels of the high roofed structure apparently carry out very swift and co-ordinated hoisting operations. In this way they could hoist at a much higher speed and efficiency than when lifting the loads with ordinary winches or pulleys as they were used in mining or at water wells. A co-ordinated and rhythmic hoisting action is visible on both types of tomb bricks: On Type A brick co-ordination takes place crosswise (Ill. 2), on Type B brick sidewise (Ill. 3). Pulleys with ropes and two buckets fixed to the end of the ropes were also used at the White Hare Well (Baitujing 白兔井) and other wells of Yun'an zhen 雲安鎮, Yun'yang 雲陽縣 District of Sichuan province, well into the nineteenth and twentieth centuries, though each pulley was only manned with one worker, instead of two pairs. Moreover, the pulley used there was drum-shaped, and not a pulley with a slender concave waist (Ill. 4 to 6).

While the high hoisting structure and the input of manpower clearly distinguish brine shaft wells from ordinary water wells, these two categories of wells showed a number of similarities. For instance, we know from early depictions or pottery models of water wells that pulleys were widely used for hoisting water. In modern terminology, two basic kinds of pulleys can be discerned, namely the ordinary wheel-shaped pulley (Ill. 7) and the rather large pulley with a slender concave waist² (Ill. 8). It is rather the latter type of pulley which can be seen on the tomb brick relieves (Ill. 9) showing Sichuan salt production. It is particularly this type of pulley that works very well in cases where wells are deep and long ropes have to be used which tend to jump off the ordinary small pulley when handled with much vigor and swing. Slender-waist pulleys were, however, not only used at salt wells, but also at water wells. A clear example of a slender-waist pulley of a water well can be seen on a tomb brick relief of Jiexiang 架祥 in Shandong from the Eastern Han period. Contrary to the illustration of the Sichuan salt works, hoisting on the Shandong picture was carried out with only one container, an earthenware vessel bound to one end of the rope, while the other end was drawn by one man (Ill. 8). An early pottery model of a water well shows that the lifting of water with two containers was also not uncommon (Ill. 10). Thus, when one container was lifted, the other was sent down. This not only meant economy in the input of force, but also resulted in continuous and swift hoisting. In physical terms, this can be expressed as follows (Ill. 11): The gravitational force of the full brine bag or bucket is m_1 , the force of the laborers lifting the full bag or bucket F_1 . The gravitational force of the empty bag or bucket is m_2 , the force of the workers sending down the empty bag or bucket F_2 . Assuming that friction is zero (which is, of course, not the case), the pulling force is: $F = F_1 + F_2 + m_2$.³ In comparison with a situation where one worker would simply draw up one bag or bucket with the help of a rope or bamboo pole (as this was done in late imperial Yunnan, Ill. 12), the arrangement as it is shown on the tomb relief bricks was certainly very conducive in raising the productivity of brine hoisting operations, at least in those cases where a well produced a continuously high quantity of brine.

1 Based on indications in the secondary literature, it seems that tomb brick relieves showing salt production has been discovered at least at four places, namely 1956 in a Han tomb in the Yangzishan 羊子山 northern suburb of Chengdu, the Ximenwai 西門外 western suburb of Chengdu, in Huapaifang 花牌坊 of Qionglai 邛崃縣 District, and in Pi 郫縣 District. While we ignore the tomb brick relief of Pi District, it is quite clear that so far we can distinguish two different types of tomb brick relieves, namely Type A which is represented by the Qionglai brick and Type B to which the bricks from Yangzishan and Ximenwai of Chengdu belong.

2 There is still much confusion and ambiguity with regard to the terms „pulley“ and „winch“ or „windlass“ in Chinese dictionaries and research articles. While Laufer (1962) distinguishes between *lulu* 轆轤, i.e. „winch“ or „windlass“, and *huache* 滑車, i.e. „pulley“, this is not systematically done in Chinese dictionaries and by Chinese authors. Thus, both Li Chenyou (1984) and Liu Shizhong (1991) speak of *lulu* 轆轤 in the sense of pulley and of *xiyao lulu* 細腰轆轤, or *shuyao lulu* 束腰, in the sense of the large pulley with its slanting concave middle section. For the “winch with the crank handle” they also use the term *lulu*, namely *qubing lulu* 曲柄轆轤. Xie Zhongliang (1985), pp. 63-64, says that the [Type B] relief shows a “fixed pulley” (*ding hualun* 定滑輪), contrasting it with a “movable pulley” (*huo hualun* 活滑輪). Moreover, when referring to the slanting concave middle section he calls the pulley *shencao hualun* 深槽滑輪 or “pulley with a deep groove”.

3 Zhang Xuejun (1986), p. 108.

With the input of animal power for hoisting brine from deep-drilled wells in the late imperial period, different kinds of pulleys were applied. These devices were particularly important also because wells became deeper, bamboo hoisting tubes longer and heavier, and hoisting rigs higher. The earliest illustration of hoisting carried out with the help of buffaloes comes from Song Yingxing's *Tiangong kaiwu* of 1637. The description of the hoisting equipment given in the text is given below. It is somewhat strange that Song Yingxing speaks of a [well] swape in the context of hoisting brine from deep-drilled wells. This certainly is indicative of the lack of a standardized terminology of hoisting devices during this period:

Above the well a [well] sweep [structure is established] and pulleys are hung up. A capstan is made to which a buffalo is harnessed. When the buffalo drags, the capstan turns, the rope is wound up by the pulleys, and the brine is hoisted.⁴

井上懸桔槔轆轤諸具，制盤駕牛。牛曳盤轉，轆轤絞繩，汲水而上。

Illustration 13 from the *Tiangong kaiwu* shows a one-leg hoisting rig with two pulleys fixed to it by a rope. Behind the hoisting rig a capstan can be seen. The huge axle of the capstan drum is held in position in some way by a structure consisting of two large, forked pillars and a long cross-beam. A buffalo is harnessed to the capstan and driven by a man holding a whip in his hand. The rope is wound around the capstan, passes over the buffalo and the driver to the lower pulley, and from there upwards to the pulley at the top of the hoisting rig. From there it goes down into the well, that is, to the bamboo hoisting tube to which it is fixed. The lines drawn in the picture itself are, however, somewhat confusing and do not make this very clear. It looks like if one rope was fixed to the top of the hoisting tube, while another rope seems to disappear in the well (?). In the picture the bamboo tube filled with brine is just coming out of the well and is grasped by a worker standing near the aperture of the well.

It is open to question how realistic this picture is. We do not know whether Song Yingxing had ever seen a deep-drilled well himself. Apart from inaccuracies in the depiction of fine details, like in the case of the junction of the capstan axle with the cross-beam, the pulleys look strange, when compared with pictures we have from later periods. They look like pulleys that were used on ships. Less doubtful is the structure of the one-leg rig. Historical documents of the early twentieth century as well as evidence from field research carried out in Northern Sichuan in the 1980s have shown that such structures did indeed exist. However, such one-leg rigs did not serve primarily the hoisting operation in its narrow sense, but were used for fixing and securing the tube after it had emerged from the well orifice. In this fixed position the leather valve at the tube's bottom could be opened and the brine emptied into a tub or reservoir.

The text to the right of the capstan drum in illustration 13 reads as follows:⁵

The rope is wound up and [the brine] hoisted from the well. When the [empty] tube is lowered into the well, the buffalo turns around [in] left [direction].

轉繩汲井。放筒下井，牛則左旋。

The bamboo tube is one zhang long.

竹筒長丈。

It is not very probable that the buffalo was also used for sending down the tube into the well. We know from later periods that the tube descended the well by its own weight and at quite high speed. No buffaloes were needed for that operation.

A much better illustration of the hoisting equipment at deep-drilled brine wells is provided by a scroll from about 1740. In the second main scene with the title "Illustration of the wells' inexhaustible [brine] supply" the hoisting of brine with the help of a buffalo-driven capstan is shown. In comparison to the

4 Pan Jixing (1989), p. 274. For a Chinese translation, see Zhong Guangyan (1988), p. 157. The English translation by Sun & Sun (1966), p. 116, is somewhat too straightforward: "After it [i.e. the hoisting tube; annotation by HUV] fills with brine, it is raised to the surface by means of a pulley fixed over the well and [a windlass; emendation by Sun & Sun] turned by an ox-powered wheel."

5 Pan Jixing (1989), p. 274. The legends in the illustrations were neither translated by Sun and Sun (1966) nor by Zhong Guangyan (1988).

woodblock print of the *Tiangong kaiwu*, this painting shows much more details and may help us to complement information that is missing in Song Yingxing's work. Some of the inscriptions on the scroll inform us about the function of the devices. From right to left, they read as follows (Ill. 14):

Capstan (*panche*).

盤車。

Commentary by HUV: As shown by a search in both the text and the commentaries of the digital version of the *Siku quanshu* (henceforth: dSKQS), the expression *panche* for the „capstan“ applied at the salt wells of Sichuan is apparently for the first time used in the scroll. In the dSKQS *panche* is often used in the sense of “wheeled cart”, for instance in paragraphs, chapters or manuals dealing with painting, not only as an object of “ruled-line” (*jiehua* 界劃) painting (see *Shanhu wang* 珊瑚網, chap. 48, ed. dSKQS), but also as a modular element in landscape painting (see, e.g., *Huashi huiyao* 畫史會要, chap. 5, ed. dSKQS) or as a principal motif in other types of paintings expressing the state of mind (see, e.g., *Zhuzhuang shihua* 竹莊詩話, chap. 9, ed. dSKQS). The *Yanjing tushuo* (chap. 66, pp. 24a-b) and the *Tiangong kaiwu* (Pan Jixing (1989), p. 274-275; Sun and Sun (1966), p. 116) are not very explicit when talking about the capstan. They both use the character *pan* 盤 in the general sense of a “[winding] wheel”. In the case of the buffalo-driven device the *Yanjing tushuo* (chap. 66, p. 24a), however, speaks of a „buffalo wheel“ (*niuche* 牛車). *Panche* as expression for the capstan is also used in *Pengxixian zhi* 蓬溪縣志 (Local gazetteer of Pengxi district), 1786 ed., quoted by Wu Tianying (1991), p. 559.

It is interesting to note that the capstan depicted in the scroll shows no brake which in later periods was used for stopping the capstan when the empty hoisting tube was sent down into the well again. Moreover, the man standing behind the capstan is holding a whip in one hand, while with the other hand is grasping the rope. He is perhaps lifting the rope whenever the buffalo is passing by. The axle of the capstan seems to have been equipped with iron shoes at both ends. It turns in a kind of stone hollow at the bottom and is held in position by a ring at the top. Like in the illustration of the *Tiangong kaiwu* the capstan is powered by one buffalo only. In the highly developed salines of Ziliujing 自流井 and Gongjing 貢井 of the late nineteenth century hoisting will be carried out with harnessing four buffaloes to the capstan.

Capstan house (*chefang*).

車房。

Commentary: The dSKQS has no entry which would point to this specific meaning.

„Earth roller“ (*digun*): It serves for guiding and controlling the rope so that it is kept even.

地滾。用以管定繩索使平。

Commentary: It is in the scroll that for the first time the term *digun* is mentioned. The rope coming forth from below of the “earth roller” passes over a fork. Both the cylindrical roller and the fork serve the stabilization of the rope and for guiding it upwards to the “heaven's roller”.

Rope (*shengsuo*).

繩索。

Tower frame (*loujia*).

樓架。

Commentary: The scroll shows a hoisting structure with three legs, which from the point of view of a developmental history occupies a middle position between the two-legs and four-legs hoisting rigs. The rather unspecific term *loujia* is already mentioned in the *Yanjing tushuo* (chap. 66, p. 24a).

Heaven's roller (*tiangun*): It enables the rope to move without obstruction.

天滾：使繩索動不滯。

Commentary: *Tiangun* is again an expression which is already used in the *Yanjing tushuo* (chap. 66, p. 24a). According to my search in the dSKQS, it has been there for the first time that this expression bears this specific meaning. The “heaven's roller” is nothing else than a pulley of the slender-waist type, as it was already applied for hoisting brine from shaft wells during the Eastern Han period at the latest. The slender waist serves the stabilization of the rope. This type of pulley differs from the top wheel as it was used in late nineteenth-century Ziliujing and Gongjing.

The bamboo tube for brine hoisting consists of two bamboo [tubes] joined together and has a length of two *zhang*.

汲水筒以兩竹接連，長二丈。

Commentary: In the *Tiangong kaiwu* (Pan Jixing (1989), p. 274; Sun and Sun (1966), p. 116) a length of „one *zhang*“ is mentioned for the hoisting tube, while the text of the scroll speaks of a length of two *zhang* which was attained by joining two bamboo tubes. This suggests a rise in the hoisting capacity around the middle of the eighteenth century. See Wu Tianying (1991), p. 562. Wu Tianying errs, however, when he writes that the *Yanjing tushuo* makes indications about the length of the hoisting tubes. The *Yanjing tushuo* (chap. 66, p. 23a) has the phrase „something more than one *zhang*“, but this refers to a bamboo tube (*tongzhu* 筒竹, literally „tube bamboo“) used for the hoisting of drilling-slurry, and not to the brine hoisting tube.

At wells with much brine 70 to 80 tubes can be hoisted in one day. At wells with less brine it are 20 to 30 tubes or only a few tubes per day.

水多之井一日可汲六七十筒。水少之井一日或汲二三十筒或數筒不等。

Commentary: According to Wu Tianying (1991), p. 562, this passage is evidence of an increase in productivity of the salt wells in Northern Sichuan in the 1840s. However, one should not forget that the productivity of individual wells could differ dramatically. For the differences in productivity of salt wells in Sichuan, see Vogel (1990), pp. 313-314.

Well eye (*jingyan*).

井眼。

Collection tub (*jianpen*).

簡盆。

Commentary: This expression is not found in the dSKQS. Wu Tianying (1991), p. 562, thinks that instead of the character *jian* 簡 the characters *jian* 筧 or *jian* 覘 should be used. The last mentioned two characters usually designated bamboo pipelines for transporting brine. However, these two last-mentioned characters are nowhere used in the scroll, even not in the case where bamboo pipelines are shown and named. They also do not show up in combination with *pen* 盆 or tub when searching the dSKQS.

The illustration and inscriptions make it clear that the “earth roller” as well as the fork served for controlling and guiding the rope, while the “heaven's roller” cared for it that the rope ran and passed without obstruction. One important function, however, is not explicitly mentioned, namely, the change of direction effected by the pulleys. During hoisting, the “heaven's roller” changed the direction of the up-coming rope from vertical to diagonal, while the “earth roller” from diagonal to more or less horizontal (and in both cases vice versa when sending the tube down into the well). No saving of effort or force, however, resulted from this arrangement.

The illustration on the scroll may help us to understand in some way a passage in the *Yanjing tushuo* of about 1587:

After the [drilling] work has been finished, the tower frame (*loujia*)⁶ is established, which may reach the height of a watchtower. Above one makes the heaven's roller (*tiangun*),⁷ which produces a sound like a winch (*lulu*). A bamboo tube (*tong*) and a rope (*suo*) are manufactured [which are used] for sucking up the [salt] water. This is the same method as it is used for the afore [mentioned] sucking up of muddy water, with the difference that hub and axle (*shuzhou*) are controlled by a wheel frame (*cheshuang*).⁸ The wooden cross-pieces of the wheel frame form a drum (*pan*), which has two ears and an inward-bent (*quchi*) form.⁹ [This device] unfolds to the left or right and upwards or downwards in opposite directions. If the [rope] bends to the earth on the left, it is stretched on the right; if [it] bends to the earth on the right, it is stretched on the left. [The wheel-drum] is turned with effort until the rope comes to an end and the tube emerges from the well. ...

The turning of the winches (*lulu*) is done by three men. Where great strength is needed, a buffalo capstan (*niuche*) of large dimension is built. Thus, strength is reduced and results doubled (*li yi er gong bei ye*).

厥工既就，始樹樓架，高可似敵樓。上為天滾，有轆轤聲。制筒索吸水，如前吸泥水法，而輪軸則管於車床也。床橫木為槃，槃有兩耳，作曲池狀。左右低昂逆施，左揖地右伸，右揖地左伸。循環用力，索盡筒出。... 轉轆轤者，蓋三人為之。力厚者，則制牛車，車狀大。力逸而功倍也。

Some passages in this text are indeed difficult to translate. Chinese historians of technology usually see in the first part the first description not only of the explicitly mentioned „heaven's roller“, but also of the „earth roller“. They think that such expressions like “left”, “right”, „bending“ and „stretching“ are referring to the functions of guiding, controlling and directing the rope.¹⁰ If this interpretation is correct, it is worthwhile to be noted that also in the case of lifting brine with human power a “tower frame” was used. We know from many cases of small-scale deep-drilled salt wells in Northern Sichuan during the twentieth century that when hoisting was carried out by human power the hoisting equipment consisted of a hoisting wheel with a horizontal axle. Moreover, a pole was set up near the well which, however, only served for fixing and securing the bamboo tube when it came out of the well mouth; it was not involved in the hoisting operation itself (Ill. 15a and 15b). Because the *Yanjing tushuo*, however, speaks of winches that are powered by three men, we may have to do it in that case not with a hoisting wheel with a horizontal axle, but perhaps with one with a vertical axle, turned by three workers running in a circle. Another possibility is that for human-powered wheels with horizontal axles sometimes also a “heaven's roller” fixed to the top of the tower frame was used. Such an arrangement is described in the local gazetteer of Nanbu District (*Nanbuxian zhi* 南部縣志).¹¹

A final point to be mentioned is that a different hoisting arrangement emerged in Ziliujing and Gongjing during the late nineteenth century where between the “earth roller” and the “heaven's wheel” an “earth wheel” was added. It was for this reason that in such an arrangement the rope did not pass below the earth roller, but above it, before it reached the capstan. Apparently no slender-waist pulleys were used anymore (Ill. 16). Moreover, in comparison to the depiction in the *Tiangong kaiwu* and the scroll of the mid-eighteenth century, a capstan drum was powered by up to four buffaloes, instead of one only.

6. Hoisting Wheels and Capstans

From the Song period at the latest, special hoisting wheels were used at the brine shaft wells of Sichuan. Shen Gua 沈括 (1031-1095) in his *Mengxi bitan* 夢溪筆談, in the chapter called “wisdom in emergencies” (*quanzhi* 權智), mentions that a large hoisting wheel (*dache* 大車) was used at the

6 This is a relatively unspecific term.

7 In the dSKQS this term is used only in the *Yanjing tushuo* in this meaning.

8 *Cheshuang* is an expression which usually is synonymous with „cart“ (*yu* 輿). See, for instance, the *Liji shuzhu* 禮記述註 (Reviews on, and commentaries to, the *Book of Rites*), by Li Guangpo 李光坡, 1708, ed. dSKQS, chap. 14.

9 Apart from the meaning of „bent pond“ the expression *quchi* designates a acu-moxa point inside of one's elbow. See Southern Materials Center (1985), p. 267. Hence, “bent inside”, “bent” or, perhaps, “concave” may be reasonable translations of this term.

10 See, e.g., Zhang Xuejun (1983), p. 193; Lin Yuanxiong et al. (1987), p. 302.

11 See Hu Daojing (1987), p. 140.

famous Ling Well (Lingjing 陵井) of Lingzhou 陵州 prefecture in Sichuan.¹² When deep-drilling was invented around the middle of the eleventh century, Su Dongpo 蘇東坡 (1037-1101) wrote in his famous note about the Sichuan salt industry that “all the pipe wells use machinery [hoists] (*jixie* 機械).”¹³ In the beginning, the hoisting wheels were probably quite small, with a diameter of perhaps a few feet only. These were wheels with a horizontal axle and powered by one or two men, just in the way as we can see them on the illustrations of the Nanlang 南閬 saline, Northern Sichuan, of the early twentieth century (Ill. 15b). Customarily, these hoists were called “dogs-climbing-up wheels” (*goupa che* 狗爬車). While no force was saved by this device, it had the advantage that the weight of the body of the laborers helped in raising the heavy brine tube.

It is interesting to note that in the *Yanjing tushuo* of the late sixteenth century the impression is given that in the case of man-powered hoisting operations a rig with a “heaven’s roller” was used, that is, that the hoisting rope passed first over the “heaven’s roller” and perhaps also below an “earth roller” before it reached the hoisting wheel. This would have been totally different to the situation as we know it to have existed at many small deep-drilled salt wells in Northern Sichuan where obviously the rope or cable passed directly from the top of the hoisting wheel down to the well (Ill. 15a and 15b).

The first report that buffaloes were used for hoisting refers to the middle of the thirteenth century when at the famous Yujing 育井 brine spring of Changning 昌寧縣 district human hoisting labor was replaced by buffalo[-driven] equipment (*niuju* 牛具). A stele was set up there in honor of the official who had instituted this change.¹⁴ The use of buffaloes for hoisting appears to have become more widespread during the late Ming period as both the *Yanjing tuoshuo* and the *Tiangong kaiwu* mention it. The picture in the *Tiangong kaiwu* (Ill. 13) as well as the illustration from the mid-eighteenth century scroll (Ill. 14) show a rather large capstan drum powered by one buffalo. Moreover, in the *Yanjing tushuo* some sort of proto-mechanical thinking is reflected. It says that there where great strength is needed, a buffalo capstan (*niuche* 牛車) of large dimension is built and that, thus, strength can be reduced and results doubled (*li yi er gong bei ye* 力逸而功倍也). Compared with the hoists manpower the capstans driven by buffaloes were not only huge, but differed also in the arrangement of the axle and thus of the whole hoisting wheel. Manpowered were mostly equipped with a horizontal axle, capstans with a vertical one.

In the highly productive saline of Ziliujing and Gongjing during the late nineteenth and early twentieth centuries, capstans were powered with two to six buffaloes, depending on the depth and productivity of the well. After three hoistings, the buffaloes were exhausted and had to be replaced by a new team. In illustration 16 taken from the *Sichuan yanfa zhi* a capstan driven probably by four buffaloes is shown. Zhang Xuejun mentions that because the buffaloes were harnessed to the capstan in such a way that their running course described a wider circle around the axis of the capstan than the circle of the rope wound around it, this arrangement resulted in economizing force, but not work.¹⁵

7. Lever

For the invention of deep-drilling, the lever was of tremendous importance. The difference between early, or more pristine, arrangements of drilling and such of later stages can be gleaned from illustrations. The first picture shows the situation at the small-scale wells in Northern Sichuan, with one man jumping on and off the lever thus raising and releasing the drill bit down in the borehole (Ill. 17). The second picture represents the situation in the highly developed salt production center of Ziliujing and Gongjing. We can see two teams of two men each jumping on and off the lever in rhythmic turns (Ill. 18). In both cases, one worker is supervising the drilling cable. His main function is to turn the cable regularly in order to keep the borehole strictly vertical and round.

It is not hard to figure out that tilt-hammers used for husking grain served as a model for this type of percussion drilling. This is confirmed by Chinese sources, like the *Tiangong kaiwu*, which explicitly

12 See Hu Daojing (1987), p. 140.

13 See *Su Shi wenji* 蘇軾文集 (Collected Writings of Su Shi [Su Dongpo]), late 11th century, punctuated and collated by Kong Fanli 孔凡禮, ed. Beijing: Zhonghua shuju, Beijing, 1986, vol. 6, p. 2367.

14 *Yuan yitong zhi* 元一統志 (Comprehensive Geography of the Yuan [Dynasty]), chap. 5, quoted by Lin Yuanxiong et al. (1987), p. 287.

15 Zhang Xuejun (1986), p. 111.

says that this type of operation resembles the pounding of grain. Although the relevant passage in the *Tiangong kaiwu* describing the drilling operation is not very clear, the parallelity with grain-pounding operations is made clear:

Down to one *zhang* [ca. 3 m] or so [the drilling equipment] can be operated by stepping on it with one's feet, like [the motion of] pounding rice in a mortar [emphasis by HUV]. When [the well] becomes too deep, one holds [the drilling device] by one's hands for stamping down.¹⁶

初入丈許，或以腳踏碓梢，如舂米形。太深則用手捧持頓下。

One obvious difference between deep-drilling and pounding of grain was the respective lengths of the lever arms. In the case of the tilt-hammers for husking grain, the hammer arm was longer than the arm on which the worker stepped. In the case of deep-drilling the arm to which the bamboo-cable and thus the drill bit was fixed was much shorter than the arm the workers jumped on and off.

No mention is made of the “lever” or the lever principle in accounts of deep-drilling, neither in the *Tiangong kaiwu* nor any later source. Nonetheless, for drilling it was important to find out, at least empirically, the right proportion between the part to which the drill was attached and the stepping board. On the one hand, force, and thus strength, could be saved with a long stepping board. On the other hand, the lift of the shorter end had to be high enough so that the drill-bit was dropped from a proper height to crush the rock.

In Northern Sichuan where wells were rather shallow and the rock easy to break, the drill-bit arm was rather short and one or two men sufficed for the stepping board. In Ziliujing and Gongjing with their deep and rather large wells the drill-bit arm was in most cases longer than in Northern Sichuan. In the case of “new wells” (*xinjing* 新井), that is, wells where drilling operations had started only recently, two men were enough for the stepping-board. In case of “deep wells” (*shenjing* 深井), however, three to four workers were employed for this job.

One figure that is given as a sort of standard length of the drill-bit arm is 1.5 *chi* (ca. 45-50 cm). There is the story that the well entrepreneur or – as he is called – capitalist Li Yongzhi 李永之 had increased this length to 1.8 or even 2.0 *chi* in order to raise the height of the drop of the drill, to speed up the drilling operation, and thus to increase his chances for deriving more profit.¹⁷

8. Conclusions

The messages I want to bring home are not very spectacular because they are well known to historians of technology. A first point to be mentioned is that more often than not a substantial gap existed between theory and practice in the production sector. Chinese well drillers nonetheless could get along quite well by empirical means in their search and exploitation of underground brine resources as long as they were able to cover the limited demands of a traditional economy, as this was the case for the inelastic salt market in late imperial China. Whenever there was demand for more salt or salt of better quality, no breakthrough in science was required under these conditions, but rather the improvement of basic inventions and innovations that had been already made quite some time ago. All this could even be better shown for the relationship between salt boiling and chemistry. Even in Europe where in the seventeenth, eighteenth and nineteenth centuries great progress has been made in the chemical analysis of mineral waters and salt chemistry, salt boiling continued to be carried out in basically empirical ways. This was not always an expression of conservatism on the part of the salt producers, but also has to do with the fact that, for instance, some of the phenomena encountered during salt boiling could not be explained scientifically until very recently.¹⁸ Another characteristic of the Chinese situation and that of many other stages in the history of various civilizations is that although mechanical principles, like that of the lever, were neither formulated in mathematical and physical terms nor explained in scientific ways, there can be no doubt that this principles were adopted empirically in many areas or production. Hence there is no difficulty in showing the wide-spread use of mechanics in an “intuitive” way and as

16 See Pan Jixing (1989), p. 274. See also the somewhat different translation in Sun and Sun (1966), p. 116.

17 Lin Yuanxiong et al. (1987), pp. 155-157.

18 For an invaluable insight into the problem of the relationship between theory and practice of salt boiling see Walter (1985), pp. 121-129. For the development of the salt chemistry see Multhauf (1978), pp. 125-143.

one of the currents of practice and thought on which a more scientific account of mechanics could build.

9. References to Secondary Literature

Bai Guangmei 白廣美 (1988). “Chuangdong, bei jingyan kaocha baogao” 川東、北井鹽考察報告 (Report on the Well Salt Investigation in Eastern and Northern Sichuan), *Ziran kexueshi yanjiu* 自然科學史研究 (Studies in the History of Natural Science), 7.3: 263-272 (1988).

Chen Wenhua 陳文華 (1991). *Zhongguo gudai nongye kejishi tupu* 中國古代農業科技史圖譜 (A Collection of Illustrations on Agricultural Science and Technology in Ancient China). Beijing: Nongye chubanshe, 1991.

Dai Nianzu 戴念祖 and Lao Liang's 老亮 (2001). *Zhongguo wulixueshi daxi: Lixueshi* 中國物理學史大係：力學史 (Series of Books on the History of Physics in China: A History of Mechanics). Changsha: Hunan jiaoyu chubanshe, 2001.

Falkenhausen, Lothar von (1999). “Excerpts from Diary, February 21 – April 2, 1999 (Research Trip to Sichuan and the Three Gorges).” MS 1999.

Herrmann, Konrad (2004). *Erschließung der himmlischen Schätze, von Song Yingxing*. Bremerhaven: Wirtschaftsverlag Nw, 2004.

Hu Daojing 胡道靜 (1987). *Xin jiaozheng Mengxi bitan* 新校正夢溪筆談 (New Corrected Edition of the *Dream Pool Essays*). Beijing: Zhonghua shuju, 1958. Reprinted by Hongkong: Zhonghua shuju Xianggang fenju, 1987.

Laufer, Berthold (1909). *Chinese Pottery of the Han Dynasty*. Leiden: E. J. Brill, 1909.

Li Chenyou 李趁友 (1984). “Handai de lulu jiqi fazhan” 漢代的輻輳及其發展 (Pulleys during the Han Period and their Development), *Nongye kaogu* 農業考古 (Agricultural Archaeology), 1: 93-95 (1984).

Lin Yuanxiong 林元雄, Song Liangxi 宋良曦, et al. (1987). *Zhongguo jingyan kejishi* 中國井鹽科技史 (The History of Science and Technology of Chinese Well Salt). Chengdu: Sichuan kexue jishu chubanshe, 1987.

Liu Shizhong 劉詩中 (1991). “Zhongguo gudai shuijing xingzhi chutan” 中國古代水井形制初探 (A First Investigation into the Forms and Construction Methods of Water Wells in Ancient China), *Nongye kaogu* 農業考古 (Agricultural Archaeology), 3: 212-220, 226 (1991).

Lü Changsheng 呂長生 (1983). “Qingdai Yunnan jingyan shengchan de lishi huajuan – ‘Diannan yanfa tu’ 清代雲南井鹽生產的歷史畫卷—《滇南鹽法圖》 (A Historical Scroll on the Production of Well Salt in Yunnan during the Qing Period: “Illustrations on the Salt [Production] Methods of Yunnan”), *Jingyanshi tongxun* (Well Salt History Newsletter), 2 (14): 19-21 (1985).

Lü Lin 呂林 (1988). *Sichuan Handai huaxiang yishu xuan* 四川漢代畫像藝術選 (Selections from the Stone Relief Art of the Han Period in Sichuan). Chengdu: Sichuan meishu chubanshe, 1988.

Multhauf, Robert P. (1978). *Neptune's Gift: A History of Common Salt*. Baltimore and London, Johns Hopkins University Press, 1978.

Pan Jixing 潘吉星 (1989). *Tiangong kaiwu jiaozhu ji yanjiu* 《天工開物》校注及研究 (An Investigation and Annotated Edition of the *Tiangong kaiwu*). Chengdu: Ba Shu shushe, 1989.

Rudolph, R. C., and Wen Yu (1953). *Han Tomb Art of West China; a Collection of First and Second Century Reliefs*. Berkeley and Los Angeles: University of California Press, 1957.

Southern Materials Center, Inc. (ed.). *Dictionary of Traditional Chinese Medicine*. Taipei: Southern Materials Center, 1985.

Sun E-Tu Zen and Sun Shiou-Chuan (1966). *T'ien-Kung K'ai-wu: Chinese Technology in the Seventeenth Century*, by Sung Ying-Hsing, University Park, Penn., and London: Pennsylvania State University Press, 1966.

Vogel, Hans Ulrich. (1990). *Untersuchungen über die Salzgeschichte von Sichuan (311 v. Chr. - 1911): Strukturen des Monopols und der Produktion*. Stuttgart: Franz Steiner Verlag (Münchener Ostasiatische Studien, Bd. 51.), 1990.

Walter, Hans-Henning. “Zur Entwicklung der Siedesalzgewinnung in Deutschland von 1500 bis 1900 unter besonderer Berücksichtigung chemisch-technologischer Probleme“, dissertation thesis, Bergakademie Freiberg, Freiberg, 1985.

Wu Tianying 吳天穎 (1991). “Liangzhong Qingdai jingyan tuji shuping” 兩種清代井鹽圖籍述評 (Comments on Two Illustrated Works on Well Salt of the Qing Period), in Peng Zeyi 彭澤益 and Wang Renyuan 王仁遠 (eds.) (1991), *Zhongguo yanyeshi guoji xueshu taolunhui lunwenji* 中國鹽業史國際學術討論會論文集 (Collected Articles of the International Symposium on the History of the Chinese Salt Industry). Chengdu: Sichuan renmin chubanshe, 1991, pp. 557—570.

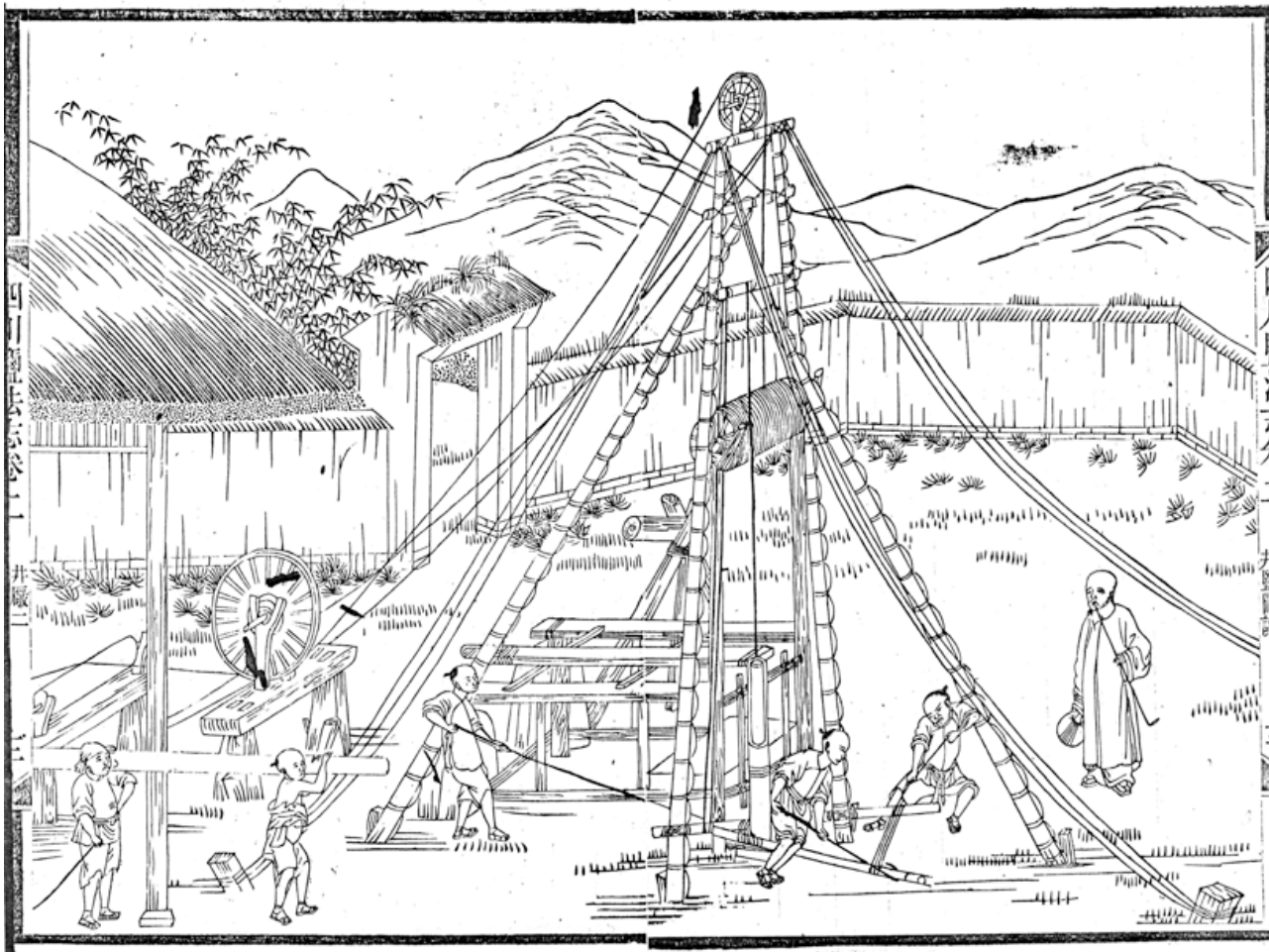
Xie Zhongliang 謝忠樑 (1985), “Handai Sichuan jingyan shengchan laodong huaxiangzhuang xintan – jian tan gudai Sichuan jingyanye de yixie wenti” 漢代四川井鹽生產勞動畫像磚新探—兼談古代四川井鹽業的一些問題 (New Research on the Sichuan Han Tomb Brick Reliefs Depicting Well Salt Production Work – with an Additional Discussion of Some Problems Relating to the Ancient Well Salt Industry of Sichuan), in: Peng Jiusong 彭久松 and Chen Ran 陳然 (eds.), *Sichuan jingyanshi luncong* 四川井鹽史論叢 (A Collection of Articles on the History of Sichuan Well Salt). Chengdu: Sichuansheng shehui kexueyuan chubanshe, pp. 62-73.

Zhang Xuejun 張學君(1983). *Gudai Zhongguo de jingyan shengchan gongyi* 古代中國的井鹽生產工藝 (The Art of Well Salt Production in Ancient China). MS Zigong 1983.

Zhang Xuejun 張學君 (1986). “Gudai Sichuan jingyan shengchan zhong de wulixue chengjiu” 古代四川井鹽生產中的物理學成就 (Achievements in Physics in the Production of Well Salt in Ancient Sichuan), *Yanyeshi yanjiu* 鹽業史研究 (Salt Industry History Research), 1: 108-116 (1986).

Zhong Guangyan 鐘廣言 (1988). “*Tiangong kaiwu*” 《天工開物》 (The *Tiangong kaiwu* [of Song Yingxing 宋應星, with commentaries and a translation by Zhong Guangyan]). 2nd ed. Hongkong: Zhonghua shuju, 1988.

10. Illustrations



Ill. 1: Inserting pipes in a deep-drilled well in Ziliujing 自流井 and Gongjing 貢井 (Furong 富榮 saline), Sichuan, late nineteenth century

Source: *Sichuan yanfa zhi*, chap. 2, pp. 11b-12a.

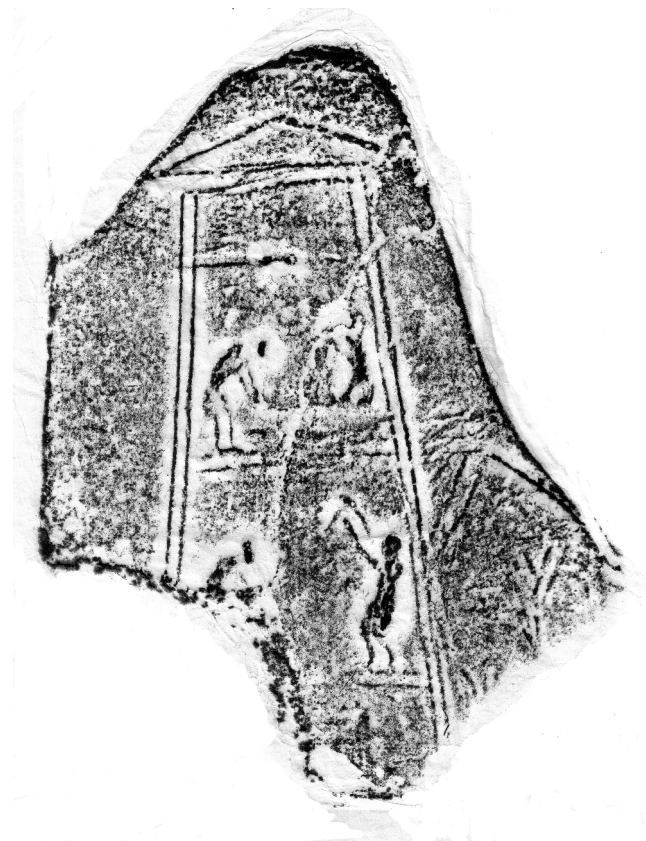
Note: The wooden pipes are inserted into the deep-drilled well with the help of a giant pair of pincers. The pipes are made of wooden logs sawn lengthwise into halves which are then hollowed out. Thereafter the halves are assembled again, bound together, wrapped with hemp and plastered with a mixture of tung oil and ashes to make them water-tight. Also the joints between the single pipes are treated in a similar way so that intrusion of sweet water into the well is prevented.



Ill. 2: Rubbing of Type A tomb relief brick showing salt production in Sichuan during the Eastern Han period

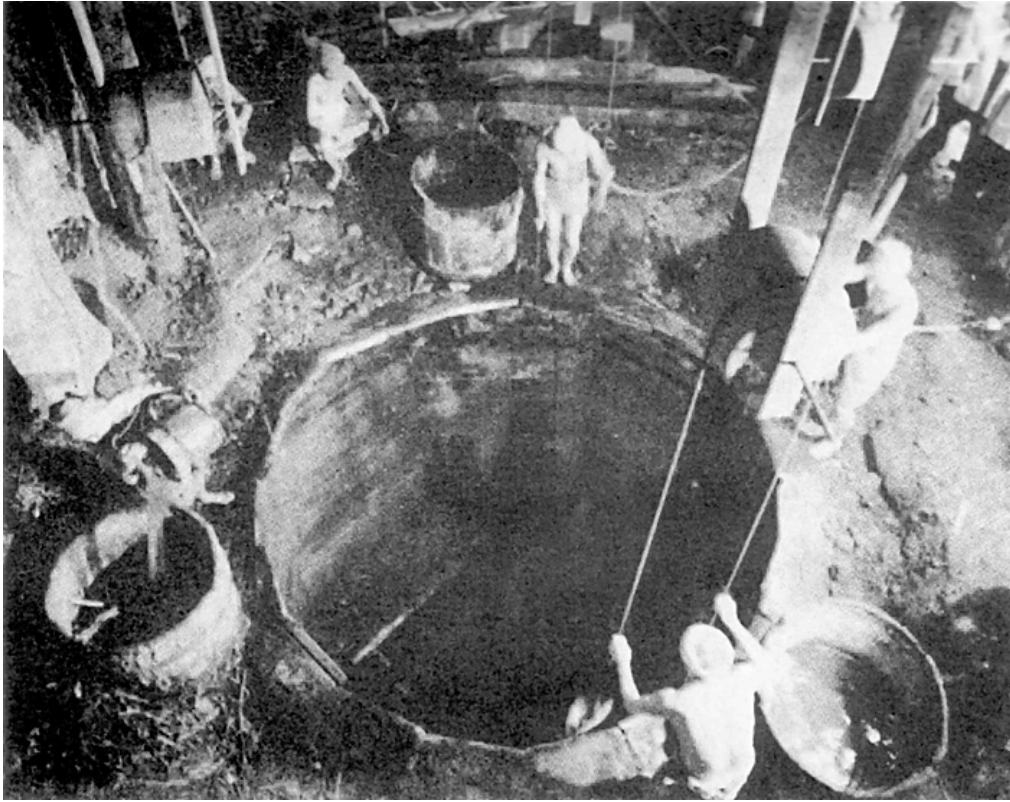
Source: Rudolph and Wen (1953), pl. 91.

Note: This tomb brick comes from Qionglai 邛崃 in Sichuan. The well shown on this rubbing is clearly a shaft well, which shows a well defined square-shaped foundation, perhaps made of brick or masonry work. The hoisting structure consists of four legs and two storeys. Each storey is manned with two labourers carrying out the hoisting operation. At the top of the hoisting tower a pulley for guiding the rope is placed. Each end of the rope is equipped with a container, so that when one container is lowered, the other is raised. Judged from the proportions of the hoisting arrangements, the length of the rope and the size of the workers, we may estimate that the diameter of the well was about 1 m and its depth about 8 to 10 m. To the right of the uppermost storey of the hoisting structure, a tank is set up for receiving the brine. It seems that a pipeline, probably made of bamboo, is winding down to the salt boiling hearth, though the line may also represent a path. In Type B, however, the pipeline is clearly visible. Both the hearth and the hoisting tower are protected by a roof. At least two people are working at the hearth which slants upwards. One person seems to oversee the work, while the other cares for the fire. A part of the relief showing the hearth is destroyed, but two round pans can still be seen. On the upper right-hand quarter of the picture, hunting scenes are depicted. Trees are growing on the mountains which cover the whole background.



Ill. 3: Detail of a rubbing of Type B tomb relief brick showing salt production in Sichuan during the Eastern Han period.

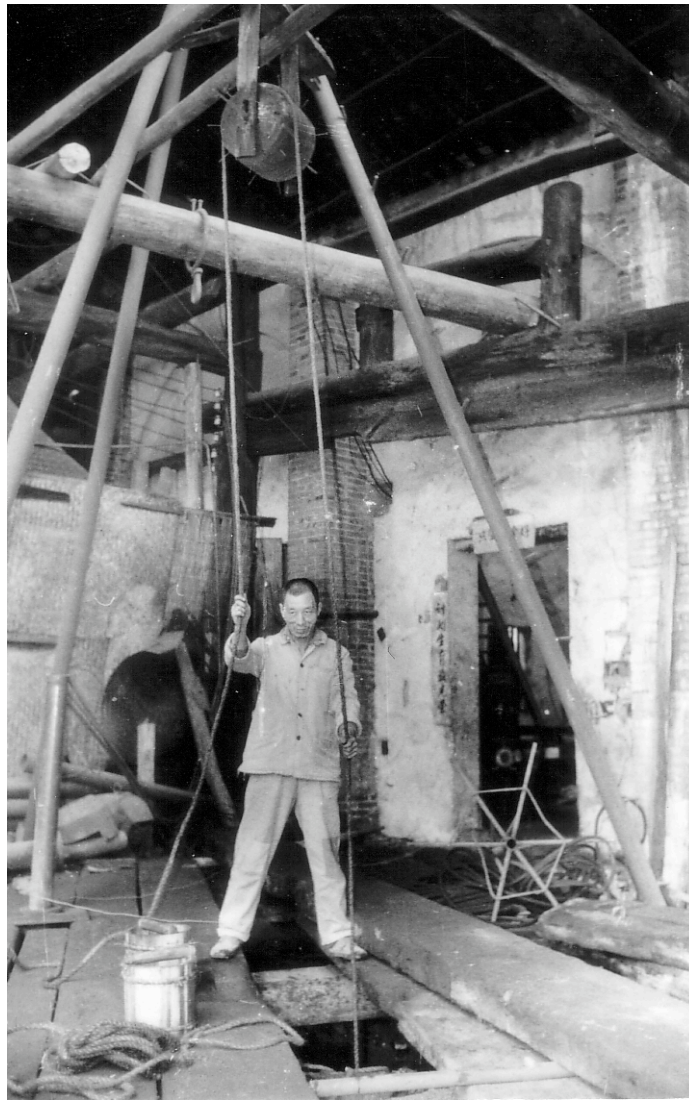
Note: This is a detail from a tomb brick relief found in Ximenwai 西門外 western suburb of Chengdu. It shows the hoisting tower operated by four men. In contrast to the brick relief of Type A where coordinated hoisting action between the four workers takes place crosswise, in this type of relief, the workers coordinate their movements sidewise. The pulley appears to be rather long and very probably belonged to the type with slender waist.



Ill. 4: Historical reconstruction of brine hoisting at or near the White Hare Well (Baitujing 白兔井) in Yun'an zhen 雲安鎮, Yunyang District 雲陽縣 in Sichuan province

Source: Lin Yuanxiong et al. (1987), plate 10.

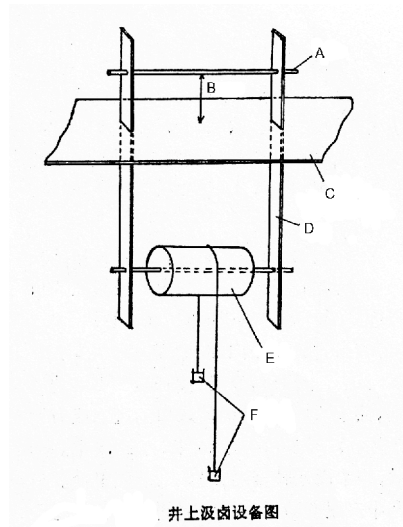
Note: One can clearly distinguish the large cylindrical pulleys as well as the large wooden tubs into which the brine is emptied. From there, brine is conveyed to the boiling houses with the help of bamboo pipelines. Several workers hoist brine from the well at the same time. In former times, the White Hare Well is said to have had twenty hoisting frames (*jiā*) which were operated at the same time. It should be mentioned that Falkenhausen (1999), p. 116, thinks that this is not the White Hare Well but another well, called Wantan Well, just adjacent to the White Hare Well.



Ill. 5: Reconstruction of brine hoisting at or near the White Hare Well
in Yun'anzen, Yunyang District of Sichuan province

Source: Photo by courtesy of Chen Ran, Zigong, Sichuan.

Note: This reconstruction shows how a pulley frame was operated by one man. The rope is conducted over the cylindrical pulley. At both ends of the rope a bucket is fastened. The hoisting at each pulley is carried out with one hand lowering the empty bucket and with the other hand pulling up the full one. Note that the cylindrical pulley is equipped with spikes at its fringes that prevent the rope to slide off the drum. It is not clear which well exactly is depicted on this photograph.



Ill. 6: The structure of the hoisting frames used at or near the White Hare Well of Yun'anzen, Yunyang District of Sichuan province

Source: Bai Guangmei (1988), p. 265.

Legend:

- A: „Thousand jin“ (*qianjin* 千斤) weight bar
- B: Place where the cushion piece (*dianwu* 墊物) is inserted for fixing the hoisting apparatus
- C: „Heavenly balance“ (*tiancheng* 天稱) which is connected to the beam of the hoisting structure called „cloud pulley“ (*yunpan* 雲盤)
- D: „Ear boards“ (*erban* 耳板), i.e. side boards
- E: Roller (*gunzi* 滾子)
- F: Buckets for hoisting brine



Ill. 7: Pottery model of a water well from the Han period

Source: Laufer (1909), ill. 14, p. 62.

Note: The well is equipped with a rather small wheel-shaped pulley.



Ill. 8: Rubbing of an Eastern Han period tomb brick relief showing a water well, Jiexiang 架祥, Shandong province

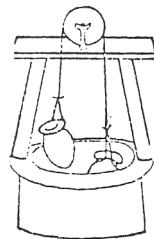
Source: Chen Wenhua (1991), p. 174.

Note: The picture shows a pulley with a slender waist. A man is pulling up (or lowering down) a earthenware vessel bound to the other end of the rope. One difference to the Sichuan salt work hoisting scene depicted in Ill. 2 is that there balanced hoisting, with a bucket or leather-bag fixed to each end of the rope, was carried out.



Ill. 9: Detail of a rubbing of Type A Eastern Han tomb brick relief from Qionglai 邛崃, Sichuan, probably showing a pulley with slender waist

Source: Lü Lin (1988), plate 10.

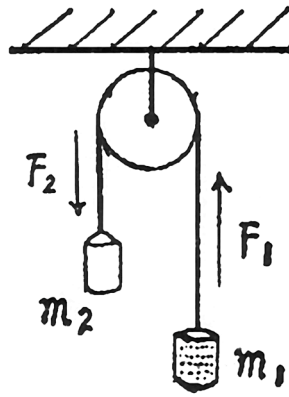


洛阳出土汉代陶水井
(据《文物参考资料》1954年9期)

Ill. 10: Drawing of a pottery model of a water well excavated in Luoyang 洛陽, Han period

Source: Li Chenyou (1984), p. 95.

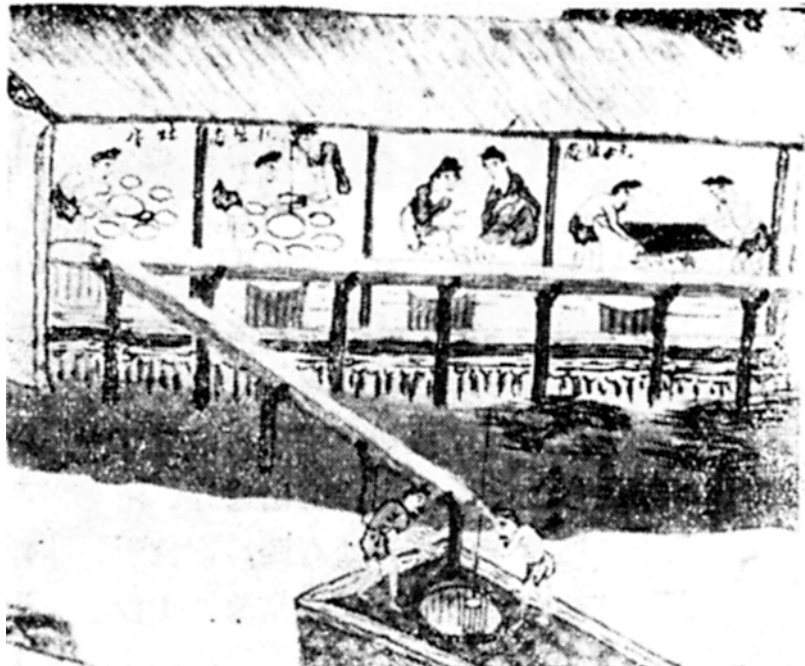
Note: A container is fixed to each end of the rope. The pulley seems to be of the wheel-shaped type rather than to belong to the slender-waist type.



Ill. 11: Diagram showing forces at work in case of balanced hoisting with a pulley

Source: Zhang Xuejun (1986), p. 108.

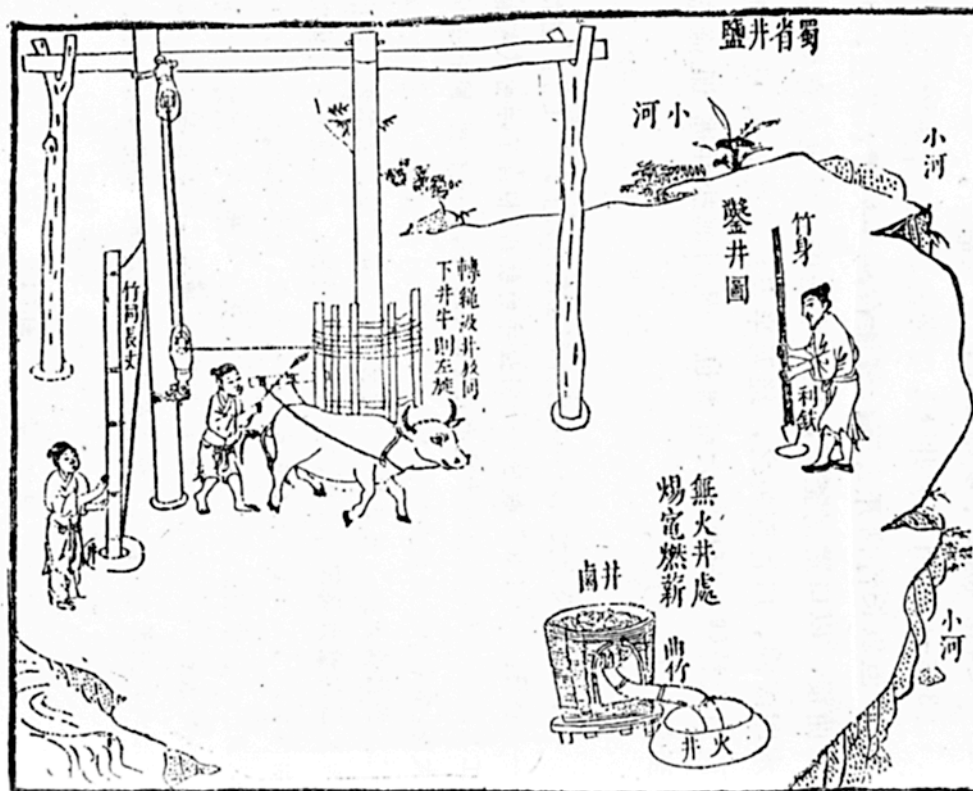
Note: The gravitational force of the full brine bag or bucket is m_1 , the force of the labourers lifting up the full bag or bucket F_1 . The gravitational force of the empty bag or bucket is m_2 , the force of the labourers sending down the empty bag or bucket F_2 . Assumed that friction is zero, the pulling force is: $F = F_1 + F_2 + m_2$.



Ill. 12: Hoisting brine with a bucket fixed to a bamboo pole at the Zijing Mowai Well 子井磨外井 of Jingdongjing 景東井, Yunnan, as depicted on a scroll of 1707

Source: *Diannan yanfu tu*; Lü Changsheng (1983), p. 111.

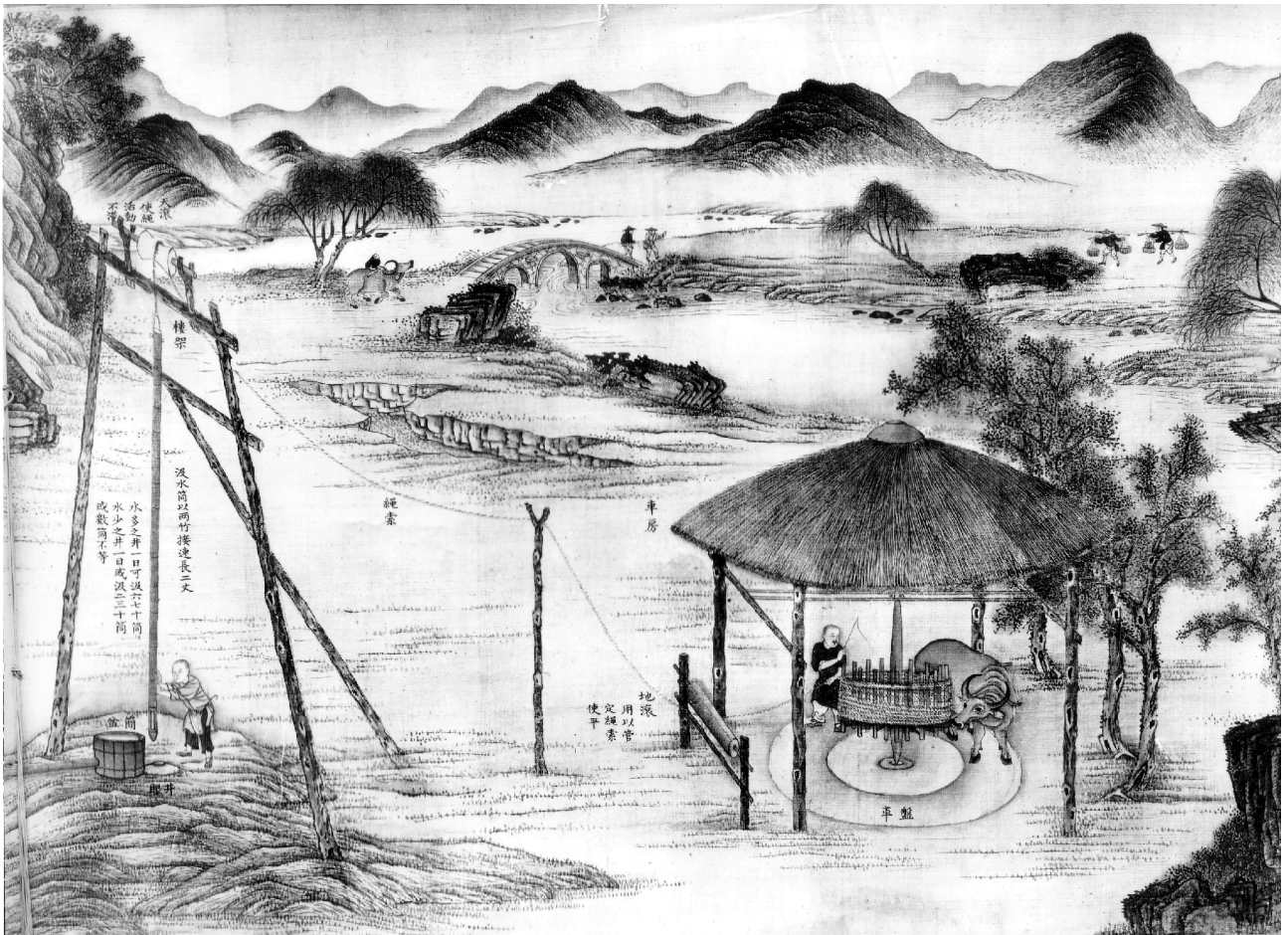
Note: A worker is hoisting brine in a bucket which is fixed to a bamboo pole. Thereafter the brine is poured into a gutter conducting the brine to the boiling house. The Zijing Mowai Well belongs to those brine wells which are located in midst of a river and which are inundated during summer and autumn so that production takes place only in winter and spring.



Ill. 13: Depiction of a brine well and a natural gas well in Sichuan in the *Tiangong kaiwu*, early seventeenth century

Source: Pan Jixing (1989), p. 276; see also Sun and Sun (1966), p. 119.

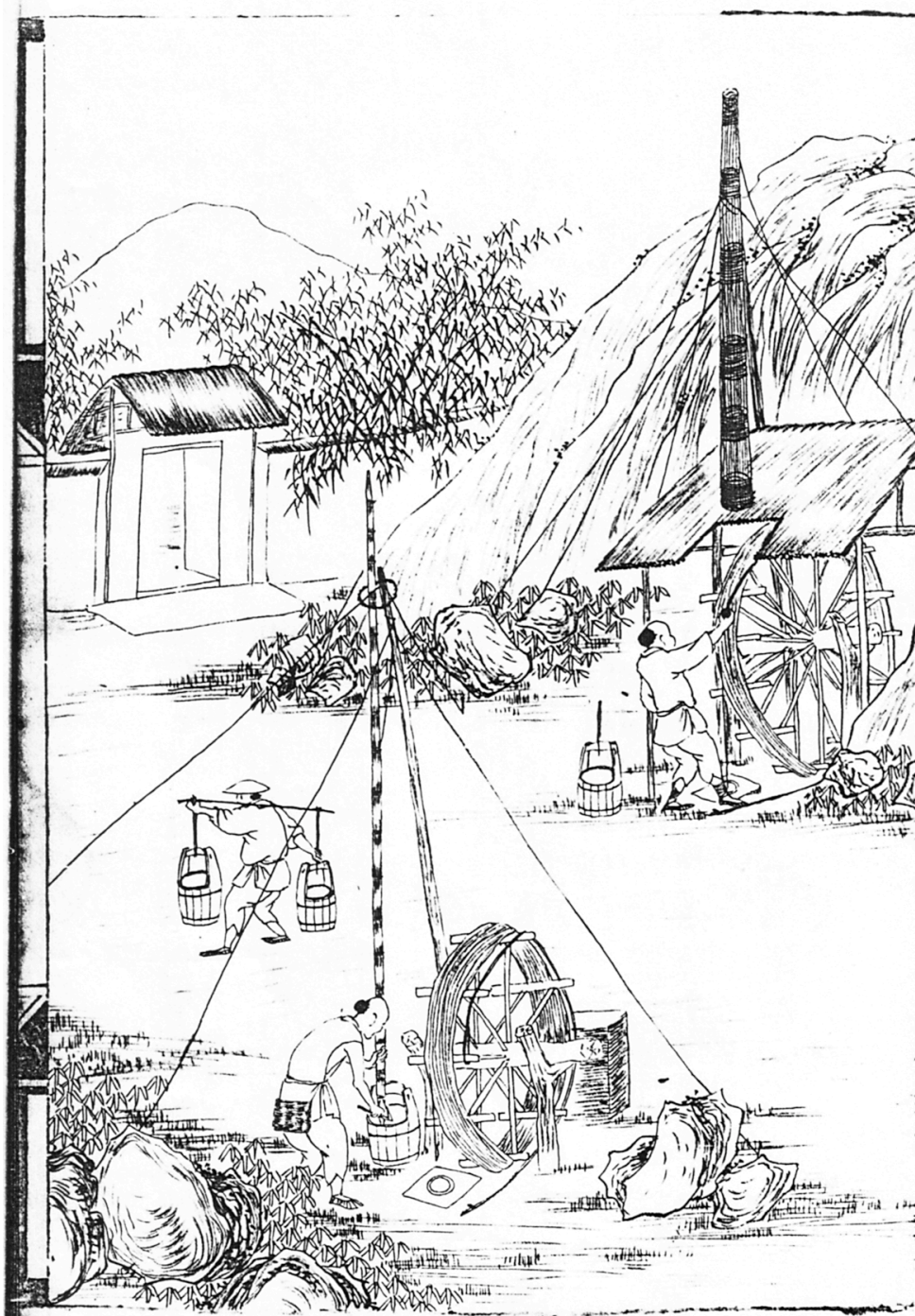
Note: This is the earliest extant illustration of a deep-drilled salt well and a deep-drilled natural gas well in Sichuan. In the lower part to the right a “fire well” is shown, in the upper part drilling is carried out. On the left side hoisting of brine takes place.



Ill. 14: Brine hoisting in Sichuan as depicted on a scroll of about 1740

Source: Courtesy of the Needham Research Institute, Cambridge.

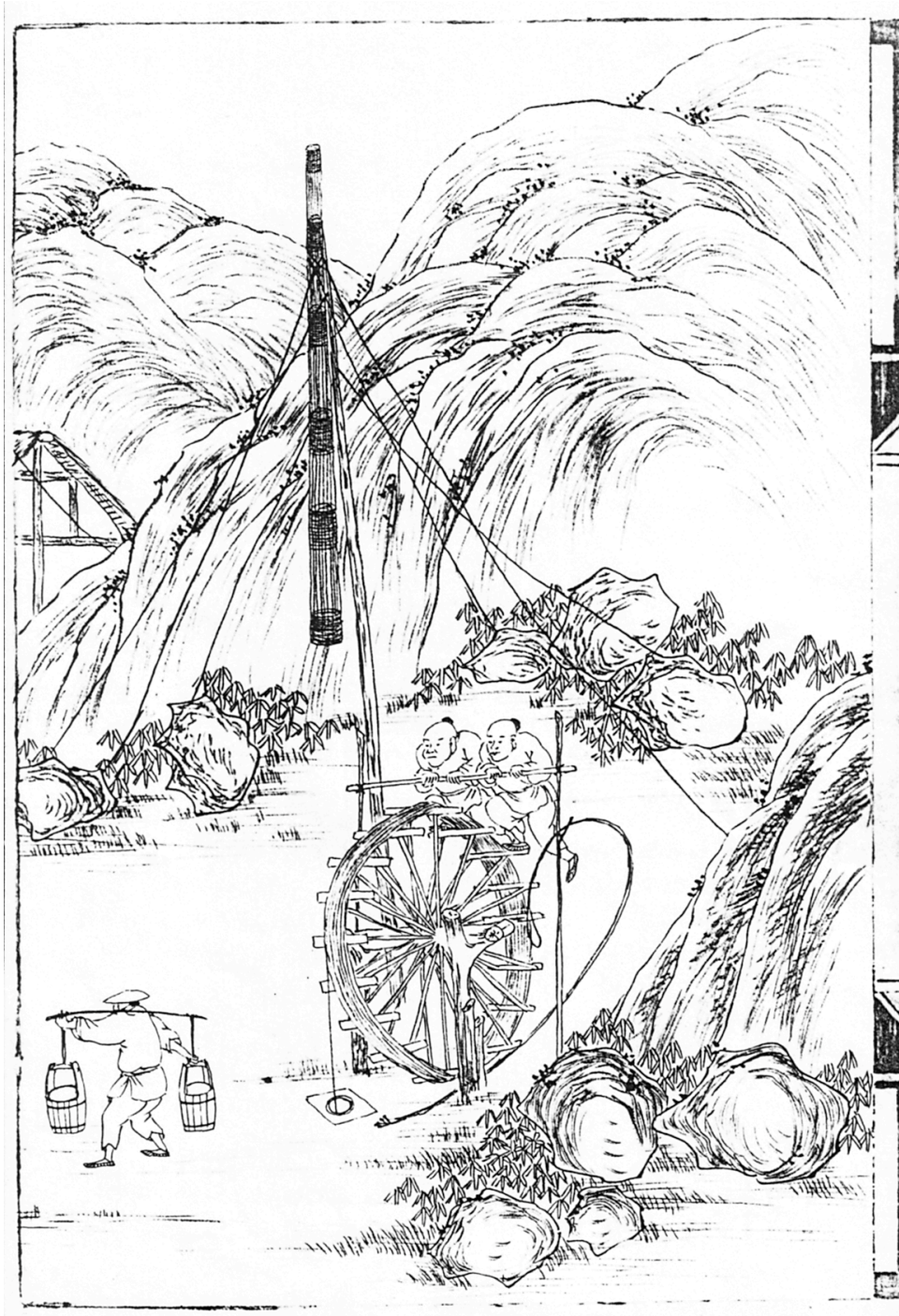
Note: From right to left a capstan drum, powered by one buffalo and covered by a roof, the “earth roller”, a guiding fork, and the three-legs hoisting rig with its slender-waist pulley (“heaven’s roller”) are shown. The tube just comes out of the small well opening and will be emptied into the tub on the left side.



Ill. 15a: Hoisting of brine at the Nanlang 南閬 saline, Sichuan, early twentieth century

Source: *Sichuan Nanlang yanwu tushuo*.

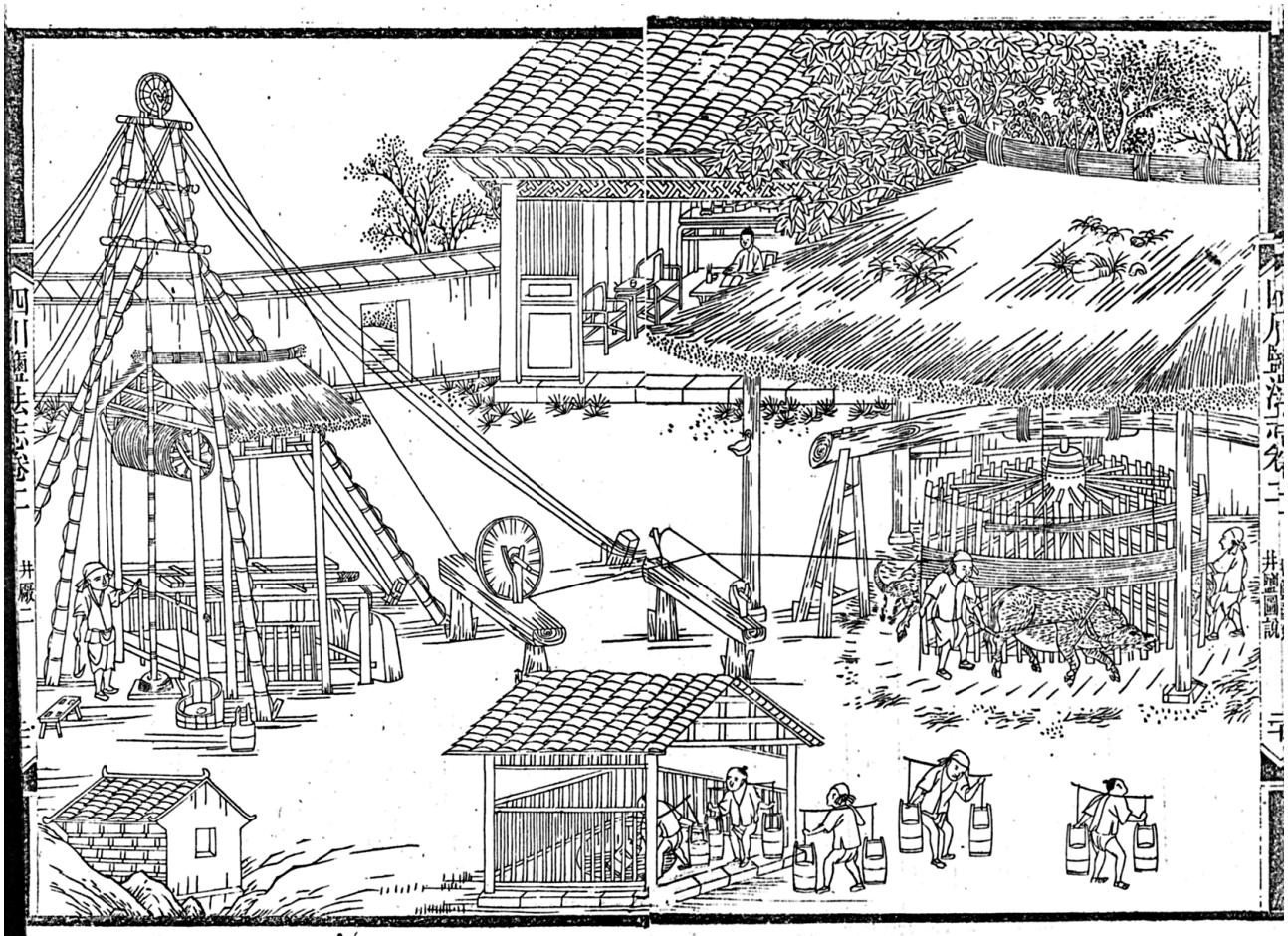
Note: Hoisting of brine at both wells is apparently powered by one man only. The hoisting apparatus is a wheel with a horizontal axle which rests on two forks. The rig has only one leg with either a ring or a basket-like structure at the top for fixing and securing the bamboo tube. At the well in the foreground the valve at the bottom of the tube bucket is opened with a hook and the brine released into a tub.



Ill. 15b: Hoisting of brine at the Nanlang saline, Sichuan, early twentieth century

Source: *Sichuan Nanlang yanwu tushuo*.

Note: Hoisting of brine is carried out in this picture with the help of a rather large wheel powered by two men. The rope leads from the wheel directly into the well, without being guided by pulleys. Like in Ill. 15a the wheel has a horizontal axis.



Ill. 16: Hoisting of brine in Ziliujing 自流井 and Gongjing 貢井 (Furong 富榮 saline), Sichuan, late nineteenth century

Source: *Sichuan yanfa zhi*, chap. 2, pp. 20b-21a.

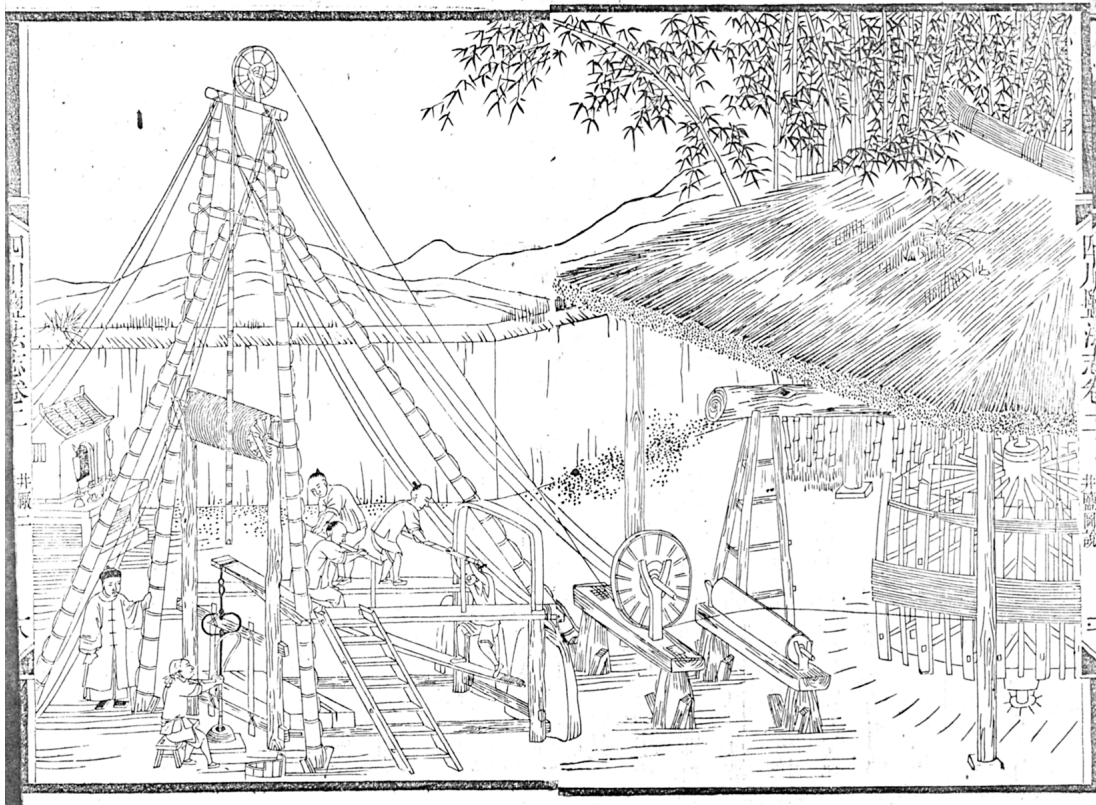
Note: This illustration shows a hoisting arrangement involving (from right to left) a capstan driven by four buffaloes, an „earth roller“, an „earth wheel“, and a „heaven’s wheel“. The brine hoisting pipe is just coming out of the well hole.



Ill. 17: Drilling a brine well at the Nanlang saline, Sichuan, early twentieth century

Source: *Sichuan Nanlang yanwu tushuo*.

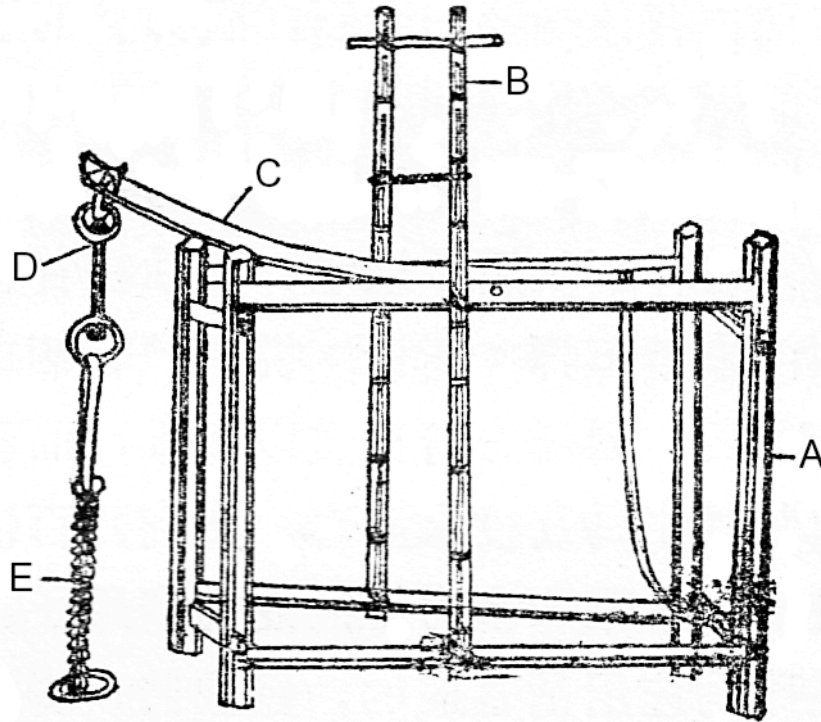
Note: The drilling apparatus is set in motion by one man who steps on, and jumps off, the lever beam. The drill is suspended from the shorter end of the stepping board. The worker at the orifice constantly turns the drill in order to make the borehole round and perpendicular. The rather complex release device of the drilling line can be seen.



Ill. 18: Drilling a brine well in Ziliujing and Gongjing (Furong saline),
Sichuan, late nineteenth century

Source: *Sichuan yanfa zhi*, chap. 2, pp. 17b-18a.

Note: The drilling is carried out by four men stepping on, and jumping off, the lever beam in coordinated action. Another worker, supervised by a foreman or drilling specialist, sits at the orifice of the well. He is responsible for turning the drill.



Ill. 19: Drilling apparatus of the Lezhi 樂至 saline, Northern Sichuan

Source: Lin Yuanxiong et al. (1987), p. 156.

Legend:

- A pounding frame (*duijiazi* 碓架子)
- B support for the hands (*fushou* 扶手)
- C stepping board (*caiban* 踩板)
- D suspension ring (*diaohuan* 吊環)
- E “whip cudgel” (*bianbang* 鞭棒)

Influences of Western Military Technology and Mechanics on Chinese Ballistics

YIN Xiaodong

1. Introduction

Early in the 16th century, two key inventions of western artillery, the breech-loading cannon with powder chamber, in China designated as *folangji* 佛郎機¹ (literally translated the “Frankish” [gun]), and the musket, in China designated as *niaochong* 鳥銃 (literally translated the “bird gun”), or as *niaozuichong* 鳥嘴銃 (literally translated the “bird-beak gun”), were brought to China from the West. Copied in China, they soon played a major role in Chinese weaponry. In the late 16th and in the 17th centuries the relations between the East and the West were intensified when Jesuit missionaries introduced western science and further technology in China. By the early 17th century, China’s government obtained more cannons from the West. In the course of the two periods of transfer of technology pertaining to the making and use of guns, western knowledge about projectile trajectories was gradually combined with traditional Chinese knowledge and experience. Some military books about artillery making and use with references also to western technology were published. These books were written partly by Chinese authors and partly in cooperation between Chinese and Jesuit authors. The integration of western and Chinese knowledge is in particular reflected in the military literature published between the 1560th and the 1680th. The investigation and analysis of the description of devices for aiming, the description of the trajectory of gun bullets, and the explanations for its shape in this literature shows that these descriptions and explanations were influenced by both Chinese and Western knowledge, especially by western military technology and mechanics. The investigation also shows that in China the interest in western military technology was focused on its practical application rather than on the accompanying theory of ballistics. The present article tries to provide an overview of how western knowledge about the projectile trajectory was introduced in China and reshaped the traditional Chinese knowledge and experience.

2. The Problem of Aiming

2.1. The Aiming Technology of Muskets

After the introduction of the musket in the 16th century several descriptions were published about the firing of these guns. The following quotation from *Chouhai tubian* 籌海圖編 [*The Illustrations Seaboard Strategy and Tactics*] written by Zheng Ruozeng 鄭若曾 (1503-1570) and published 1562 is representative of such descriptions:

都御史唐順之云虜所最畏於中國者，火器也。天助聖明，除兇滅虜。而佛朗機、子母砲、快鎗、鳥嘴銃，皆出嘉靖間。鳥嘴銃最後出而最猛利。以銅鐵為管，木橐承之，中貯鉛彈。所擊人馬洞穿。其點放之法一如弩牙發機，兩手握管，手不動，而藥線已燃。其管背施雌雄二桌。以目對桌，以桌對所欲擊之人，三相直而後發。擬入眉鼻無不著者。捷於神鎗而准於快鎗。火技至此而極。是倭夷用以肆機巧於中國習之者也。²

1 The designation *folangji* 佛郎機, a phonetical transcription of the “Franks,” is a general Chinese designation for “western” peoples and countries including Europe, Turkey, and the Arab world. The term *folangji* 佛郎機 was probably introduced into China by Chinese Muslims who lived in the east and south coastal area of China. After the introduction of western weapons into China, the term was also used for a special type of gun; see Dai Yixuan 戴裔煊, *Mingshi Folangji jianzheng* 《明史·佛朗機》箋正 [*Annotation and correction for (Mingshi. Folangji)*]. Published in 1984. Beijing: Zhongguo shehui kexueyuan chubanshe, p. 1.

2 Zheng Ruozeng, *Chouhai tubian* 籌海圖編. [*The Illustrations Seaboard Strategy and Tactics*]. Published in 1562. *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 [*General Collection of Works on Science and Technology in Ancient China*]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint] Chap. 13, p. 38a, b. Also see Tang Shunzhi 唐順之 (1507-1560), *Jingchuan waiji* 荊川外集. [*Other Collections of Jing Chuan*]. Published in 1573. Shanghai: Shangwu yinshuguan. 1936. [Reprint]. Chap. 2.

The duyushi-officer (*duyushi* 都御史³) Tang Shunzhi 唐順之 said that what the Tartars⁴ fear most about China are the firearms. Heaven helps the Holy Wise [Emperor] to get rid of the disaster and to wipe out the Tartars. In fact, the breech-loading cannon (*folangji* 佛朗機), the son-and-mother gun (*zimupao* 子母砲), the swift gun (*kuaiqiang* 快鎗), [and] the musket (*niaoziuchong* 鳥嘴銃) all have appeared during [the reign of] Jiajing 嘉靖. The musket has been the last to appear but is the fiercest. [One] uses copper [and] iron to make [its] tube, [and uses] a wooden sleeve to support it [=the tube] (*tuo* 橐, the original meaning of this character is bag open on both ends), [and] inside [it = the tube] the lead pellet is put. People and horses who are hit [by the pellets] are pierced through. The method of firing it [= the musket] is much like [the method of using] the tooth of the crossbow for triggering the mechanism. Two hands hold the tube, the hands do not move, and the gunpowder fuse has already been ignited. Its tube back set-up female male two sights. [On] the back [= the top] of its [= the musket's] tube [one] sets up two sights (*nie* 梟), male and female. [One] uses the eye to point to the sights, [and] uses the sights to point to the people [one] desires to hit. The three are [made] straight with each other and afterwards [one] triggers. [If one] intends to penetrate the eyebrow [or] the nose [of the enemy] there is no [possibility of] not hitting. [The musket] is quicker than the magic gun (*shenqiang* 神鎗) and more precise than the swift gun (*kuaiqiang* 快鎗). The technology of firearms achieved this [state] and [this] is the acme. This is what the Japanese foreigners (*woyi* 倭夷⁵) use to demonstrate the mechanical skills that [they] learned from China.⁶

According to this description, in which Zheng Ruozeng 鄭若曾 quotes a remark of the official Tang Shunzhi 唐順之 (1507-1560), the firing of the musket is widely the same as the firing of a crossbow. In ancient China, the term *nie* 梟, literally a “pillar instrument,” designates a gnomon, that is, an instrument for measuring the shadow cast by the sun.⁷ In the present context, however, it designates the back-sight and the fore-sight of the musket. This becomes obvious if we compare the description with similar descriptions in other military treatises as, for instance a description in a section entitled *The explanation of the musket* (*niaoichongjie* 鳥銃解) of chapter 5 on *The explanation of ordnance* (*junqijie* 軍器解) of Qi Jiguang's 戚繼光 (1528-1587) treatise *Lianbing shiji zaji* 練兵實紀雜集 [Miscellaneous Records concerning Military Training], published in 1571:

目照之法，銃上後有一星，口上有一星。以目對後星，以後星對前星，以前星對所擊之物，故十發有八九中。⁸

The method of aiming with eyes [is that] there is a [back]-sight on the rear of the gun, [and] there is a [fore]-sight on the muzzle. Use the eye to point to the back-sight, [and] use the back-sight to point to the fore-sight, [and] use the fore-sight to point to the thing to hit, therefore have eight or nine hits out of ten fires.

According to such descriptions, the firing of a musket follows the traditional Chinese aiming method designated as “three points linked by a line,” that is, to bring back-sight, fore-sight, and target into one line. In ancient China, the same method was used to shoot an arrow, bringing the *wangshan* 望山 with the arrowhead and the target into one line. The devices for aiming the arrow to be shot are designated as *dou* 斗 and *xing* 星. The *dou* 斗 is a small, quadratic hole and the *xing* 星 is a quadratic frame with a line that can be adjusted. After a sighting shot, this line has to be adjusted according to the aiming criterion, that is, the sinking of the arrow has to be compensated by an elevation of the shot⁹ (see figure

3 Here the term *duyushi* 都御史 is a title of the officer of *duchayuan* (都察院). The organization *duchayuan* (都察院) is the central control organization in the Ming and Qing dynasties in China.

4 The term *lu* 虜 is a form of designation for the Tartars or Mongols living in rural areas of northern and western regions of ancient China. Here the term is translated as Tartars because at that time it were the Tartars who always invaded Ming China.

5 The term *wo* 倭 is an old designation for Japan and *yi* 夷 is usually used in ancient China to designate foreign countries and foreigner. Thus the term *woyi* 倭夷 refers here to the Japanese enemy or to Japanese pirates.

6 *Tuo* 橐, the original meaning of this character is bag open on both ends

7 *Hanyu da zidian* 漢語大字典. [Dictionary of Chinese]. Chengdu: Sichuan cishu chubanshe, Hubei cishu chubanshe 1995, p. 3047.

8 Qi Jiguang. *Lianbing shiji zaji* 練兵實紀雜集. [Miscellaneous Records concerning Military Training]. Published in 1571. *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 [General Collection of Works on Science and Technology in Ancient China]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint]. Chap. 5, p. 21b.

9 Yi Degang 儀德剛, *Zhongguo chuantong gongjian zhizuo gongyi diaocha yanjiu ji xiangguan lixue zhishi fenxi* 中國傳統弓箭製作工藝調查研究及相關力學知識分析. [Study on Traditional Chinese Bow and Arrow Making and

1). This was the traditional aiming method of the crossbow for bringing the *dou* 斗, the *xing* 星, and the target into one line. The method shows that the people at that time were aware of the fact that at long distances the sight-line deviates from the trajectory of the arrow (see figure 2¹⁰).

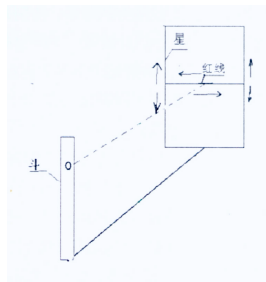


Figure 1

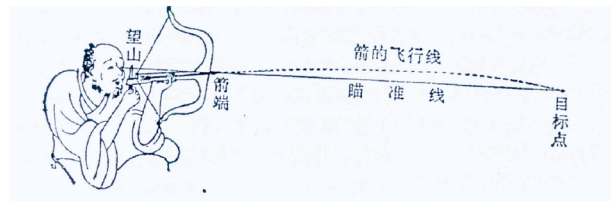


Figure 2

In contrast to the fore-sight of the crossbow, the fore-sight of the musket cannot be adjusted, but this does not affect its efficacy. The reason is that the tube of the musket is long and straight. Its tube is up to 6 *chi* 尺 long. If the proper gunpowder is used, the fired bullet is powerful so that the path of the bullet is a long straight line at the beginning. If the target is within this range it can easily be hit. Thus, since the tube of the musket is long, its fore-sight needs not to be adjusted. When the musket or gun is fired it should be aimed straight at the target without any adjustment (see figures 3¹¹, 4¹², and 5¹³).



Figure 3

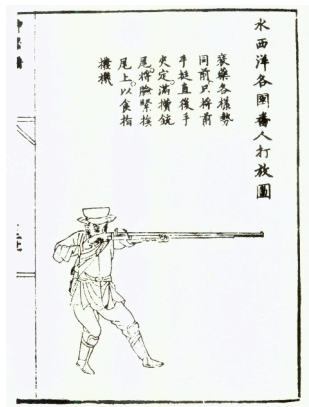


Figure 4



Figure 5

The comparison of aiming a musket and aiming a crossbow shows that both methods are similar. They both apply the principle of “three points linked by a line.” Moreover, some terms related to the aiming of a bow or a crossbow in ancient China, are used also for the description of aiming a musket and, in a similar way, also for the description of aiming a cannon. The intuitive knowledge comprising a rough, overall picture of the shape of the projectile trajectory was independently acquired by warriors in China

[Using Skill and related Mechanical Knowledge]. Hefei: PHD dissertation of University of Science and Technology of China 2004. p. 162.

- 10 Zhongguo keji daxue deng (Mengxi bitan) bianyi zu 中國科技大學等《夢溪筆談》編譯組. [The group for translating and editing for Dream Pool Essays (Mengxi bitan) in University of Science and Technology of China], Mengxi bitan yizhu 《夢溪筆談》譯註. [The Interpretation and Annotation for Dream Pool Essays (Mengxi bitan)]. Hefei: Anhui kexue jishu chubanshe 1979, p. 3.
- 11 Song Yingxing 宋應星(1587-1666?), *Tiangong kaiwu* 天工開物. [The Exploitation of the Works of Nature] Published in 1637. Beijing:Zhonghua shuju 1952. [Reprint]. Vol. 2, chap. 15, p. 35b.
- 12 Zhao Shizhen 趙士楨 (1553?-1611?), *Shenqi pu* 神器譜. [Treatise on Extraordinary Weapons]. Published in 1598. Zhongguo kexue jishu dianji tonghui 中國科學技術典籍通彙 [General Collection of Works on Science and Technology in Ancient China]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint]. p. 21a.
- 13 Zhao Shizhen, *Shenqi pu*. 1994, p. 19a.

and in the western world. This common knowledge was obviously the basis that made it possible for the Chinese to easily adapt the knowledge to use western firearms.

2.2. The Aiming Technology of Western Cannons

The western cannons introduced in China in the early 16th century were provided with a trunnion which made it easier to control the angle of elevation. At the same time some devices to operate the cannon including the gunner's quadrant and the geometrical square were also introduced. The geometrical square was used to determine the distance from the cannon to the target and the gunner's quadrant to determine the elevation of the cannon. The treatise *Xifa shenji* 西法神機 [*Western Masterpieces of Firearms*], written around 1623 by Sun Yuanhua 孫元化 (1581-1633) and published 1662, contains a description of how to use these two instruments:

銃頭低昂，合於天度。別有器量二種：一方，方度二十四；一圓，圓度九十。方器以量敵營之遠近，圓器以量銃頭之高低。平時先以方器就所據之臺，量來路高下幾何、遠近幾何，宜用何銃。每里即立一表，或樹或石。次以圓器，就所用之銃試擊之。視銃頭高幾度者至何處，低幾度者至何處。臨時視敵所至，即依所定度數擊之。有此器具，有此算法，故任所處而百變不窮，一成不誤，故敢任敢言也。¹⁴

The lowness and highness of the head of cannon, should conform to the degrees (*du* 度) of the heaven. In addition, there are two kinds of measuring devices: one is quadratic [device], [and] the degree of the quadratic [device] [is] 24. The other is circular [device], [and] the degree of the quadratic [device] [is] 90. The quadratic device is used to measure the distance from the enemy camp, and the circular device is used to measure the height of the head of the cannon. At ordinary time, using the quadratic device in advance, from the platform [one] holds, [one] measures how much the height of the path of coming [of the enemy] is, how much the distance [the path of coming of the enemy] is, and which [kind of] cannon should be used. Every one *li* 里, stand a [marking] rod, or a tree or a stone. Next, use the circular device, test to hit it (= the marking rod) with the help of the used cannon. [one] examines where [the ball of the cannon] reaches [when] the head of the cannon is some degree high; [and examines] where [the ball of the cannon] reaches [when] the head of the cannon is some degree low. At the time [when the war happens], examining [the place] where the enemy reaches, then [one] hits them according to the number of the selected degree (*du* 度). Having these appliances, [and] having this method of calculation, therefore, no matter what situation [one] is in, [he has] most changeful [treatments], [and] will make no mistake. Therefore [he] dares to take responsibility and dares to express [his opinions].

The quadratic device (*fangqi* 方器) mentioned in this quotation is the geometrical square and the circular device (*yuanyi* 圓器) is the gunner's quadrant. The treatise *Xifa shenji*, as well as the treatise *Binglu* 兵錄 [*Records of Military Art*], both contain detailed descriptions of the construction of these instruments:

諸銃點放平仰步數仍悉開於各銃之下。既知銃高幾度得至遠步幾何矣，然人於步之遠近從何測驗？則又當另置一器。其器以銅板為之，見方六寸，上端有兩耳，厚三分，見方一寸。橫豎於板面之上，距兩端各一寸。見方之中鑽一細眼，彼此相平。板面先畫一方楞，方楞角端為勾股交運規心。心繫一線，線末用錘。循規作四分之一，規分十二度。亦如量銃法。用時務立表於地，而以銅板端之耳兩見方細眼對視器所指之表，以線所直幾何度即知當用銃高幾何度也。¹⁵

The number of *bu* 步 for all cannons' horizontal elevation firing is still set under each canon. Knowing the long distance [measured] in *bu* 步 that a canon can reach [based on] degree of elevation, then how do people measure the distance in *bu* 步 [to the target]? Then another device should be set again. The device is made with copper plank 6 *cun* 寸 square. The upper portion has two ears, [which] are 3 *fen* 分 thick, and 1 *cun* 寸 square. Erect them transversely (=Set them orthogonally) to the plank, 1 *cun* 寸 in from both ends [of the plank]. Drill a fine hole in both ears, and position the holes parallel [within a straight line of sight]. Draw a square frame (*fangleng* 方楞) on the surface of the plank in advance, and the end of the corner (=is the vertex which is on the side

14 Sun Yuanhua, *Xifa shenji* 西法神機. [*Western Masterpieces of Firearms*]. 1662, chap. 1, p. 28b, 29a.

15 Sun Yuanhua, *Xifa shenji*. 1662, chap. 2, p. 21a, b.

with two ears) of the square frame is the center of a circle, namely the intersection of *gougu* 勾股 (=two legs of a right triangle). Hang a string at the center, [and] use a bob (*chui* 錘)[to be hung] at the end of the string. Make a quarter of the circle, [and] divide the [demisemi] circle into 12 degrees (*du* 度). [The usage of the quadratic device] is also like the method of measuring the cannon. When using [the quadratic device], [one] must stand a [marking] rod (*biao* 表) on the ground, and use fine holes in the two square ears on the upper portion of the copper plank to point to the [marking] rod (*biao* 表) that the device directs at, then [one] knows how many degrees of elevation for the cannon should be used according to how many degrees where the string meet the device. (See Fig 6, 7)

凡神器俱隨樣大小做厚牆鐵圍神車，架銃耳于上，銃力最猛，車輪圓轉，以殺其勢。點放之法有二：一曰平放，一曰仰放。故銃尾處預備木墊聽用，以便高下。製一量器，用四分規之一，規分十二分。以銅為之，規分之端有銅柄，量時以銅柄插入銃口。規心穿一線墜，看線所至分數，即可知銃彈到處。每高一分則銃彈到處較平放更遠，推而至于六分，遠步乃止，高七分彈反短步矣。¹⁶

Make magical cannon-carriages with thick iron walls according to the sizes and the styles [of the firearms] for all magical firearms. Support the trunnion of the cannon on [the cannon-carriage]. The power of the cannon is the most violent, [while] the wheels are round and turn, [which] can reduce its power (it=the cannon, its power means the backward influence of the cannon) . There are two methods of firing: one is named as horizontal fire, the other is named as fire with elevations. Therefore at the end of the cannon wood pillows are prepared to be used, which is convenient to raise and lower [the cannon] (=for the convenience of adjusting the height of the cannon) . Make a measuring device (*liangqi* 量器), which uses a quarter of a circle, [and] divide the [demisemi] circle into 12 *fen* 分. Use copper to make it (it=the measuring device), [and] there is a copper handle at the end of the divided circle (namely gunner's quadrant). When measuring, use the copper handle to insert into the gun muzzle. The center of the [demisemi] circle is threaded through by a string with a bob, [and through] examining the number of *fen* 分 where the string reaches, then [one] can know where the ball of the cannon reaches. [If] every *fen* 分 increased, then the place where the ball of the cannon reaches is farther than [the range of] horizontal fire. Deduce [the above relation] until 6 *fen* 分, only then the increased distance stops [increasing]. [When the elevation] increases to 7 *fen* 分, the ball [reaches] shortened distance instead. (See Fig 8¹⁷)

The geometrical square was not only used to determine the distance to the target, but also for measuring the height of the enemy's platform. The geometrical principle of these applications requires a knowledge of similar triangles (see figures 6 and 7)¹⁸, but both treatises are only concerned with the rules of how to use these instruments and do not explain them.

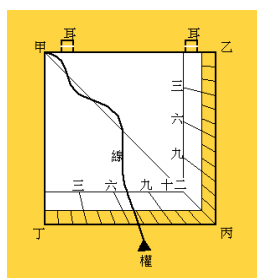


Figure 6

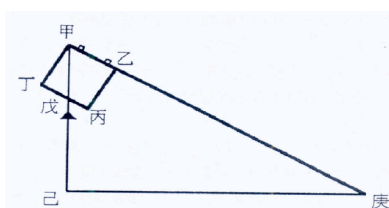


Figure 7

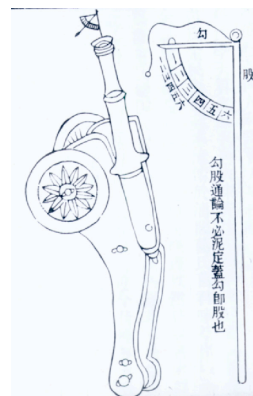


Figure 8

16 He Rubin 何汝賓(?-?), *Binglu* 兵錄. [*The Records of Military Art*]. Published in 1628. *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 [*General Collection of Works on Science and Technology in Ancient China*]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint]. Chap. 13, p. 3a.

17 Sun Yuanhua, *Xifa shenji*, 1662. cha 2, p 26a.

18 Huang Yinong 黃一農, "Cannon and Wars in China in 1600-1900: A Case Study on the Targeting Techniques Introduced from Europe" *Qsinghua Journal of Chinese Studies*. Vol. 26 (1996), no. 1, pp. 31-70.

The treatises *Xifa shenji* and *Binglu* not only describe western devices for aiming but also contain western knowledge concerning the determination of the range of shots, the method of trial shots, and calculations based on geometrical considerations. On the other hand, traditional Chinese knowledge about mathematics and astronomy is applied, represented for instance by terms such as ‘two legs of a right triangle’ (*gougu* 勾股) and ‘small pole (*biao* 表)’.

3. Description and Explanation of the Trajectory of Projectiles

The military literature published between the 1560s and the 1680s contains several descriptions and explanations of the trajectory of projectiles. These descriptions and explanations indicate that the Chinese authors, on the one hand, conceived of the projectile trajectory in terms of traditional Chinese knowledge, but, on the other hand, adapted western knowledge about the trajectory. This will be shown in the following by analyzing relevant texts.

Three representative texts written at different times by authors with different background knowledge have been chosen to be analyzed. The first text is the chapter 5 containing “*Explanation of the swift gun*” (*kuaiqiang jie* 快鎗解) of the treatise *Lianbing shiji zaji* written in 1571 by the famous general Qi Jiguang 戚繼光 (1528-1587). The second text is chapter 1 containing “*Illustration and Explanation of the gun platform*” (*chongtai tushuo* 銃臺圖說) of the treatise *Xifa shenji*, written around 1623. The third text concerns “*The preconditions and principles of the traveling path of a heavy thing in the air*” (*zhongwu kongzhong xingdao zhi shili* 重物空中行道之勢理) of the treatise *Qiongli xue* 窮理學 [*The Learning of Making a Thorough Inquiry into Principles*], written before or in 1683 by Ferdinand Verbiest (1623-1688).¹⁹

3.1. The Trajectory of Small Chinese Firearms

入藥線之後，用竹木筒內藥，每次一筒，用槩杖築實。下鉛子一枚，不宜用二三枚。二三枚者舊弊。彼殊不知：一錢藥一錢（鉛）子，則去直，中途不落地，可以計步命中。藥多子輕，則未出腹而化如水；藥少子重，則出腹至半途，必墜地。激之再發，不惟不可中，且中不殺人。²⁰

After the gunpowder fuse is put into [the tube], [and] bamboo or wood tubes are used to contain the gunpowder. Every time a tube [of gunpowder is loaded], [and] a ramrod (*shuozhang* 槩杖) is used to ram [the gunpowder] solid. A lead pellet is put into [the tube], [and] it is not suitable to use two or three [lead pellets]. [Putting into] two or three lead pellets is an old faulty practice (*jiubi* 舊弊). They (they=the users) hardly realized: [if] 1 *qian* 錢 gunpowder [is used to fire] 1 lead pellet, then [the lead pellet] goes straight, [and] does not fall to the ground in the midway, [so that it] can hit [the target by] counting *bu* 步. [If] the gunpowder is much [while the lead] pellet is light, then [the lead pellet] would melt as water [when it] has not gone out of the belly [of the tube]. [If] the gunpowder is little [while the lead] pellet is heavy, then [the lead pellet] goes out of the belly [of the tube and] necessarily falls to the ground in the midway. [If the lead pellet which falls to the ground] rebounds to fire again, [it] dose not impossible to hit [the target], Even [it] hits, [it can] not kill people.

The first part of this text deals with the right proportion of the amount of gunpowder to the weight of the lead bullet. The following part describes what happens if the amount of gunpowder and the weight of the bullet are not in the right proportion. Two aspects of this text deserve special attention.

First, the author who has the experience of actual combat knew that the amount of gunpowder should be in a specific relation to the weight of the bullet. This knowledge was obtained by practice. The text says that it is an “old faulty practice” to put two or three bullets into a gun barrel having filled it with gunpowder. This remark shows that such procedure was actually practiced. Furthermore, the text shows

19 Ferdinand Verbiest 南懷仁, *Qiongli xue* 窮理學 [*The Learning of Making a Thorough Inquiry into Principles*], written before or in 1683. The only known manuscript, located at Beijing University, is incomplete and contains only 14 chapters. According to its table of contents, the text originally contained 60 chapters. The section discussed here and in the following belongs to the chapter *Xingxing zhi litui bajuan* 形性之理推八卷 [Chapter Eight concerning] *Principles and Inferences of Shape and Quality*.

20 Qi Jiguang, *Lianbing shiji zaji* 練兵實紀雜集. [*Miscellaneous Records concerning Military Training*]. Published in 1571. *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 [*General Collection of Works on Science and Technology in Ancient China*]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint], chapter 5, p. 24b, 25a.

that the Chinese people believed that a bullet possesses the greatest power to kill or wound people as long as it hits them directly. This belief was based on experiences with shooting arrows in ancient China. The text argues that once the bullet hits the ground and is rebounded it can no longer hit the enemy or it is, at least, unable to kill a person even if it hits him. This shows that the Chinese people knew about the relation between the power of a projectile and the conditions of firing it. Such knowledge about conditions and effects of shooting was based on experiences gained in longtime observations and practices.

It is such type of experience that caused the author to describe the trajectory of a projectile using terms with an intuitive meaning. The term *zhui* 墜 is typical for this type of concept. Its meaning is “to fall” or “to let drop.” It does not imply any details of the curve and lets it open, whether the projectile falls down vertically or not. It is, however, unlikely that the author assumed that a lead bullet might fall down vertically at the end of its trajectory. This is evident from the use of the term *zhui* 墜 in other contexts. The term was, in fact, frequently used in ancient China to designate, for instance, the falling down of arrows and of falling stars. A sentence from the *Chuci* 楚辭 [*Songs of Chu*], written by Qu Yuan 屈原 (340 B.C.-278 B.C.), may serve as an example. The text reads: *shi jiao zhui xi shi zheng xian* 矢交墜兮士爭先. It says that two armies shoot arrows against each other and the arrows fall down while the soldiers still strive to approach the position of the enemy. The ancient Chinese people also used the term *zhui* 墜 to describe the motion of a falling star. They knew from observation and experience that in both cases the end of the trajectory is not a vertical line. The knowledge of these facts is based on experience and is expressed by a term which denotes this intuitive knowledge.

Up to the late 16th century, nobody seems to have cared about the question of how precisely and why a bullet travels in this way. Let us now look at the situation about half a century later.

3.2. The Introduction of Western Knowledge about the Trajectory

The first Chinese book on western military technology, *Xifa shenji* 西法神機 [*Western Masterpieces of Firearms*], written around 1623, introduced the western technology of gun making in China. Its author Sun Yuanhua 孫元化 learned western mathematics and military technology from the Jesuits. Moreover, he practiced the making and the use of western guns. As a consequence, his description of the projectile trajectory differs to a certain extent from earlier Chinese descriptions. Such a description is contained in the first chapter of his book.

夫銃之行也，全用其直勢，亦半用其曲勢。曲勢過半，不能殺人矣。²¹

The travel [of ball] of a cannon, fully uses its straight potential (*shi* 勢) (the trajectory of a fired ball is composed of a former straight path and a latter bent path, here straight power means the potential of straight path.), also half uses its bent potential (*shi* 勢) (here bent potential means the potential of bent path). [If] more than a half of bent potential [is used], [then the ball of the cannon] is not able to kill people.

It is mentioned in this text, that the trajectory of a bullet shot from a western gun is composed of a straight path and a curved path. The trajectory begins with *zhishi* 直勢, followed by *qushi* 曲勢. If the bullet has passed half of the *qushi* 曲勢, the power to kill people will be weakened. It is obvious that the character *qu* 曲 in this sentence designates the bent part of the trajectory. The term is missing in *Lianbing shiji zaji* 練兵實紀雜集, discussed before. But the viewpoint concerning the power of a bullet is similar in both texts. It is assumed that the effect is related to the phase of the trajectory at which it hits the target.

The text is not accompanied by an illustration which could help to further clarify the meaning of the terms. What precisely do the terms *zhishi* 直勢 and *qushi* 曲勢 designate? What shapes do these terms represent? In order to answer this question it is worth while to consider the possible meanings of the character *shi* 勢 which is common to both terms. In the present context this character designates the state or tendency of the moving projectile. Occasionally its meaning is close to that of the term *li* 力 designating force or power. In general, however, it is used to express the condition or situation of

21 Sun Yuanhua, *Xifa shenji*. 1662. Chap. 1, p. 28b.

something. It has been shown that the character *shi* 勢 had in ancient times a variety of different meanings.²² One of these meanings was close to that of indicating that something is moving.

Some indications of the later use of the term *shi* 勢 are provided by an encyclopedia collection of military treatises from the Song dynasty, *Wujing zongyao* 武經總要 [*Collection of the Most Important Military Techniques*], compiled in the 11th century and published between 1040 and 1044 by Zeng Gongliang 曾公亮 (999-1078). This collection of military treatises uses numerous compound terms containing the character *shi* 勢 such as *deshi* 得勢, *shizhong* 勢重, *xingshi* 形勢, *chengshi* 乘勢, *shuishhi* 水勢, *fengshi* 風勢, and *huoshi* 火勢. Their respective meanings are “power and influence, force, situation,” “circumstance,” “condition,” “state of affairs, tendency,” “trend,” “opportunity, flow of water,” and “force of the current of a river, the way the wind blows, fire.”

The volumes 10 to 13 of the collection concern the ordnance. An investigation of the use of the term *shi* 勢 in these volumes shows, that its meaning varies even in this narrowly defined context. Some examples may illustrate this, the first concerning a ship with an upper deck (*louchuan* 樓船), the second concerning the defense of a city (*shoucheng* 守城), and the third concerning a Chinese bamboo weapon called “bamboo fire snipe” (*zhuhuoyao* 竹火鷓).

若遇暴風，則人力不能制，不甚便於用，然施之水軍，不可以不設，足張形勢也。²³

If [the ship with an upper deck] encounters storm wind, then the manpower can not control [it (it=the ship with an upper deck)], [and] is not very convenient to be applied. But [if] being applied to the navy, [the ships] cannot but be equipped, [and] are adequate to enlarge the prestige and influence (*xingshi* 形勢).

凡守之道，敵來逼城，靜默而待，無輒出拒，俟其矢石可及，則以術破之。若遇主將自臨，度其便利，以強弩叢射，飛石併擊，斃之，則軍聲阻喪，其勢必遁。²⁴

Generally, the law of defending [the city is that] [when] the enemy comes and approaches the city, [The defender] waits in silence, [and] does not go out to repel [the enemy] immediately. [The defender] waits till the enemy [enters the range that] the arrow and stone can reach, then uses tactics to defeat it (it=the enemy). If [the defender] meets [the situation] that the commanding general attacks by himself, [the defender should] considers its advantages, [and] uses powerful crossbows to shoot clusteringly, [uses] slungshots to hit altogether, [and] kill him (him=the commanding general), then the army prestige [of the enemy] is dejected, [and] its tendency (*shi* 勢) [is] necessarily fleeing. ([In] this situation, [the enemy] necessarily flees.)

竹火鷓：編竹為疏眼籠，腹大口狹，形微脩長。外糊紙數重，刷令黃色。入火藥一斤，在內加小卵（卵）石，使其勢重。²⁵

Weave bamboo to make a cage with sparse holes. Its belly is big [while] its mouth is narrow, [and] its shape is small and slender. Paste several layers of paper on the exterior, [and] paint to make [it] yellow. 1 *jin* 斤 gunpowder is put into [the cage], [and] add small oval-shaped stones inside, [to] make the potential (*shi* 勢) heavy.

These examples show that in the 11th century the term *shi* 勢 was used with divers meanings. The term was not yet focused on one specific aspect of the context in which it was used. This situation seems to have changed in the early 17th century. This change is indicated by the way the term *shi* 勢 is used in the three important military treatises written in the first part of the 17th century. These three books are *Xifa shenji* 西法神機 [*Western Masterpieces of Firearms*], written around 1623, *Bing Lu* 兵錄 [*Records of Military Art*], published around 1628, and *Huogong qieyao* 火攻挈要 [*Essentials of Gunnery*], published in 1643 by Johann Adam Schall von Bell (1592-1666) and Jiao Xu 焦勳 (?-?)²⁶.

22 See the contribution of ZOU Dahai in this volume.

23 Zeng Gongliang (ed.), *Wujing zongyao* 武經總要. [*Collection of the Most Important Military Techniques*]. Published in 1510. In: Hua jueming (ed.) *Zhongguo kexue jishu dianji tonghui*, Jishu juan 5 中國科學技術典籍通彙，技術卷五 [*General Collection of Works on Science and Technology in Ancient China, Technology Vol. 5*], Zhengzhou: Henan jiaoyu chubanshe 1994, pp. 41–122. [Reprint in part]. Chap. 11, p. 7.

24 Op. cit. chap. 12, p. 1b.

25 Op. cit. chap. 12, p. 57a, b.

26 Schall von Bell composed the text (*shou* 授) and Jiao Xu 焦勳 wrote it down (*shu* 述).

The term *shi* 勢 is used in all three books, 14 times in the *Xifa shenji*, 13 times in *Bing Lu*, and 7 times in *Huogong qiyaoyao*. In most of these cases, the term *shi* 勢 refers to the state of motion or to the effect of an action. The meaning of the term is now focused on the conceptualization of movements. This indicates an interesting shift of the meaning of the term in the 17th century, for which the given example from *Xifa shenji* 西法神機 [*Western Masterpieces of Firearms*] is characteristic. The author Sun Yuanhua 孫元化 used the traditional term *shi* 勢 to represent properties of the projectile trajectory which had not been addressed in traditional Chinese military treatises.

3.3. Later References to the Western Theory of the Projectile Trajectory

During the reign of Kang Xi 康熙 (reg. 1661–1722), the Belgian Jesuit Ferdinand Verbiest using his knowledge of western military technology cast more than three hundred cannons for the Chinese army. In 1683 he published his treatise on the principle of western technology *Qiongli xue* 窮理學 [*The Learning of Making a Thorough Inquiry into Principles*]. He dedicated this book to the emperor Kangxi 康熙 and explained that his intention to write the treatise was to present the western theory of the calendar and to demonstrate why and how knowledge can be deduced from basic theories.²⁷ Among other topics he discussed in his treatise the foundations of 17th century ballistics.

總論重物之動，有依兩道而行。一曰因性之道，即上往下之行；二曰強性之道，即或橫或上而行。其因性之道者，即從上往下作正垂線而行。緣下為其本所也，天下之物，各有本所。物之性，亦各喜得本所。每物不在其所，則必與性相反，且別物得以攻之。故各就本所，乃各物之所喜向也。假如火本炎上，使之入水，則非本所，便就滅息。重之性下，水土其本所也。且物性直捷，重之垂下，不作迂曲。況天下之物性最巧，直線之途必短。迂曲之線，其途甚長。物喜短捷之便，故不肯拂性而迂曲也。

重物強性而行，由兩彼此相反之力而動。一曰本性之內力向下行；二曰：逆本性之外力向上行。蓋凡重物逆其本性而強受動，則施動者必通施猛墜之力于重物之體，以強帶而動。若無如此之力隨重物之體而帶動之，則重物既已離動者之手，即因本性垂線之直道而下行。然既不能全順本性之動而直下行，又猛墜之力，既不能全勝重性之逆行，而擊之直上行。則重物空中之道，非依直線往上往下，惟依曲線，而仿佛圭竇形之線。一半往上，一半往下行矣。²⁸

The power and reason of the traveling path of a heavy thing in the air: ‘Generally discussing the movement of a heavy thing, [it] has two paths to travel along. One is the path relying on the quality, namely the travels [from] up to down; the other is the path [of] the quality being forced, namely travels forward or upward (horizontally or up). Its (its=a heavy thing’s) [traveling along] the path relying on the quality is [its] making a travel [along] a vertical line from up to down. The reason [is that] down [place] is its original place. Each thing under Heaven has [its own] original place. The quality of thing is that each of them is also fond of getting its original place. [If] every thing is not at its original place, it is necessarily inverse of its quality, moreover, it can be can attacked by other things. Therefore, to moves towards its original place respectively is what each thing likes. For example, fire originally flames upwards as soon as it is made enter water, which is not its original place it goes out. The quality of weight is to go down, water and soil are its original places. Moreover, the quality of things is straightness and directness, a heavy object comes down vertically instead of acting deviously. Moreover, the quality of all things under Heaven is the most artful. The route of straight line is necessarily short. The route of a devious line is very long. The thing is fond of shortness and directness, therefore it does not like to move deviously disobeying its quality.’²⁹

27 Ferdinand Verbiest, *Jincheng Qiongli xue zoushu* 進呈窮理學書奏 [*The Respectful Dedication of the Book Qiong Li Xue*], written in 1683. In: Xu Zongze 徐宗澤 (1886–1947) (ed.), *Mingqing jian yesu huishi yizhu tiyao* 明清間耶穌會士譯著提要 [*Summary of the Translations and Works of Jesuits in the Ming and Qing Dynasties*]. Beijing: Zhonghua shuju 1989. [Reprint], p. 191.

28 Ferdinand Verbiest 南懷仁, *Qiongli xue* 窮理學 [*The Learning of Making a Thorough Inquiry into Principles*]. Chap. *Xingxing zhi litui bajuan* 形性之理推八卷 [*Chapter Eight [concerning] Principles and Inferences of Shape and Quality*], pp. 16a, b.

29 Here part of this paragraph is the same as a paragraph of *yuanxi qiqi tushuo* 遠西奇器圖說 [*The Record of the Best Illustration and Description of the Extraordinary Device of the Far West*], written by Wang Zheng 王徵 (1571–1644) and Johann Terrenz (Schreck) (1576–1630). Published in 1627. cha1, p 36b.

[When] a heavy thing travels [with] the quality being forced, [it] is moved by two reciprocal forces. One is traveling down [relying on] the internal force of original quality. The other is traveling up [relying on] the outside force against the original quality. [This is] because [if] every heavy thing is forced to move against the original quality, then the person who exerts the movement always necessarily exerts fierce force on the body of the heavy thing, and moves [it] by forcing drive. If there is no such force on the body of the heavy thing to drive and move it, then after the heavy thing has already departed from the hand of the person who exerts the movement, then it travels down along the straight path of a vertical line relying on original quality. However neither can [the heavy thing] follow entirely the movement of original quality to travel down straight, moreover, nor can the fierce force win completely the quality of weight, and travels conversely, and hits it (it=the heavy thing) to travel up straight. Then the path of the heavy thing in the air [goes] up and down instead according to a straight line, [but] only according to a curved line, [which] is similar to the line [outline] of the shape of a *guidou* 圭寶. One half [of the path] goes up, [and] the other half [of the path] goes down.

Verbiest uses the term *gui* 圭 which in ancient China designated a certain ritual object made of jade and used in sacrifices and funerals. The shape of this object consists of a rectangle and a triangle which forms a peak at the top (see figure 9³⁰). Verbiest describes the curved line of the projectile trajectory as similar to the shape of such a *gui* 圭. The curve he had in mind must have been parabolic although he did not even explicitly state that the trajectory is symmetrical, like a *gui* 圭. In another book, *Xinzhi lingtai yixiang zhi* 新製靈臺儀象志 [A Record of Newly-Built Astronomical Instruments at the Observatory of Beijing] published in 1674, he had depicted trajectories of a crossbow and a cannon (see figure 10³¹). These trajectories are roughly symmetrical as they should be from the viewpoint of early modern mechanics. Verbiest's comparison of the projectile trajectory with the shape of an ancient ritual object shows how he tried to combine Chinese and western knowledge.



Figure 9. A ritual object made of jade and used in sacrifices and funerals. Verbiest used it to illustrate the curved line of a projectile trajectory.

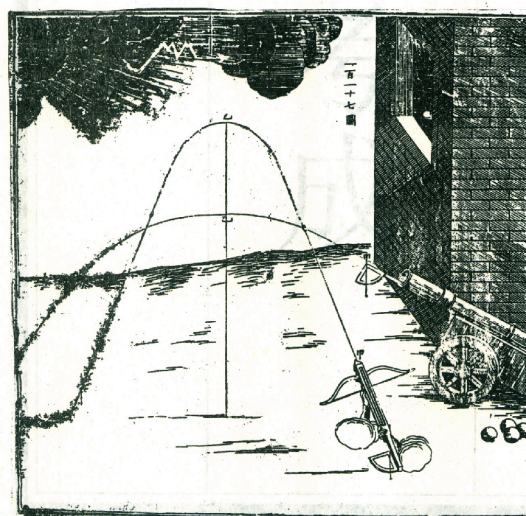


Figure 10. Trajectories of a crossbow and a cannon depicted in Verbiest's *Xinzhi lingtai yixiang zhi*.

30 Source: <http://www.zh5000.com/05/yuti/03/03-0001.htm>. The image is entitled *Qing yu shuangling guwen gui* 青玉雙菱谷紋圭 [Blue jade Gui decorated with double diamonds in the shape of grain veins]. The object dates to the Ming Dynasty and has the dimensions 26.3*6.5*0.8 cm.

31 Ferdinand Verbiest 南懷仁, 新製靈臺儀象志 [A Record of Newly-Built Astronomical Instruments at the Observatory]. Published in 1674. In: *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 [General Collection of Works on Science and Technology in Ancient China]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint]. attached volume: *Lingtai yixiang zhi tu* 靈臺儀象志圖. [Figures of the Astronomical Instruments at the Observatory]. Fig 117.

4. Conclusion

Let us summarize briefly the results of the present investigation.

1. The description of aiming techniques at different times reflects the changing knowledge background. In the first period considered here when the western musket was introduced into China, the experience gained from shooting arrows was applied and the aiming technique was described using terms dating back to ancient Chinese traditions. In the second period when western cannons were introduced into China, the western method of combining geometry and calculation methods was accepted, but they were described partly with terms taken from ancient Chinese mathematics and astronomy.
2. The texts which have been analyzed were written at different times by authors who had different knowledge backgrounds. Qi Jiguang 戚繼光, for instance, was a famous general who never acquired any western knowledge. Sun Yuanhua 孫元化, by contrast, not only extensively studied western mathematics and technology, but applied this knowledge as a practitioner in manufacturing artillery weapons. Finally, Ferdinand Verbiest was a Jesuit who possessed substantial western scientific knowledge. When these three authors were confronted with phenomena related to the projectile trajectory, they consequently described them in different ways using different terminologies. These differences reflect the development from the description of experiences in natural language to their representation within a scientific theoretical framework, a process that clarified and structured the knowledge about the projectile trajectory.
3. The gradual understanding and mastery of knowledge about the projectile trajectory depended on both the use of concepts from ancient Chinese traditions and the adoption of western knowledge. Based on both conceptual traditions, a new body of knowledge was formed. The Chinese authors tried to describe the trajectory with concepts of their own tradition. The term *shi* 勢 played a special role. Its meaning changed so that it could capture the developing knowledge about the projectile trajectory.

5. Chinese sources

1. Zheng Ruozeng 鄭若曾 (1503-1570), *Chouhai tubian* 籌海圖編. [*The Illustrations Seaboard Strategy and Tactics*]. Published in 1562. *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 [*General Collection of Works on Science and Technology in Ancient China*]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint]
2. Qi Jiguang 戚繼光 (1528-1587), *Lianbing shiji zaji* 練兵實紀雜集. [*Miscellaneous Records concerning Military Training*]. Published in 1571. *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 [*General Collection of Works on Science and Technology in Ancient China*]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint]
3. *Zhongguo keji daxue deng Mengxi bitan bianyi zu* 中國科技大學等《夢溪筆談》編譯組. [*The group for translating and editing for Dream Pool Essays (Mengxi bitan) in University of Science and Technology of China*], *Mengxi bitan yizhu* 《夢溪筆談》譯註. [*The Interpretation and Annotation for Dream Pool Essays (Mengxi bitan)*]. Hefei: Anhui kexue jishu chubanshe 1979.
4. Song Yingxing 宋應星(1587-1666?), *Tiangong kaiwu* 天工開物. [*The Exploitation of the Works of Nature*] Published in 1637.
5. Zhao Shizhen 趙士楨 (1553?-1611?), *Shenqi pu* 神器譜. [*Treatise on Extraordinary Weapons*]. Published in 1598.
6. Sun Yuanhua 孫元化 (1581-1633), *Xifa shenji* 西法神機. [*Western Masterpieces of Firearms*], Published in 1662
7. He Rubin 何汝賓(?-?), *Binglu* 兵錄. [*The Records of Military Art*]. Published in 1628.
8. edited by Zeng Gongliang 曾公亮 (999-1078), *Wujing zongyao* 武經總要. [*Collection of the Most Important Military Techniques*]. Published in 1510. *Zhongguo kexue jishu dianji tonghui* 中

- 國科學技術典籍通彙 [*General Collection of Works on Science and Technology in Ancient China*]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint].
9. edited by Xu Zongze 徐宗澤, *Mingqing jian yesu huishi yizhu tiyao* 明清間耶穌會士譯著提要 [*Summary to the Translation and Works of Jesuits in Ming and Qing Dynasty*]. Beijing: Zhonghua shuju 1989. [Reprint]. P191 Ferdinand Verbiest 南懷仁, *Jincheng Qiongli xue zoushu* 進呈窮理學書奏 [*The Memorial to the Throne to Submit the Book Qiongli Xue*].
10. Ferdinand Verbiest 南懷仁 (1623-1688), *靈臺儀象志* [*A Record of Astronomical Instruments at the Observatory*]. Published in 1674. *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙 [*General Collection of Works on Science and Technology in Ancient China*]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint].
11. Ferdinand Verbiest 南懷仁 (1623-1688), *Qiongli xue* 窮理學 [*The Learning of Making a Thorough Inquiry into the Reason*]. Published in 1683. the incomplete book in Beijing University.

The Use of Galileo's Theory of the Strength of Materials by the Jesuits in China

CHEN Yue

The Beijing Observatory, which is situated next to the Chang'an Avenue, was built in 1442. It was used for astronomical observations for a period of nearly 500 years. Today six astronomical instruments are standing on the platform on top of its tower. These six instruments were built by the Belgium Jesuit Ferdinand Verbiest (1623-1688) during 1669 and 1673. In 1674, he published his *Xin Zhi Ling Tai Yi Xiang Zhi* 新制靈台儀象志 [A Record of Newly-built Astronomical Instruments at the Observatory of Beijing], in the following *Yi Xiang Zhi*, in which he describes these six astronomical instruments and discusses theoretical problems related to them. In particular, the book contains a section on mechanical knowledge with the title "The principles for the strength of new instruments," which is related to Galileo's theory of the stability of materials. The present article focuses on this section.

In the 17th century, western mechanical knowledge was introduced into China together with other western knowledge and technologies concerning the calendar, astronomy and mathematics etc. In particular, *Yuanxi Qiqi Tushuo Luzui* 遠西奇器圖說錄最 [The Records of the Best Illustrations and Descriptions of Extraordinary Devices the Far West] published in 1627 for the first time systematically introduced western mechanical knowledge. Verbiest's book, published half a century later, while mainly dealing with astronomical instruments and stars tables, still introduced European mechanical knowledge not yet covered by earlier Chinese publications. He argued that this would be necessary to give "principles" (*li* 理) for the way he constructed his astronomical instruments. He considered it not to be sufficient to describe these constructions but provided also their theoretical background in western mechanical knowledge. He may have intended to prove in this way that the construction of his instruments was reasonable.

1. The Section on the Strength of Materials

The section "The principle for the strength of new instruments" counts about 1700 Chinese characters. This section can sensibly be divided into two parts. The first part, which constitutes about one third of the whole content, is a general introduction to the mechanical knowledge about materials. In the second part, which consists of one long paragraph, five statements, which can be considered propositions, may be discerned. In these five propositions, it is discussed how to deduce the weight that wires or columns in upright or horizontal position can sustain.

The five drawings corresponding to the propositions are reproduced in Figs. 1–5. They show that only in the first two propositions there are situations of testing hanging wires or upright columns, while the others all focus on testing columns parallel to the horizon. In the following, the contents of the five propositions shall be discussed in more detail.

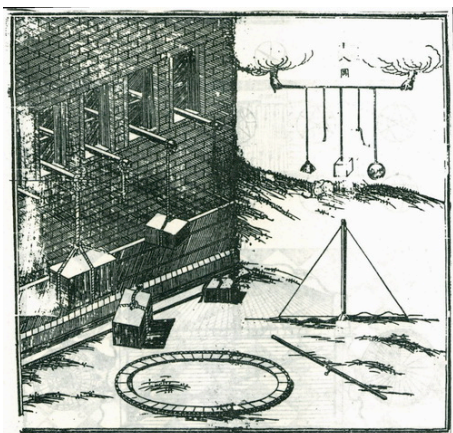


Figure 1

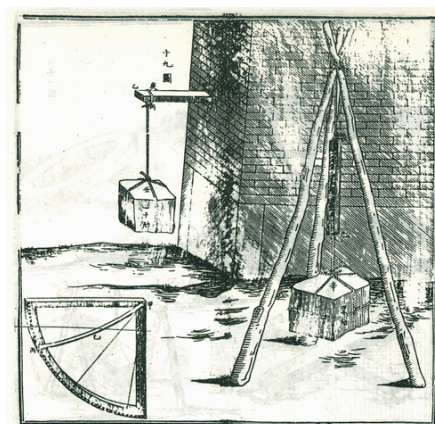


Figure 2

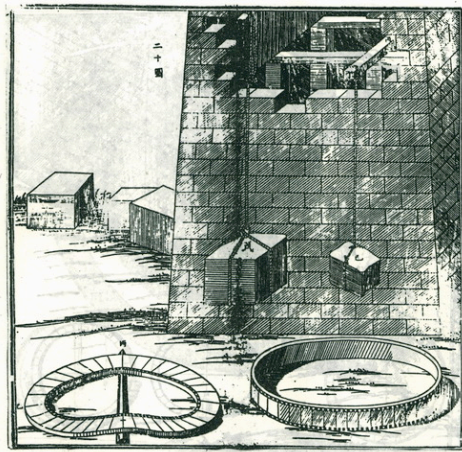


Figure 3

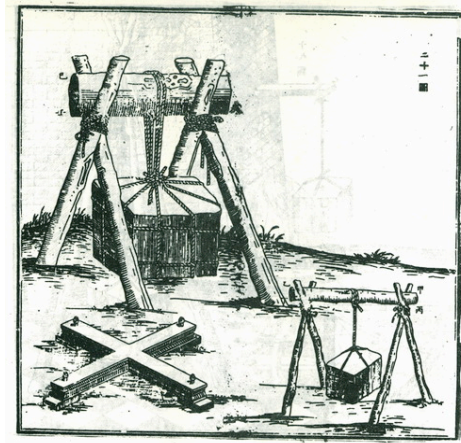


Figure 4

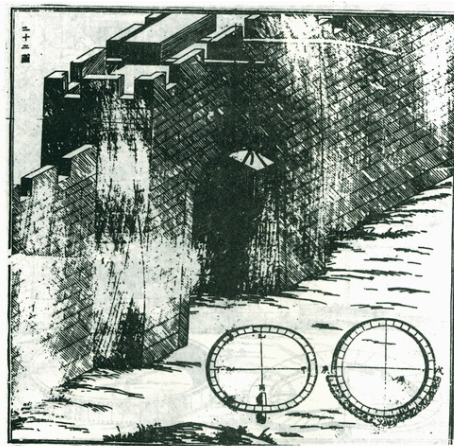


Figure 5

2. The Introductory Part of the Section

The introductory part of the section on the strength of materials contains the following passage:

夫欲儀制之堅固，不在乎尺寸之加廣，銖兩之加重，而徒以粗厚名也。大率在于儀徑長短之尺寸與儀體輕重之銖兩相稱而適均，乃為得耳。蓋儀之徑愈長，則儀愈難承負。儀體既重，若又加銅以圖堅固，則徑反弱而自下垂。如赤道黃道經緯諸規，兩端懸于南北兩極之軸。若銖兩加倍，則東西兩半太重，必自下垂而不合乎天上所當之平面圈矣。若豎立之，則上下兩半又下垂，而圓圈又類卵形矣。其長圓之徑表兩端定處，則中心太重，必自下垂而離南北之徑線。

The desired strength of the instrument's construction does not rely on the increase of the *measure* (*chicun* 尺寸) of width [or] the increase of the heaviness of the *weight* (*zhuliang* 銖兩), and taking the thickness (粗厚) into consideration is in vain. Generally, [the strength] relies on the measure of the length of the instrument's diameter and the *weight* of the instrument's body being matched and harmonious, only then [the principle] is achieved. The reason is [that] the longer the diameter of an instrument is, the more difficult [it is] to support the instrument. The instrument's body is already heavy, if again copper is added so as to seek strength, then the diameter weakens instead and sags. [For] such various rings of the equatorial and the ecliptic, the two ends are suspended at the axis of the south and north two poles. If the *weight* is doubled, then the eastern and western two halves are too heavy, [and] certainly sag and do not accord with the corresponding celestial plane rings. If they are erected, then the upper and lower two halves also sag and [causing] the circular rings moreover to be an egg shape. The two ends of the oval's *diameter* which has a marker are fixed [in their]

places, then the center is too heavy, [and] definitely sags, and deviates from the line of the southern and northern diameter.

In his introduction, Verbiest firstly emphasizes that it was not advisable to strengthen such things as poles and circles merely by adding material, since this would only result in distorting them and making them useless. This kind of opinion was not directly expressed very often in Chinese sources before *Yi Xiang Zhi*, while a special criterion system, which is called a *Caifen system* 材份制, contains information about materials and sizes of timbers in constructing.¹ This system was widely used by the 7th century at the latest.² Here Verbiest explicitly pointed out the wrong viewpoint that things would be stronger the more material was used.

There are two further key ideas of significance in the introduction. The first of them is expressed in the following sentences which concern the necessary way that showing the stability of an instrument:

今更取五金所以堅固之理以明之。夫五金等材堅固之力，必從人之所推移而見，又必從壓之以重物而始見之。

Now, [I] will further take the principles of the strength of the *five metals* (*wujin* 五金) and elucidate them. The force of the strength of the *five metals* and other materials can certainly be seen from a person's pushing and moving, and also only after pressing it with a heavy object.

According to Verbiest's point of view, the force of strength is a quality of materials which can be described by external forces. If somebody pushes and moves a solid body such as a thing made of one of the *five metals* or even presses a heavy thing on it, he can determine the force of strength, that is to say, the force of strength emerges. This way Verbiest conveys the idea of the "force of the strength of materials" which is quite similar with the one of explanations of the concept *li* 力 in the Chinese tradition.³

The second key idea expressed in the introductory part of this section is the general notion of different forces that columns can support and their relation to each other:

姑借方、圓柱所承之力以類推焉。凡形之長者，必有縱徑、有橫徑。其縱徑之力與橫徑不同。儀之中，有方柱、圓柱；有長方各梁柱；有長遠表。其中有豎立者，有與地平線平行者，有橫斜用者。縱徑、橫徑各有說焉。今先論縱徑之力以定橫徑所承之力。

[We will] now consider the force that rectangular and circular columns [can] support and categorically deduce [them]. Those that are long in shape must have *lengthwise diameters* (*zongjing* 縱徑) [and] has a *transverse diameters* (*hengjing* 橫徑). The [resisting] force of the lengthwise diameters are different from [the force of] the transverse diameters. Within an instrument, there are

1 Several sentences being found in chapter 64 in *Jinshu* 晋书, appear the similar meaning to Verbiest's opinion. They are shown as follows: "The saying says: burying [is] hiding. Hiding desires [that] it (the coffin) is deep and firm. The outer coffin is big, as a consequence, [it is] a hard act to be strong and is no good." (語曰：葬者，藏也。藏欲其深而固也。槨大，則難為堅固，無益。), *Jinshu* 晋书 (the book of Jin Dynasty, was written by a group of scholars including Fang Xuanling 房玄齡 (A.D.579~648) who held the post of the minister of public works, in A.D.650, *Qinding Siku Quanshu* 欽定四庫全書 (Complete Collection in Four Treasuries), 256-91, 1773).)

2 *Ying Zao Fa Shi* 營造法式, which was written by Li Jie 李诫, was officially issued as an architectural law in 1103. Before this book, criteria of timbers in architecture were established along with the development of human-beings' practical knowledge about timber, material, structure and mechanics. According to researches on the historically remained buildings, most Chinese scholars believe that the *Caifen system* was at the height of its use from the 7th century to the 12th century.

3 In *Wai Chu Shuo Zuo Shang* chapter of *Han Fei Zi*, a Chinese book written in the third century B.C., contains following sentences: "MO Zi made a wood glider, which spent him three years to finish it, it flew only one day and damaged. His disciples said: "You are so skillful to make a wood glider fly." Mo Zi said: "I am not so skillful as a carriage-crossbar maker is. He can use only a small piece of wood and spend less than one morning time to finish a carriage-crossbar, which can draw 30 dan load to reach a long distance. It has much *li* 力 and can be used for many years. Now I made a glider, which spent me three years to finish, but flew only one day and damaged." Hui Zi heard it and said: "Mo Zi was greatly skillful, he was skillful in making carriage-crossbars but was clumsy in making glider." (《韓非子·外儲說左上》：墨子為木鸞，三年而成，蜚一日而敗。弟子曰：“先生之巧，至能使木鸞飛。”墨子曰：“吾不如為車輓者巧也。用咫尺之木，不費一朝之事，而引三十石之任致遠，力多，久於歲數。今我為鸞，三年成，蜚一日而敗。”惠子聞之曰：“墨子大巧，巧為輓，拙為鸞。”) Here, *li* means the force. The great efficiency of the carriage-crossbar is represented by the "much *li*". This indicates that *li* mentioned in above source possesses the meaning similar to the strength of a particular project.

rectangular columns (*fangzhu* 方柱) and *circular columns* (*yuanzhu* 圓柱); there are various long *rectangular* (*changfang* 長方) beams [and] columns; there are markers that are long and faraway. Among them, there are those that are upright [and] those that are parallel to the horizon, [and] there are those that are used transversely and obliquely. The *lengthwise diameter* [and] the *transverse diameter* each have their own descriptions. Now [we will] first discuss the force of the *lengthwise diameter* to determine the supporting force of the *transverse diameter*.

In this part, Verbiest deals with the strength of objects with a specific shape such as rectangular columns and circular columns. If we compare this specification with the beginning where he deals with materials of indefinite shape or size, we can conclude that the strength of materials depends on the kind of material as well as on its shape. In particular, the force of strength of an object with a specific shape varies with the direction it is pushed or pressed.

Verbiest's idea that 'the force of strength' denotes a general quality of materials and 'the force that columns support' denotes a specific force depending on the shape of a material body raises the question of the relation between these two notions. However, this problem will be addressed only after the propositions of the following passages in Verbiest's text have been discussed. But before discussing these propositions, the importance of the last sentence in the text quoted above should be emphasized. This sentence is important not only because it announces that the force in the direction of the width of a column (the *transverse diameter*, see Fig.6) can be deduced from the force in the direction of its length (the *lengthwise diameter*, see Fig.6), but also because it indicates the total scope of these propositions.⁴

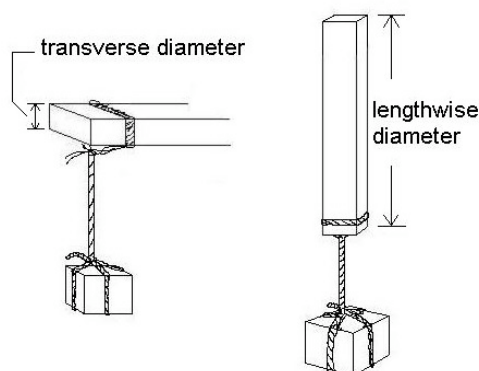


Figure 6: The transverse diameter and the lengthwise diameter

3. The Five Propositions of the Section

3.1. The First Proposition

The following text contains implicitly the first proposition:

西士嘉理勒之法曰，觀于金銀銅鐵等垂線繫起若干斤重漸。次加分兩，至本線不能當而斷。如金及銀之垂線，其橫徑一厘，試加斤兩至二十三斤而斷。又同徑之銅鐵線，試加斤兩至十八斤而斷。因此法而推論，曰：有金銀立柱于此，其橫徑有六厘，必得八百二十七斤之分兩能當之；銅鐵柱，必得六百四十七斤之分兩能當之；有同徑之烏木等材料之立柱，約得一百一十八斤之分兩能當之，如八圖。蓋凡兩柱大小之比例，為其兩橫徑再加之比例，而其堅固之比例，必與之相同。譬十如有金線于此其橫徑為一厘。若能當二十斤，則一分徑之金線必能當二（十）[千]斤矣。蓋一厘之徑與一分之徑如一分之徑與一寸之徑，則一厘之徑與一寸之徑，如二十斤與二千斤同是再加倍之比例。從此而推方圓等柱以其橫徑之所當分兩若干。

⁴ The *transverse diameter* and the *lengthwise diameter* are not defined in Verbiest's book. The former one is mentioned in the first four propositions and the latter one is mentioned only in the second proposition. According to the usage of these two concepts in the propositions, the *transverse diameter* means the vertical line of a column, which parallels to the horizon, or the diameter of a string or a cylinder, while the *lengthwise diameter* means the longest line of a column or the length of a cylinder.

The method of the western scholar, Galileo (Jialile 嘉理勒), says: observe plumb wires [made] of gold, silver, copper, iron and other materials which are tied down with a weight of several *jin* 斤. Gradually add weight (*fenliang* 分兩) [to the load] till the wire cannot bear [it] and breaks. For example, plumb wires of gold and silver have a *transverse diameter* of one *li2* 厘. Test to add the *weight* to 23 *jin* [before the wire] break. [Likewise], copper or iron wires of the same diameter, were tested until the added the *weight* reached 18 *jin* and broke. [One can] deduce from this method and say: Given a gold or silver upright column with a *transverse diameter* of six *li2*, the weight [of the load] that it can bear must be 827 *jin*; a copper or iron column [of the same width] must be able to bear 647 *jin*; Given an upright column of the same width [made] of such materials as ebony, the load it can bear is approximately 118 *jin*, as in figure 18. The reason is that, every ratio of two columns' sizes is the duplicate ratio of their two *transverse diameters*, and the ratio of their strengths must be the same with it. For example, given a gold wire with a *transverse diameter* of one *li2*. If [the wire] can bear [a load of] 20 *jin*, then a gold wire with a diameter of one *fen* 分 can definitely bear [a load of] 2000 *jin*. Since [the ratio of] a diameter of one *li2* to a diameter of one *fen* is proportional to [the ratio of] a diameter of one *fen* to a diameter of one *cun* 寸, then [the ratio of a maximum load borne by a column with] a diameter of one *li2* to [a maximum load borne by a column with] a diameter of one *cun* is proportional to the likewise duplicate ratio of 20 *jin* to 2000 *jin*. From this [one can] deduce the weight [of a maximum load that] a rectangular and circular columns can bear according to their *transverse diameters*.⁵

At the beginning of this quotation, Verbiest referred to Galileo's work as the source of his exposition of a theory on the strength of materials. In fact, the quotation starts in the same way as Galileo's presentation of the first of his "two new sciences" in the First Day of his *Discorsi*:⁶

Salv. I cannot refuse to be of service, provided that memory serves me in bringing back what I once learned from our Academician [Galileo] who made many speculations about this subject, all geometrically demonstrated, according to be called a new science. For though some of the conclusions have been noted by others, and first of all by Aristotle, those are not the prettiest; and what is more important, they were not proved by necessary demonstrations from their primary and unquestionable foundations.

Since, as I say, I want to prove these to you demonstratively, and not just persuade you of them by probable arguments, I assume that you have that knowledge of the basic mechanical conclusions that have been treated by others up to the present which will be necessary for our purpose.

First of all, we must consider what effect is at work in the breaking of a stick, or of some other solid whose parts are firmly attached together; for this is the primary concept, and it contains the first simple principle that must be assumed as known. To clarify this, let us draw the cylinder or prism AB, of wood or other solid and coherent material, fastened above at A, and hanging plumb; at the other end, B, let the weight C be attached. It is manifest that whatever may be the tenacity and mutual coherence of the parts of this solid, provided only that that is not infinite[ly strong], it can be overcome by the force of the pulling weight C, of which the heaviness [gravità] can be increased as much as we please, and that this solid will finally break, just like a rope. And just as we understand that the resistance of a rope is derived from the multitude of hempen fibers that compose it, so in wood there are seen fibers and filaments stretched out lengthwise which render it even more resistant to breakage than hemp of the same length would be. In a stone or metal cylinder, the coherence of parts seems still greater, and depends on some other cement than that of filaments or fibers. Yet even these [cylinders] are broken by a sufficient pull.

The five propositions of Verbiest's exposition are all taken directly or indirectly (possibly through an author paraphrasing Galileo) from Galileo's theory as it is primarily elaborated in the Second Day of the *Discorsi* either as direct reformulations of Galileo's propositions or as implicit consequences of them.

5 *jin* is a unit of weight, 1 *jin* equals 0.5 kg. *Li*, *fen*, *cun* are units of length, 10 *li* equals 1 *fen*, and 10 *fen* equals 1 *cun*.

6 Galileo, *Two New Sciences, Including Centers of Gravity and Force of Percussion*, a new translation with introduction and notes, by Stillman Drake, the University of Wisconsin Press, published in 1974. p 15.

Verbiest's text quoted above contains the proposition that if you increase a weight hanging at a wire there will come a point so that the wire breaks and that the ratio of the strength of two wires or circular columns with different diameters made of the same material equals the square of the ratio of their diameters.

This proposition says, by using the method of Galileo, the weight borne by wires of different materials can be measured for a specific diameter, and then the weight borne by columns of other diameters can be deduced. Verbiest gives numerical values, possibly empirical ones, of the weight that wires as well as columns of different materials could bear. Thus, he establishes a relation between materials and weights. In this way, a certain kind of column of a certain material corresponds to a certain maximum weight it can bear. That is to say, an external measure conveys a quality of materials and columns, namely 'the force of strength' and 'the force that columns support' is represented by the maximum weight hung under the wires or column. These relations are illustrated by Fig.7.

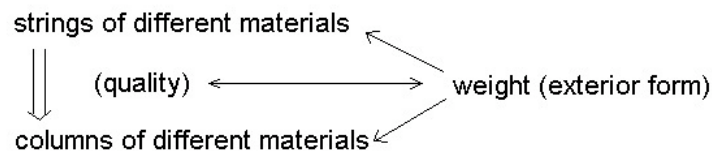


Figure 7

3.2. The Remaining Four Propositions

The second proposition is contained in the following text:

如十九圖，有方柱豎立為戊己，其縱徑僅足拉斷之斤兩，即辛繫在于己。又有方柱甲乙丙丁於地平線平行。其大小于豎立之方柱戊己相同。其橫徑僅足拉斷之斤兩，即壬繫在於丙。題曰：辛之斤兩于壬之斤兩，如戊己柱之縱徑于甲丙柱之橫半徑。蓋丙丁線槓杆之類，其支磯在丁，其用力在丙。由此論之：試令本柱之橫半徑丙庚有其縱徑甲乙四分之一。而辛之斤兩為四千斤。則壬之斤兩不過一千斤。而原柱依其橫徑必墜斷矣。

As in figure19, there is a rectangular column standing upright as E (戊 *wu*) and F (己 *ji*), that [each] has a *lengthwise diameter* which is only sufficient for a [certain] weight to stretch and break [it], depicted as H (辛 *xin*) tied at F. There is also a rectangular column, ABCD (甲 *jia* 乙 *yi* 丙 *bing* 丁 *ding*), parallels to the horizon. Its size (*daxiao* 大小) is the same as the upright rectangular column EF. Its *transverse diameter* is only sufficient for the *weight* [that can] stretch and break [it], depicted as I (壬 *ren*) tied at C. The proposition says: (the ratio of) the *weight* of H to the *weight* of I is proportional to the lengthwise diameter of column EF to *half transverse diameter* (*hengbanjing* 橫半徑) of column AC. This is because the wire CD can be classified as a lever, whose fulcrum is at D, [and whose] exerted force at C. From this (one can) surmise the following: test by setting half the *transverse diameter* CG of the original column to be one fourth of its *lengthwise diameter* AB. Let the *weight* at H be 4000 *jin*. Then the *weight* at I can be no more than 1000 *jin*. The said column will definitely break and fall along its *transverse diameter*.⁷

7 *Jia* 甲, *yi* 乙, *bing* 丙, *ding* 丁, *wu* 戊, *ji* 己, *geng* 庚, *xin* 辛, *ren* 壬, *gui* 癸, which were used commonly in ancient China, are the ten Heavenly stems to designate marks of order. In *Yi Xiang Zhi*, *ji* 己 is always written as 巳, which may be treated as mistaken writing.

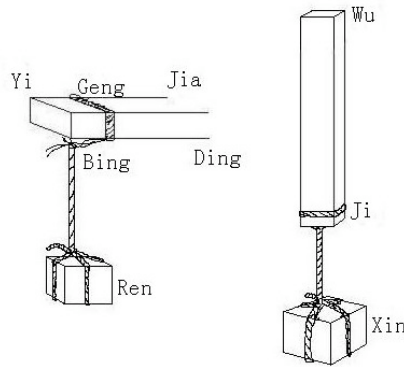


Figure 8

The proposition contained in this text states for a column placed horizontally being fixed to a wall at its one end with a load fixed at its other end that the ratio of the weight that the column can bear if placed horizontally to the weight it can bear if placed vertically equals the ratio of the column's half *transverse diameter* to its *lengthwise diameter*. This proposition is found as the first proposition of Galileo's treatment of the strength of materials in the Second Day of his *Discorsi*. The proposition connects the strength of a column in upright position to its strength in horizontal position. It allows to derive the force of strength of the horizontal column from the force of strength of the upright column (see fig.12). The proposition realizes what Verbiest announced in the last sentence of the introductory part quoted at the beginning.

Deducing model in the second proposition:

the upright column → the horizontal column

The last sentence in the introduction:

the force of lengthwise diameter → the force of transverse diameter.

This second proposition concerns the relation between upright columns to horizontal columns, while the remaining three propositions all deal with horizontal columns sustaining weights.

The third proposition is contained in the following text [see Fig. 3 and its illustration Fig. 9]:

又有兩長方之柱，見二十圖甲乙丙丁。而甲乙之厚面及丙丁之寬面兩面于地平線平行。與兩柱之一端各有繫于本力相稱之斤兩，如戊與己。若再加之斤兩，則兩柱必不能當而墜斷矣。題曰：甲乙柱厚面之橫徑於丙丁柱寬面之橫徑加倍之尺寸若干，則戊之斤兩于己之斤兩加倍若干。解曰，甲乙柱厚面之橫徑與丙丁柱寬面之橫徑如五與一。因而若己之重一百斤，則戊之重五百斤矣。

Again there are two rectangular columns, see figure 20, [column] AB [and] CD. And [as for] the thick face of [column] AB and the wide face of [column] CD, these two faces are parallel to the horizon. At the end of each of the two columns is separately tied a weight equivalent to the [column's maximum] original force, as in E and F. If more *weight* is added, then the two columns will definitely not be able to bear [it] and will break and fall. Proposition: if the *transverse diameter* of the thick face of column AB [in relation] to the *transverse diameter* of the wide face of column CD is multiplied by a certain measure, then the *weight* of E [in relation] to the *weight* of F is [also] multiplied by the [same] measure. Explanation: the *transverse diameter* of the thick face of column AB to the *transverse diameter* of the wide face of column CD is 5 to 1. Thus, if the weight at F [is] 100 *jin*, then the weight at E [is] 500 *jin*.

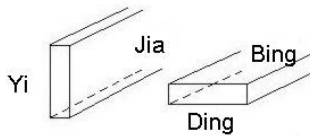


Figure 9

The third proposition states that for a rectangular column fixed to a wall at its one end the ratio of the maximum weights it can bear standing on edge to the maximum weights it can bear lying flat equals the ratio of the width to the thickness. This proposition is found as the second proposition in Galileo's treatment of the strength of materials.

The fourth and the fifth propositions are contained in the following text [see Fig. 4 and its illustration Fig. 10 and Fig. 5 and its illustration Fig. 11, respectively]:

有兩柱，見二十一圖甲乙丙丁、戊己庚壬。其長短等，其粗細不等。其粗柱之堅固與細柱之堅固，有己壬之橫徑與乙丁之橫徑三加之比例。如乙丁有己壬三分之一，而細柱之堅固能當三千斤，則粗柱之堅固能當八萬一千斤。

There are two columns, see figure 21, ABCD [and] EFGI (*wu ji geng* 庚 *ren*). Their *length* (*changduan*, 長短) is equal, [and] their *thickness* (*cuxi*, 粗細) is not equal. [The ratio of] the strength of the thick column to the strength of the thin column is the triplicate ratio of the *transverse diameter* of FI to the *transverse diameter* of BD. For example, [if] BD is one third of FI, and the strength of the thin column can bear 3000 *jin*, then the strength of the thick column can bear eighty-one thousand *jin*.

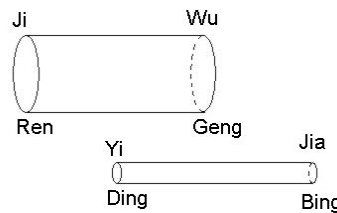


Figure 10

因此而推圓柱之長應加若干之尺寸。以知其不能當本體之重，以知其橫繫于空中時，若釘此一端於壁，則彼一端自弱而重，垂下必橫斷矣。如甲乙柱，見二十二圖，橫懸於空中。其長徑五尺，於地平線平行。其本體之重有六百斤。若再加一千斤之重，繫在于丁，則圓柱墜斷。今（球）[求]應加若干尺寸，以知其自垂而斷之處。依本法之理以論之，若于本柱加一丈五尺，共得二丈，則本柱不能當本體之重，自垂而橫斷矣。總而論之，甲乙柱之斤兩與本柱之斤兩並其所繫于丁斤兩之加倍，如五尺與二丈一尺七寸之比例。今於二丈（乙）[一]尺七寸再加本柱之長五尺，而三倍之。其積數共得八丈零（乙）[一]寸。若此數並五尺之數中，取中比例數，得二丈即所求甲乙柱之尺寸矣。從圓或方柱之理可推他類；從五金之柱形可推他形并材料。又筋系藤等繩堅固之力同一比例之理。

Relying on this, [one can] deduce how much *length* should be added to the length of the circular column. Knowing that [the column] is unable to bear the weight of [its] own body, [and] knowing that when it is tied horizontally in the air, if one end is nailed to the wall, then the other end will itself be weak and heavy, [and] will sag and definitely break transversely. For example, column AB, see figure 22, hangs horizontally in the air. Its lengthwise diameter [is] 5 *chi*, [and it] parallels the horizon. The weight of its own body is 600 *jin*. If a weight of 1000 *jin* is added, tied at *ding*, then the circular column will break and fall. Now, [let us] seek how much *length* should be added [to the column], [in order] to know the place where it will itself sag and break. Relying on the principle of this method and discussing it [further], if to the original column is added [a length of] 1 *zhang* 丈 5

chi, for a total [length] of 2 *zhang*, then the column will not be able to bear the weight of [its] own body, [and] will sag and break transversely. To sum up, [the ratio of] the *weight* of column AB to the sum of its own *weight* plus two times of the *weight* tied at D, is proportional to the ratio of five *chi* to two *zhang* one *chi* seven *cun*. Now add two *zhang* one *chi* seven *cun* to the column's original length of five *chi*, and triple it. Its product comes to a total of eight *zhang* one *cun*. If 5 *chi* is added to this number, [and] the mean proportion [of the sum] is taken, [then] resulting two *zhang* is the sought length of the column AB. From the principles of circular and rectangular columns can be deduced other types; from the shapes of columns [made] of the *five metals* can be deduced other shapes and materials. Moreover, the force of strength of cords [made] of tendon, rope, hemp, and other materials [follows] the same principle of ratios.

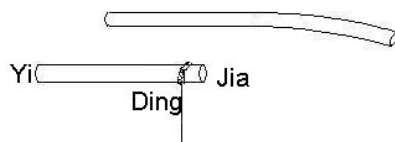


Figure11

The two propositions concern columns that are horizontally fixed to a wall under two different conditions. In the first case the diameter of the column to which a weight is attached is varied, in the second case the length is varied. In the second case the length of the column is increased to its maximum at which the column breaks due to its own weight. These propositions correspond to propositions four and propositions three and ten in Galileo's treatment of the strength of materials.

After this brief review of the contents of the five propositions, we can summarize their structure, as shown in the following table.

Proposition 1	Proposition 2	Proposition 3, 4, 5
upright strings or columns	upright columns ← horizontal columns	horizontal columns

4. Conclusion

As we have seen, *Yi Xiang Zhi*, which is commonly regarded as a book on astronomical instruments, contains some knowledge about mechanics and machines. The mechanical knowledge of materials is mainly introduced in a section whose title *The principles for the strength of new instruments* gives readers the impression that it discusses problems about the strength of instruments. Actually, however, what is mostly presented in this section are basic propositions of Galileo's theory of the strength of materials as it was developed in the First Day and the Second Day of the *Discorsi*. Although it is unclear whether Verbiest really used this mechanical knowledge and its methods in order to design parts of instruments, his work of introducing this kind of knowledge remains a significant achievement. The theoretical mechanical knowledge about the strength of materials and the methods for calculating this strength were completely new to Chinese readers. Although ancient Chinese were good at workmanship in constructing and engineering and the workmanship had a long history, Chinese people rarely measured the strength of materials or studied the relationship between their strengths and the size of objects. Chinese craftsmen always did their work on the basis of experiences, which were handed down from generation to generation. In contrast to the Chinese tradition, Verbiest's presentation of mechanical knowledge concerning the strength of materials was an intellectual outcome of the European Renaissance. It was new to Chinese people. Furthermore, due to the detailed explanation of propositions on the mechanical knowledge concerning the strength of materials, the instruments constructed by Verbiest were seemingly constructed on more reasonable grounds than the Chinese

instruments. Verbiest could thus hope that they would be highly estimated and thereby improve his career in China.

As to the origin of the knowledge presented in the section, the first proposition possibly comes from the beginning of the Galileo's *Discorsi*, and the second, the third and the fourth proposition separately come from the first, the second and the fourth proposition in the Second Day of *Discorsi*, while the fifth proposition probably comes from both the third and the tenth proposition in the Second Day. However, the question whether the mechanical knowledge of materials introduced by Verbiest comes directly from Galileo's *Discorsi* or via some intermediate source available to Verbiest still needs further study. It is also possible that Verbiest read books or texts rewritten or recompiled on the basis of Galileo's work by other Jesuits.

The theory of the strength of materials and the methods of calculation in the five propositions of the section discussed in this article, which combine the theory of the strength of materials with practical aspects of instrument making, should have been taken up by Chinese scholars and circulated among them. Actually, however, the number of readers of *Yi Xiang Zhi* was very limited and the excellent Chinese tradition in workmanship was sufficient for practical construction and engineering, so that theoretical knowledge about the strength of materials was not required urgently and hardly any Chinese scholar did further research on the topic at that time. Thus, the theoretical knowledge about the strength of materials introduced by Verbiest was ignored by Chinese readers and was seldom mentioned by other Jesuits until the next wave of introduction of European knowledge in the latter half of the nineteenth century.

Western Surveying in 17th Century China and Japan

FUNG Kam-Wing

1. Jesuit's Language Policy and Cultural Accommodation

When Europeans began to span the global world by regular sailing routes, Dominican and Franciscan Friars as well as Jesuit Fathers had already followed the path to preach Christianity firstly in the South Asia, South East Asia and then in the Far East.¹ The Society of Jesus recognized that to become a competent user of the local language was the most fundamental tool of communication in their ministry. Therefore, the Constitutions of the Society of Jesus, completed in 1552, suggested that every Jesuit missionary should realise the duty of his in “endeavouring to learn the vernacular language well” as an essential preparation.² In his January 14, 1549 letter sent from Cochin to Ignatius de Loyola (1491-1556), General of the Society of Jesus, St. Francis Xavier (1506-1552) commended his colleague Henrique Henriques (1520-1600) because of his mastery of written and spoken Tamil he was able to work alone with great profit and “his sermons and private conversations have made him a marvellous object of love and veneration to the native Christians.”³ When Alessandro Valignano (1539-1606) landed in Goa in 1574 as the Visitor of the Indian and Far East Provinces, he implemented a definite language policy as follows: “The desirability of having versions of the scriptures in various native tongues, as well as native catechisms and other Christian literatures for the masses.”⁴ In a letter of February 7, 1583 sent from Sciaochin 肇慶 (Zhaoqing, northern Guangdong 廣東) to the Society General Claudio Acquaviva (1543-1615), Michele Ruggieri (1543-1607) writes:

I have translated the catechism. ...I have learnt the court language (lingua cortegiana) that Chinese people call mandarin(mandarina). But in Macao, people speak Cantonese that makes me difficult to practise. It is essential for me to go to the inland of China to practise.⁵

A co-worker of Ruggieri, Matteo Ricci (1552-1610) actively followed Valignano's strategy of “cultural accommodation”. Ricci's method of cultural accommodation was not an inflexible policy but an adaptation attitude developed on a trial-and-error basis. He observed that Chinese were a very intelligent and reasonable people and placed high priority on moral principles and ethical behaviour, and filled with admiration for science and technology.⁶

1 Joaquim Romero Magalhães, *The Portuguese in the 16th Century World : Areas and Products* (Lisboa, 1998), pp.61-84; K.W.Fung 馮錦榮, M. S. Yeung 楊文信, “Zhongguo Jiangnan diqu diming di yinyi wenti – yi Helan ren Joan Blaeu di *Xin Zhongguo ditu wei li* 中國江南地區地名的音譯問題——以荷蘭人 Joan Blaeu 的《新中國地圖》為例——” (The System of Chinese Place Names Transliteration : with reference to Dutch Cartographer Joan Blaeu's *Novus Atlas Sinensis*), in *Zhongguo yuyan xuebao* 中國語言學報 (Journal of Chinese Linguistics, Beijing), Vol.10 (2001), pp. 176-186.

2 Ignatius Loyola, *The Constitutions of the Society of Jesus* (Translated by George E. Ganss; St. Louis : Institute of Jesuit Sources, 1970), n. 402.

3 Francis Xavier, “Carta de Xavier A S. Inácio de Loyola, Cochim, 14 de Janeiro de 1549”, *Epistolae S. Francisci Xaverii aliaque eius scripta* (Edited by George Schurhammer, S.J. and Joseph Wicki, S.J., Romae: Monumenta Historica Soc.Iesu, 1944) II, pp. 20-28; Henry James Coleridge, *The Life and Letters of St. Francis Xavier* (London: Burns and Oates, Portman Street, 1874), Vol.2, pp.67-75; Ines G. Zupanov, *Jesuit Experiments and Brahmanical Knowledge in 17th Century South India* (Delhi: Oxford University Press, 1999), pp.10-25.

4 Josef Franz Schutte, *Valignano's Mission Principles for Japan* (Translated by John J. Coyne, S.J., St. Louis: Institute of Jesuit Sources, 1980), p. 51, Point 31; Konrad Schilling, *Das Schulwesen der Jesuiten in Japan (1551-1614)* [Münster: Druck der Regensbergischen Buchdruckerei, 1931], pp.13-18; J. E. Moran, *The Japanese and the Jesuits: Alessandro Valignano in Sixteenth-century Japan* (London: Routledge, 1993; p. 145), pp.134-135; Edward J. Malatesta, “Alessandro Valignano, Fan Li-An (1539-1606): Strategist of the Jesuit Mission in China”, *Review of Culture* (Macao), No. 21 (1994), pp. 35-54.

5 Michele Ruggieri, “Al P. Claudio Acquaviva Prep. Gen. S.I. a Roma”(Sciaochin, 7 febbraio 1583), P. Pietro Tacchi Venturi S.I. ed. *Opere Storiche del P. Matteo Ricci S.I.* (Macerata: Premiato Stabilimento Tipografico, 1913), Vol. 2, pp. 410-419; Albert Chan 陳綸緒 S.J., “Two Chinese Poems written by Hsü Wei 徐渭 (1521-1593) on Michele Ruggieri, S.J.”, *Monumenta Serica*, Vol. 44 (1996), pp. 317-337.

6 Matteo Ricci, “A Giambattista Roman a Macao” (Sciaochin, 13 settembre 1584), *Opere Storiche del P. Matteo Ricci S.I.*, Vol. 2, pp. 36-49.

Christophori Clavii Bambergensis Astrolabivm, (Romae, 1593, [Pé-táng Library, No.1291])

De Horologijs [Horologiorvm nova descriptio](Romae, 1599, [Pé-táng Library, No.1302])

Compendivm brevissimvm describendorvm Horologiorvm Horizontalium ac Declinantium (Romae, 1603, [Pé-táng Library , No.1294])

In 1603, Ricci completed the Chinese translation of Clavius' *Calendrivm Gregorianvm* (Romae, 1603, [Pé-táng Library, No.1306]).

Matteo Ricci's collection also very likely included Austrian astronomer Georg von Peurbach (1423-1461) and Johannes Regiomontanus (1436-1476) co-authored *Epitome in Almagestum Ptolomei* (Venice, Joh. Hamman de Landoia, 1496) and *Novae theoricæ planetarvm, Georgij Purbachij, Germani...* (Coloniae Agrippinae, apud Haeredes Arnoldi Bircmanni, 1581).¹²

It is worthy to note here that when Ricci presented himself before Ming Emperor Shenzong 神宗 (r. 1572-1620) in January 1601, he wrote a memorial to the throne stating as follows: "(In my country,) the making of instruments and observation of heavenly bodies as well as examination of shadows by sundial, all are tallied with traditional Chinese ways (*yu Zhongguo gufa wenhe* 與中國古法脗合)."¹³

3. Chinese Researches on Western Surveying during the Ming-Qing Transition

China has a long tradition in practising the art of land survey. From the descriptions in *Zhou li* 周禮 and *Wujing zongyao* 武經總要 as well as surviving pictures, artifacts, we know about the principal surveying instruments employed by the ancient Chinese, namely, sighting or reference poles, *biao* 表; the set-square, *ju* 矩; the plumb line, *xian* 縣; the water level, *zhun* 準; and ropes and cords, which were later replaced by measuring tapes, *bu che* 步車.

Ming scholar-official Xu Guangqi's 徐光啟 (1562-1633, baptized Paul[Paolo] in 1603 by João de Rocha [1566-1623]) first encounter with Christianity was his meeting with Italian Jesuit Lazzaro Cattaneo (1560-1640) at Shaozhou in 1596.¹⁴ In 1600, he met Ricci in Nanjing 南京. Xu appeared to be acquainted with Western surveying from the Jesuits in the years between 1600 and 1603. In an essay entitled *Liangsuan hegong ji ceyan dishifa* 量算河工及測驗地勢法 (Methods of measuring the distance on an inaccessible point along the river and topographical measurement, 1603) submitted to Liu Yiguang 劉一燝 (*jinshi* 1595), the county magistrate of Shanghai, Xu discussed various problems in traditional Chinese surveying and Western surveying for the repair of local waterways.¹⁵ On traditional Chinese surveying, Xu mentioned various methods such as "*gougu liangshen fa* 勾股量深法" (method measuring depth by *gougu* [the right angle triangle ; base and altitude]theory) and "*zhongju zhongbiao gouguceiliang fa* 重矩重表勾股測量法" (surveying method using double set-squares, double sighting or reference poles by *gougu* theory). And on Western surveying, Xu translated the newly introduced Western geometric-square or quadrant in Chinese term as *judu* 矩度.¹⁶ He also

12 See K. W. Fung, "Li Madou jiu cangshu zhong ti xifang keji shuji zai Lingnan diqu ti liubo 利瑪竇舊藏書中的西方科技書籍在嶺南地區的流播" (The Transmission of Western Scientific Books in the Library Collection of Matteo Ricci in 16th and 17th Century Guangdong), Zhao Chunchen 趙春晨, He Dajin 何大進, Leng Dong 冷東 eds., "中西文化交流與嶺南社會變遷" (East and West Cultural Exchanges and Social Transformation in South China)[Beijing: Chinese Social Sciences Press, 2004], pp. 616-627; idem, "The Transmission of Georg von Peurbach's *Theoricæ novae planetarum* in Sixteenth and Seventeenth Century China", Orchiston, W., Stephenson, R., Débarbat, S., and Nha, I.-S. eds., *Astronomical Instruments and Archives From the Asia-Pacific Region* (Seoul: Yonsei University Press, 2004), pp. 81-86.

13 黃伯祿 (Pierre Hoang, 1830-1909) ed., *Zhengjiao fengbao* 正教奉褒 (Zi-Ka-Wei, Chang-hai: Catholic Mission Press, 1904), pp.4-5.

14 Fang Hao 方豪 (1910-1980), *Zhongguo Tianzhujiao shi renwu zhuan* 中國天主教史人物傳 (Biographies of the History of Chinese Catholic)[Beijing: Zhonghua shuju, 1988 reprint], p.102; Wang Zhongmin 王重民 (1903-1975) ed., *Xu Guangqi ji* 徐光啟集 (The Collected Works of Xu Guangqi)[Shanghai: Shanghai guji chubanshe, 1984], p. 4.

15 Wang Zhongmin 王重民 (1903-1975) ed., *Xu Guangqi ji* 徐光啟集 (The Collected Works of Xu Guangqi)[Shanghai: Shanghai guji chubanshe, 1984], pp.57-62.

16 The diagram of a quadrant appeared in Spanish Friar Fray Juan Cobo's (1546?-1592) *Bian zheng jiao zhen chuan shi lu* 辯正教真傳實錄 (Apologia de la Verdadera Religion, Manila, 1593) but without specific Chinese translation. See Fidel Villarreal, O.P. ed. *Pien Cheng-Chiao Chen-Ch'uan Shih-Lu: Apologia de la Verdadera Religion por Juan Cobo*, O.P. Manila, 1593 (Introduction by Alberto Santamaria, O.P., Manila: Universidad de Santo Tomás

illustrated how to use a geometric-square to find the width of a river by using “*diping ce yuan fa* 地平測遠法” (method to find the horizontal distance between two points) and to read the shadow scales *zhijing* 直景 (Latin: *umbra recta*). In 1607, Xu and Ricci jointly translated Clavius’s *Euclidis elementorum libri XV* (Romae, 1591; Pé-táng Library, Nos.1297,1298) into Chinese under the title *Jihe yuanben* 幾何原本.¹⁷ One year later, they translated “Vol. 3 Examples of rectangular-linear-dimensions by suspended, stable, stationary but moved Geometric Quadrant [Liber III “ Earundem rectorum linearum dimensione per Quadratum Geometricum tum pendulum, tum stabile , etiam per unicum stationem agit] ” of Clavius’s *Geometriae Practicae* (Romae, 1604; Pé-táng Library , No.1300) into Chinese with the title *Celiang fayi* 測量法義 (Principles of Mensuration; published by Shi Jichang 施繼昌 in the years between 1608 and 1609).[Figures. 1a, 1b, 2a, 2b, 2c] In *Celiang fayi*, the main content includes: (1) Method of construction of the geometric-square; (2) Conversion of shadow scales between *zhijing* and *daojing* 倒景 (Latin: *umbra versa*) [*umbra recta ac versa in quot partes in hoc opere dividi*] and (3) Fifteen mathematical examples of application of geometric- square or quadrant surveying. For the construction of a portable quadrant or geometric square, it normally involves three basic features for its use as a surveying instrument: (1) two pinnule-bearing sighting vanes fixed to one of its sides, (2) a plumb bob or line suspended from the intersection of its two sides, and (3) the division of its quarter arc limb into 90°. Apart from these features, a shadow square was generally included and it might be inscribed within the arc of the geometric square, or the arc inscribed within the square. The shadow square comprised a square with two adjacent sides divided into equal parts (i.e. shadow scales) and was used to measure ratios (which were commonly regarded as tangents or co-tangents, expressed as fractions) rather than angles in degrees. The scale measuring ratios (tangents) from 0/12 to 12/12 (angles from 0 to 45°) is the “*umbra versa*” (Arabic: *al-mankūs* ; Latin: *umbra versa* , *conversa* , back or contrary shadow); that measuring ratios from 12/12 to 12/0 (angles from 45° to 90°) the “*umbra recta*” (Arabic: *al-zill al-basīt*; Latin: *umbra recta*, *umbra extensa*, direct or right shadow). The common corner of the undivided sides carried either the pivot of an alidade or the suspension point of a plumb line.¹⁸

In the prologue to the *Celiang fayi* , Xu gives an account of Chinese *gougu* , a system in conformity with Euclidean system in terms of mensuration:

“ Concerning that particular (Western mensuration) method, is this different from *gougu* system of mensuration in the *Zhoubi suanjing* 周髀算經 and *Jiuzhang suanshu* 九章算術 (Nine Chapters on the Mathematical Art)?” “They are not different.” “If (they are) not different, why is it so important?” “(It is) because of its principle. (People) like Liu Hui 劉徽 (3rd century A.D.) and Shen Gua 沈括 (1031-1095) have already discussed about (*gougu*) mensuration, they explain (the theory

Press, 1986), p. 263. On Fray Juan Cobo, see Fray Juan Cobo, *El Libro Chino Beng Sim Po Cam* 明心寶鑑; *Espejo Rico del Claro Corazon* (Edición e Introducción del P. Luis G. Alonso Getino [1877-?], Madrid : Imprenta del Asilo de Huérfanos del S. C. de Jesús , 1924), pp.1-20; Edwin Wolf, “Doctrina Christiana : An Introductory Essay”, *Doctrina Christiana: The First Book Printed in the Philippines*, Manila 1593 (Facsimile edition, Washington D.C.: Library of Congress, 1947), pp.1-50; Fidel Villarroya, O.P. ed. *Pien Cheng-Chiao Chen-Ch’uan Shih-Lu: Apologia de la Verdadera Religion por Juan Cobo*, O.P. Manila, 1593 (Introduction by Alberto Santamaria, O.P., Manila: Universidad de Santo Tomás Press, 1986), pp.1-46, 263; Francisco Antolin (1745-1796), *Noticia delos infieles igorrotos en lo interior de la Isla de Manila, de sus minas de oro, cobre y su comercio...* (translated by William Henry Scott; Manila : University of Santo Tomas Press, 1988), pp. 30-38. On brief introduction of *Bian zheng jiao zhen chuan shi lu* José Antonio Cervera Jiménez, “Spanish Friars in the Far East: Fray Juan Cobo and His Book *Shi Lu*”, *Historia Scientiarum* (Tokyo) Vol. 7, No.3 (1998), pp.181-198; K. W. Fung, “Chugoku chishikijin no seiyo sokuryogaku kenkyu -- Minmatsu kara shinmatsu ni o ke ru 中國知識人の西洋測量學研究 -- 明末から清末における” (Chinese Researches on the Western Science of Surveying during the Ming-Qing Transition), Hazama Naoki 狭間直樹 ed., *Seiyo kindai bunmei to Chuka sekai* -- Kyoto Daigaku Jinbun Kagaku Kenkyujo nanaju shunen kinen shinpojium ronshu 西洋近代文明と中華世界 -- 京都大學人文科學研究所 70 周年記念シンポジウム論集 (*Modern Western Civilization and the Universe of China: Proceedings of International Symposium in Celebration of the Seventieth Anniversary of the Institute for Research in Humanities*, Kyoto University) [Kyoto: Kyoto University Press, Feb.,2001], pp. 354-373.

17 Peter M. Engelfriet, *Euclid in China: the Genesis of the first Chinese Translation of Euclid’s Elements Books I-VI (Jihe yuanben; Beijing, 1607) and its Reception up to 1723* (Leiden: Brill, 1998), pp.132-138.

18 Edmond R. Kiely, *Surveying Instruments* (New York: Bureau of Publications, Teachers College, Columbia University, 1947), pp.75-83; Nan L. Hahn, “Medieval Mensuration : *Quadrans Ventus* and *Geometrie Due Sunt Partes Principales...*”, *Transactions of the American Philosophical Society*, Vol.72, Part 8 (1982), pp. 1-204, esp. Introduction, pp. ix-Ixxxv.

of) a single gnomon but not double gnomons ... And there is no application.” “If it is the case, wouldn’t it be that those (aforementioned) people are unable to discuss, or even Li Shou 隸首 and Shang Gao 商高 are also not able to discuss? Is the *Zhou bi suanjing* not suggesting application?” “It is not just the application, there should be elaboration among applications. The *Jihe yuanben* is resourceful in application and unlimited in elaboration Extending the (Western mensuration) methods to the urgent issues like renovation of water control and agriculture would certainly be bring about great benefits, therefore, I put it in the first place.”¹⁹

It seems that Ricci also orally taught Li Zhizao 李之藻(1565-1630, baptized Leon in 1610) to improve that translated text and accordingly re-titled as “*celiang sanlü fa* 測量三率法” and incorporated into Chapter six of *Tongwen suanzhi tongbian* 同文算指通編 (Extended Chapters for the Integrated Manual of Calculation, 1614).²⁰ [Figure 1c] Xu then further compared similarities and differences in terms of theory, methods and instruments between Chinese *gougu* surveying and Western geometric-square surveying, and along this line he completed *Celiang yitong* 測量異同 (1608) and *Gougu yi* 勾股義 (1609). In the preface to the *Gougu yi*, Xu Guangqi again emphasized the practical use of surveying. He admired the contribution of the Great Yu 大禹 who employed the surveying method to bring the floods under control and survey the territory of China. He stated that learning Chinese and Western surveying systems, and of an integration of the two (*yu yi tong bian* 於以通變), could serve the practical purposes, such as taming the rivers in the Northwest and repairing water works in the Southeast, to relieve the urgent needs.²¹

In 1630, the second year of the Calendar Reform (1629-1635), Xu worked with Italian Jesuit Jacobus Rho(1592-1638) to adapt Giovanni Antonio Magini’s (1555-1617) *De Planis Triangulis* (Venetiis, 1592; Pé-táng Library , No.2149) and *Trigonometria Sphaericorum, et Astronomica, Gnomonica, Geographica* (Bononiae, 1609; Pé-táng Library , No.2150), Clavius’s *Geometriae Practicae*, Tycho Brahe’s (1546-1601) *Astronomiae Instauratae Progymnasmata* (Prague Bohemiae, 1602; Pé-táng Library , No.1602) and translated them into Chinese under the title *Celiang quanyi* 測量全義 (Complete Meanings of [Western] Surveying, ten *juans* 卷). [Figure 1d] This newly translated work was presented to Emperor Sizong 思宗(r.1628-1644) in 1631 as part of the *Chongzhen lishu* 崇禎曆書 (Chongzhen Reign Period Treatises on Astronomy and Calendrical Science, 1634).²²

In any case, Xu Guangqi’s view on Western surveying , particularly the related issues on “Technological synthesis or integration East and West (*huitong Zhongxi* 會通中西)”, “Western Learning originated from China (*Xixue yuan chu Zhongguoshuo* 西學源出中國說)” and “The Discussion of Practical Learning(*shixue* 實學)”, influenced his contemporaries.²³ From then on, Chinese

19 Shanghai Cultural Relics Conservation Committee ed., *Xu Guangqi zhuyi ji* 徐光啟著譯集 (The Collected Writings and Translated Works of Xu Guangqi) [Shanghai: Shanghai guji chubenshe, 1983], Vol.8, *Celiang fayi*, pp.1a-1b.

20 K. W. Fung, “Christopher Clavius (1538-1612) and Li Zhizao 李之藻 (1565-1630)”, p.155.

21 *Xu Guangqi zhuyi ji*, Vol.8, *Gougu yi*, pp.1a-3a.

22 Bai Shangshu 白尚恕, “*Celiang quanyi di ben wenti chutan* 《測量全義》底本問題初探” (A Preliminary Investigation to the Original Copy of *Celiang quanyi*), *Ke xue shi ji kan* 科學史集刊(Beijing), No.11(1984), pp.143-159; Christopher Clavius, *Geometriae Practicae* (Romae: Ex Typographia Aloisij Zannetti, 1604) Liber III “*Earundem rectorum linearum dimensione per Quadratum Geometricum tum pendulum, tum stabile, etiam per unicum stationem agit*”, p.16. On personal correspondence between Clavius and Magini, see Ugo Baldini , Pier Daniele Napolitani edited, *Christoph Clavius: Corrispondenza* (Pisa: Università di Pisa, 1992) , Volume II (1570-1592), Parte I: Lettere e testi , pp.126-127. On Calendar Reform during Chongzhen Reign Period, the definitive study is Hashimoto Keizo, *Hsü Kuang-Ch’I and Astronomical Reform: The Process of the Chinese Acceptance of Western Astronomy 1629-1635* (Osaka: Kansai University Press, 1988); K.W. Fung, “Mingmo Qingchu shidafu dui Chongzhen lishu zhiyangjiu 明末清初士大夫對《崇禎曆書》之研究” (Scholar-officials’ Study on *Chongzhen lishu* [Chongzhen Reign Period Treatises on Astronomy and Calendrical Science] during the Ming- Qing Transition), *Bulletin of Ming-Qing Studies* 明清史集刊 (Hong Kong), Vol.3 (1997), pp.145-198, esp. pp.151-161.

23 On the issues of *huitong Zhongxi* and *Xixue yuan chu Zhongguoshuo*, see K.W.Fung, “Mingmo Qingchu Fang shi xuepai zhi chengli ji qi zhuzhang 明末清初方氏學派之成立及其主張” (The Formation of the School of the Fang Family and their view at the End of Ming and Beginning of Qing Dynasties), Yamada Keiji 山田慶兒 ed., *Chugoku kodai kagaku shiron* 中國古代科學史論 (Researches on History of Ancient Chinese Science)[Kyoto: the Institute for Research in Humanities, Kyoto University, 1989], pp.139-219; *Idem.*, “Ming mo Xiong Mingyu fuzi yu Xixue 明末熊明遇父子與西學” (On Xiong Mingyu and his son’s Western Learning in the Late Ming Period), Lau Pin-min 羅炳綿, Lau Kin-ming 劉健明 ed., *Ming mo Qing chu Huanan diqu lishi renwu gongye yantaohui lunwenji* 明末清初華南地區歷史人物功業研討會論文集(*Proceedings of the Conference on the Achievements of Historical Figures in South China during the Late Ming and Early Qing*)[Hong Kong: History Department, Chinese University of Hong

researches on Western surveying and its instrumentation began to flourish in the Qing period. Here I would like to discuss three prominent figures, namely, Chen Jinmo, Fang Zhongtong and Chen Xu, so as to examine the process of reception of Western Learning during the early Qing period.

(1) Chen Jinmo 陳蓋謨(1600?- 1688?):

Chen Jinmo was Huang Daozhou's 黃道周 (1585-1646) student and was acquainted with Fang Yizhi 方以智 (1611-1671). Chen was one of the early readers of Huang Daozhou's *San yi dongji* 三易洞璣 (The Three [Levels of] the Yi as Astronomical Instruments, 1629) and expressed his views on the Book of Changes with Huang through letters. *San yi dongji* consists of elaborate, abstract correlations among *Yijing* 易經 hexagram lines, calendrical units, harmonic measures, and astronomical formations based on Huang's understanding of ancient divinatory methods as recorded in pre-Qin texts. In his earlier work *Yi benxiang* 易本象 (The Real Images in *Yijing*, 1609), Huang had already displayed his unique temporal schema (*linian* 歷年) based upon sixty-four hexagrams about history in the past and in the future.²⁴ In 1634, Chen collected his correspondences with Huang on the Book of Changes and astronomical studies under the title *Suan qian* 石肅庵槩 (Correspondences of [Chen] Suan) and published a celestial cartographical book entitled *Xiang lin* 象林 (The Copse of Heavenly Bodies), in which Chen examined those celestial degrees of constellations being cited in the *Sanyi dongji*. Apart from Master Huang, Chen is likely to have received intellectual influences from Xu Guangqi's works such as *Celiang fayi* and *Gougu-yi*. Chen conducted research on the integration of Chinese surveying with the related Western studies and published his third book under the title *Du ce* 度測 (Investigation on [Shadow] Scales, 2 *juans*) in 1640. [Figure 1e] In the section of “*quanqi* 詮器” (Explanation of instrument) in Chapter one, Chen stated that the European mensuration system in the *Celiang fayi* was advanced but it was basically a development of the *gougu* theory in the *Zhou bi suanjing*. He argued that *judu*, the geometric square, might be re-named *heju* 合矩 because the geometric square was composed of two set-squares that already mentioned in the *Zhou bi suanjing*. He also designed a similar geometric square with a suspended pointer (*xuanzhen* 懸針) and called it *Suanjudu* 石肅庵矩度.²⁵

(2) Fang Zhongtong 方中通(1634-1698):

Fang Zhongtong was a famous mathematician in the Late Ming and Early Qing period. The second son of Fang Yizhi, a great thinker and Ming loyalist who was acquainted with German Jesuit Johann Adam Schall von Bell (1597-1666). It is significant to note here that Fang Yizhi and his father Fang Kongzhao 方孔炤 (1591- 1655) were experts on *Yijing* learning. They jointly composed the *Zhouyi shilun hebian* 周易時論合編 (Combined Commentaries on Critique of Time in Yi Learning, published in 1660). In fact they were strong supporters of the view that the Chinese, in ancient times, had been quite advanced in their exploration of exact sciences, but this preoccupation was regrettably taken over by the West in their times. They emphasized the concept of “time” in the Book of Changes and held a

Kong, 1993], pp.117-135; *Idem.*, “You Yi ji qi *Tianjing huowen qianhouji* 游藝及其《天經或問前後集》” (You Yi and his *Tianjing Huowen Qian houji* [The Former and Latter Parts of Queries on the Classics of Heaven]), Wang Yusheng 王渝生 ed., *Di qi jie Zhongguo kexueshi huiyi wenji* 第七屆中國科學史會議文集 (*The Colloquia of the 7th International Conference on the History of Science in China*) (Zhengzhou: Daxiang chubenshe, 1999), pp.286-301; Wang Yangzong 王揚宗, “Ming mo Qing chu ‘Xixue Zhongyuan’ shuo xinkao 明末清初 ‘西學中源’ 說新考”, Liu Dun 劉鈍, Han Qi 韓琦 ed., *Ke shi xin zhuan* 科史薪傳 (The Torch of Learning in History of Science and Technology) (Shenyang: Liaoning Jiaoyu chubenshe, 1997), pp.71- 83. On the issue of “shixue”, Ogawa Haruhisa 小川晴久, “Jitsu gaku gainen ni tsu i te 實學概念について”, *Nihon chugoku gakkai ho* 日本中國學會報, Vol.33 (1981), pp.131-138; *Idem.*, “Xixue yu Rujiao shixue di xianjiedian 西學與儒教實學的銜接點” (A Conjunctive Point of Western Learning and Confucian Practical Learning), Ge Rongjin 葛榮晉 ed., *Zhong Ri shixue shi yanjiu* 中日實學史研究 (Chinese and Japanese Scholarship on the Studies of *Shixue* History) [Beijing: Chinese Academy of Social Sciences Press, 1992], pp.247-263; K. W. Fung, “Fang Yizhi yu *Sanpu Meiyuan* 方以智與三浦梅園” (Fang Yizhi and Miura Baien), *Journal of Oriental Studies* 東方文化 (Hong Kong), Vol. 33, No.2 (1995), pp.230-257.

24 K. W. Fung, “Min matsu ni okeru Ekigaku no tenkai—Ko Do-shu no *Ekishosei* o meguute 明末における易學の展開—黃道周の『易象正』をめぐって” (The Development of *Yijing* Studies in Late Ming —concerning the *Yixiang zheng* of Huang Daozhou), *Chugoku Shisoshi Kenkyu* 中國思想史研究 (Kyoto), No.12(1989), pp. 29-61.

25 Chen Jinmo, *Du ce* (Manuscript copied in the Late Ming and Early Qing period), *juan* 1, pp.18a-19b. Also see K. W. Fung, “Mingmo Qingchu Chen Jinmo zhi Xixue 明末清初陳蓋謨(1600?-1688?)之西學” (On Chen Jinmo's (1600?-1688?) Western Learning during the Ming and Qing Transition) a paper presented in “二十一世紀中國學術研究前瞻國際研討會” (International Conference on “Chinese Studies in the 21st Century : A New Vision”) to celebrate the 90th anniversary of the University of Hong Kong, January 17-19, 2001, organized by Department of Chinese, the University of Hong Kong, 15pp.

balanced view to adopt advantages of Western Learning from all quarters. In the “*fanli* 凡例” (Introduction) of *Zhouyi shilun hebian*, they held an opinion that referring to the advocated by Shao Yong’s 邵雍(1011-1077), i.e. the cycle (*yuan* 元, 12 *hui*, 1296000 years)-epoch (*hui* 會, 30 *yun*, 10800 years)-revolution (*yun* 運, 12 *shi*, 360 years)- generation (*shi* 世, 30 years). They offered the following account:

Concerning our cycle, the sage king Yao 堯 is corresponding to the end (*mo* 末) of *si* 巳 epoch; the times of Duke Zhou 周公 and Confucius is corresponding to the beginning (*chu* 初) of the *wu* 午 epoch; our present times is in the mid of the *wu* epoch (*zhengwu* 正午), thus all the doctrines (or teachings)[*wanfa* 萬法] are manifested in their beauty.²⁶

Fang Kongzhao had keen interest in Western Learning. He had chance to read the unpublished *Chongzhen lishu* and wrote a synopsis on it under the title of *Chongzhen lishu yue* 崇禎曆書約 (A Summary of *Chongzhen lishu*). In the preface to the *Chongzhen lishu yue*, Fang Kongzhao highly praised Matteo Ricci and his introduction of European astronomy (including Aristotelian concept of concentric heavens and the round earth) and other sciences belonged to “*zhice* 質測” (material investigation). He even cited Confucius’ saying, “I have heard that, when the officers of the son of Heaven are not properly arranged, we may learn from the wild tribes all round about.” (*Zi yue tianzi shiguan xue zai siyi* 子曰天子失官學在四夷), which is recorded in the seventeenth year of Duke Zhao 昭公 in *Zuozhuan* 左傳.²⁷ The story of Confucius learning from a barbarian, the Viscount of Tan 鄆, became an influential concept in the perspective of learning Western sciences during the Ming-Qing Transition. And Fang Yizhi has this to say in the General Discourse of his *Wuli xiaozhi* 物理小識 (Little notes on the principles of the phenomena):

Using the Far West (*yuanxi* 遠西) as a Viscount of Tan, extending the Great Yu and Duke Zhou’s 周公 set-square mensuration.²⁸

Having been influenced by his father and grandfather, Fang Zhongtong showed his great interest in studying Western mathematics under the supervision of Polish Jesuit Johannes Nickolaus Smogulecki (1610-1656), who was probably a Copernican, in Nanjing during the period 1649 -1653. Fang Zhongtong’s prose writings and poems also recorded the episode of the introduction of Copernican heliocentric theory to Chinese intellectual circle under Smogulecki’s direction. When Fang Zhongtong visited Beijing and met Adam Schall von Bell in 1659, he wrote a poem entitled *Yu Xiyang Tang daowei xiansheng lun lifa* 與西洋湯道未先生論曆法 (A Discussion of Western calendrical sciences with Jesuit missionary Adam Schall von Bell). The poem tells us that Fang Zhongtong learned Keplerian method on Mars triangulation movement from Smogulecki and he discussed it with Adam Schall von Bell.²⁹

His encyclopedic mathematical work *Shudu yan* 數度衍 (Development of Calculations and Measure, 26 juans, 1687) was basically a synthesis between traditional Chinese mathematics and Western mathematics. [Figure 1h] Careful comparison between *Shudu yan* and major Jesuit translated works, for instances, *Tianxue chuhan* 天學初函 (First Collection of Writings on Learning from Heaven, 1628), *Chongzhen lishu* and its revised edition *Xiyang xinfa lishu* 西洋新法曆書 (Western New Method Treatises on Astronomy and Calendrical Science, 1645), reveals the obvious fact that John Napier’s (1550-1617) rods or bones and Galileo Galilei’s (1564-1642) proportional or sector compasses had been used by Fang Zhongtong and these instruments were widely researched by Fang’s friends who were scientists but of anti-Manchurian stand.

In the introductory section of “*fanli*” of his *Shudu yan*, Fang gave an account of his idea on traditional Chinese mathematics and Western mathematics:

26 Fang Kongzhao and Fang Yizhi, *Zhouyi shilun hebian* (1660 published edition, 1980 reprint), “Zhouyi tuxiang jibiao 周易圖象幾表” (Diagrams and Tables of Image-and-Number in Yi Learning), “*fanli*”, pp.1a-1b.

27 Fang Kongzhao and Fang Yizhi, *Zhouyi shilun hebian*, “Zhouyi tuxiang jibiao”, *juan* 7, pp.1a-2a.

28 Fang Yizhi, *Wuli xiaozhi* (published in 1664), p.1a.

29 K. W. Fung, “Fang Zhongtong ji qi Shudu yan – jian lun Ming Qing zhi ji Nabaier、Gebaini、Kaipule、Jialiliu zhi lisuan zuopin zai Hua liubo di qingxing – 方中通及其《數度衍》——兼論明清之際納白爾、哥白尼、開普勒、伽利略等之曆算作品在華流播的情形——” (Fang Zhongtong and his *Shudu yan*: with special reference to the transmission of scientific works by John Napier, Nicolaus Copernicus, Johannes Kepler and Galileo Galilei in the Ming-Qing Transition), *Lun Heng* 論衡 (Hong Kong) Vol.2 (1995) No.1, pp.123-204.

That Western mathematics is the most refined, is only because China has lost its (mathematical) tradition. Now, I have intended to bring Western Learning back to the *Jiuzhang* , and to bring back the *Jiuzhang* to the *Zhoubi*. The *Zhoubi* only discusses the *gougu*, but the *Jiuzhang* has entirely come forth from *gougu*. Therefore I have placed the chapter on *gougu* at the beginning.³⁰

Fang’s conception of “Bringing Western Learning back to the *Jiuzhang*” (*Xixue gui Jiuzhang* 西學歸九章) is probably inherited from Fang Kongzhao’s idea. In the preface to the *Chongzhen lishu yue*, Fang Kongzhao writes as follows:

Those (astronomical) researches (introduced by Matteo Ricci) were already discussed by Chinese sages and intellectuals in ancient times. In general , simple and creative idea started from very earliest time and it developed in details. (Traditional Chinese mathematics such as Zu Chongzhi 祖冲之 [c.429-c.500] and Zu Geng 祖暅’s *zhuanshu* 畝術(the art of measuring geometrical shapes), *zhuishu* 綴術(the “stitching” method) and other methods all originated from *gougu*.³¹

In the section of “*qice* 器測 (Investigation on instrument):*judu*” under the topic of *Gougu* in Chapter 7 of *Shudu yan*, Fang Zhongtong not only examined the structure of an geometric square, but also worked out a convenient and illustrative account of “the conversion of *daojing* into *zhijing*”(*daojing bian zhijing tushuo* 倒景變直景圖說). The lengthy description is tabulated as follows:³² [Figure 2f]

Daojing	11°	10°	9°	8°	7°	6°	5°	4°	3°	2°	1°
Zhijing	13°1'	14°4'	16°	18°	20°5'7"	24°	28°8'	36°	48°	72°	144°

His significant discourse on quadrant or geometric square surveying was later cited by Mei Wending 梅文鼎 (1633-1721) in an unpublished manuscript entitled *San jia celiang he ding* 三家測量合訂 (A bound volume of Textbook on Surveying by the Three Great Mathematicians) preserved in Momijiyama Bunko 紅葉山文庫 Library in Tokyo, Japan.³³

(3) Chen Xu 陳訐(styled Yanyang 言揚, 1650-1732):

Chen Xu learned traditional Chinese mathematical computation such as methods of roots extraction and Western surveying from Huang Zongxi 黃宗羲 (1601-1695), the renowned Ming scholar and loyalist, in 1676. By 1677, Chen wrote his first mathematical book entitled *Kaifang faming* 開方發明 (Innovation on Roots Extraction). Thereafter, he was enlightened by Huang Zongxi’s advice and his son Huang Baijia’s 黃百家 (1643-1709) *Gougu juce jieyuan* 句股矩測解原 (Explanations Illustrating Problems on *Gougu* and *Judu* Mensuration, 1678?) [Figures 1f, 2d], he then wrote his surveying books with a balanced view on both East -West theory and practice , namely, *Gougu shu* 句股述 (Narration on *Gougu* [Mensuration], 2 *juans*, 1678) [Figures 1g, 2e], *Juce* 矩測 (*Judu* Mensuration ,1679) and *Gougu yinmeng* 句股引蒙(The *Gougu* Primer, 5 *juans*, 1722).³⁴ In 1679, Huang Zongxi wrote a preface for Chen’s *Gougu shu*, which provides some information about Huang’s attitudes towards Western mathematics and surveying. Huang writes as follows:

30 Fang Zhongtong, *Shudu yan* (Revised edition, 1890), “fali”, p.1a.

31 Fang Kongzhao and Fang Yizhi , *Zhouyi shilun hebian*, “Zhouyi tuxiang jibiao”, *juan* 7, pp.1a-2a; K.W. Fung, “Ming mo Qing chu shidafu dui Chongzhen lishu zhiyangjiu 明末清初士大夫對《崇禎曆書》之研究” (Scholar-officials’ Studies on *Chongzhen lishu* during the Ming-Qing Transition), *Bulletin of Ming-Qing Studies* 明清史集刊 (Hong Kong), Vol.3 (1997), pp.145-198.

32 Fang Zhongtong, *Shudu yan*, *juan* 7, p.13b.

33 Kobayashi Tatsuhiko 小林龍彦, “Momijiyama Bunko ni shuzosareru Bei buntei no chosaku ni tsu i te 紅葉山文庫に收藏される梅文鼎の著作について” (On Mei Wending’s Work in Momijiyama Bunko Library), *Kagakushi Kenkyu* 科學史研究 (Journal of history of science)[Tokyo], Vol.41 (No.221) [Spring 2005], pp.26-34.

34 K. W. Fung, “ Qing chu Xiyang celiangxue di fazhan – yi Chen Xu ji qi Gougu Shu wei li 清初西洋測量學的發展——以陳訐(1650-1732)及其《句股述》為例——” (The Development of Western Surveying in the Early Qing Period: with special reference to Chen Xu [1650-1732] and his *Gougu shu*), in Sin Chow Yiu 單周堯, Lee Cheuk Yin 李焯然 and Wong Yoon Wah 王潤華 eds., *Dongxi wenhua chengchuan yu chuanguangxin* 東西方文化承傳與創新 (East-West Studies: Tradition, Transformation and Innovation: A Festschrift in Honour of Professor Chiu Ling Yeong on the Occasion of his Retirement from the Chair of Chinese, The University of Hong Kong)[Singapore: The Centre for the Arts, National University of Singapore and Global Publishing Co. Ltd., 2004], pp.136- 160.; *idem.*, “Min matsu shin sho ni okeru Ko Hyaaka no shogai to chosaku 明末清初における黄百家の生涯と著作” (Huang Baijia [1643-1709]: His Life and Works), *Chugoku Shisoshi Kenkyu*, No. 20 (1997), pp.61-92.

The *gougu* learning, culminating in the “inscribed circle” (*rongyuan* 容圓), the “measurement of circles” (*ceyuan* 測圓) and the “cutting of the circle” (*geyuan* 割圓), are all inherited art of the Duke of Zhou and Shang Gao. It was one of the Six Arts. Thereafter, scholars ceased to discuss it. Practitioners of occultism (*fangji jia* 方伎家) appropriated it...Chen Yanyang from Haichang 海昌, on the basis of one word of mine, gave up work and composed a book on *gougu*. As to the abstract numbers and the abstract principles: one by one he has made them manifest. ... Subsequently, I refrained from paying attention to it. Now, on the instigation of Yanyang I had as yet to complete my former work, in order to be able to hand it over to him in its entirety. Yanyang has extended it further, and moreover, he made the Westerners return to us our “fields of Wenyang” (*yi shi Xiren gui wo Wenyangzhi tian* 亦使西人歸我汶陽之田).³⁵

The expression “fields of Wenyang” is an allusion to “*Qiren gui wo Wenyangzhi tian* 齊人歸我汶陽之田” in the second year of Duke Cheng 成公 in *Zuozhuan* 左傳. Perhaps this expression meant that the Westerners had occupied the “fields of mathematics”, which had now been returned to China. Chen Xu later extended Huang’s idea in a balanced view in his *Gougu yinmeng*:

Mathematics was one of the Six Arts. *Gougu* was a chapter in *Jiuzhang suanshu*. The concepts of “*ji* 積” and “*mi* 冪” in the *Zhoubi suanjing* in the ancient time and nowadays concepts of “*sanjiao* 三角” and “*baxian* 八線”, all are methods of *gougu*. *Gougu* method is the origin of all methods of surveying ... Concerning mensuration, Western methods have been published and in fact, they are not different from Chinese methods. ...But for the making of instrument, Western method is recommendable.³⁶

In *Gougu shu*, Chen Xu also paid considerable concern on conversion of *zhijing* and *daojing* scales. Chen had discussed this issue with Huang Baijia who was known to have acquired the skill of using Western surveying method to draw topographical map with contours. He received Huang Baijia’s critique on calculation of “Measuring altitude by double geometric squares” and incorporated Huang’s comment in *juan 3* of *Gougu yinmeng*.³⁷

4. Japanese Confucian Mathematicians and Surveying

In 1605, after reading Ming mathematician Ke Shangqian 柯尚遷’s *Quli waiji bu xue li liuyi fulu Shuxue tonggui* 曲禮外集補學禮六藝附錄數學通軌 (Rules of Mathematics, Appendix to Outer Chapter of Minutiae of Rites for supplementing with Learning Six Arts of Rites, 1578 Ming edition), the influential Tokugawa 德川 Confucian scholar Fujiwara Seika 藤原惺窩 (1561-1619) wrote his disciple Hayashi Razan 林羅山 (1583-1657) a private letter re-emphasizing the relation between the traditional Confucian six arts arising from the *Zhouli* 周禮 (namely, *li* 禮 [propriety], *yue* 樂 [music], *she* 射 [archery], *yu* 御 [chariotting], *shu* 書 [writing] and *shu* 數 [mathematics]) and the significance of learning mathematics among Confucian scholars.³⁸

Japanese surveying in the seventeenth century was derived from two main streams. One is known as *choken jutsu* 町見術 (methods for measuring the length of a town), based on ancient Chinese mathematical treatises such as *Zhou bi suanjing* 周髀算經 and *Jiuzhang suanshu* 九章算術, dating from the 7th century; the other is known as “*kiku jutsu* 規矩術” (methods using “carpenter’s ruler” or “carpenter’s squire” for surveying) which was introduced by missionaries or foreigners in the seventeenth century.³⁹ The actual technique was a synthesis of the two traditions.

35 Chen Xu, *Gougu shu* (1683 published edition), pp.1a-1b.

36 Chen Xu, *Gougu yinmeng* (Wenyuan ge Siku quanshu 文淵閣四庫全書, 1772-1782, Taipei, 1984 photo-facsimile), the introductory “*fanli*”, pp.1a-2b.

37 Ibid, *juan 3*, p. 31a; K. W. Fung, “Min matsu shin sho ni okeru Ko Hyaaka no shogai to chosaku”, pp.72-73, 83.

38 Abe Yoshio 阿部吉雄 (1905-) ed., *Nihon no Shushigaku* 日本の朱子學 (Chu Hsi Confucianism in Japan) [Tokyo: Meitoku Shuppansha, 1975], Vol.2, pp.114-115. On the discussion of Confucian six arts and mathematics in Early China, see K. W. Fung, “Kongzi yu Zhongguo keji wenhua 孔子與中國科技文化” (Confucius and Chinese Scientific Culture), International Confucian Association ed., *Guojì rúxué yánjiū* 國際儒學研究 (Beijing, Chinese Academy of Social Sciences), Vol.4 (1998), pp. 249-264.

39 Mikami Yoshio 三上義夫 (1875-1950), *Nihon sokuryo jutsu shi no kenkyū* 日本測量術史の研究 (Studies of the History of Japanese Surveying) [Tokyo: Koseisha Koseikaku, 1948], pp.1-4; Oya Shinichi 大矢真一 (1907-), “sokuryo 測量” (Surveying), in Yajima Suketoshi 矢島祐利 (1903-) ed., *Nihon kagaku gijutsushi* 日本科學技術史 (A History of Science and Technology in Japan) [Tokyo: Asahi Shinbunsha, 1962], pp.285-300; Matsuzaki Toshio 松

By the beginning of seventeenth century, Western surveying and related instruments such as quadrant, marine astrolabe had been brought to Japan by Jesuits through Nagasaki 長崎.⁴⁰ In Ikeda Koun 池田好運's (fl.ca. 1616-1636) *Genna kkaiki* 元和航海記 (Book of the Art of Navigation in the Genna Era, 1618), he mentioned he was taught the art of navigation and use of marine astrolabe by a Portuguese captain Manuel Gonzalez (Gonçalves) in 1616 while he sailed to the Philippines with him for two years.⁴¹ Following the incident of burning Jesuit publications on 23 July 1626 at Nagasaki and the *sakoku* 鎖國 (national seclusion) policy adopted by the Tokugawa Shogunate in 1635, foreign ships such as Portuguese or Dutch ships would no longer be allowed to enter Japanese ports except Nagasaki, the introduction of Western surveying and related instruments were then stopped.⁴² However, as Tokugawa Shogunate gained political stability, exact land-surveying was pressing demand for map-making, constructing water-works, and building canals. It was until the legendary Dutch medical practitioner Caspar Schamberger (1623-1706) came to Japan in 1647 or 1648 and taught the art of Western surveying, particularly on “*bundo yo jutsu* 分度餘術” (techniques of protraction) and “*tenmon kikugenpo jutsu* 天文規矩元法術” (methods using “carpenter’s ruler” or “carpenter’s squire” for astronomical observation and land surveying), to Higuchi Gonemon 樋口権右衛門 (styled Kentei 謙貞, also known as Kobayashi Yoshinobu 小林謙貞; 1601- 1683), an astronomer at Nagasaki.⁴³ According to Hosoi Kotaku’s 細井広沢 (1658-1735) *Hiden chiiki zuho daizen* 秘傳地域圖法大全 (Complete Book of the Secret Art of Surveying and Mapping, 1717), he considered the Western surveying originated from the art of navigational pilot and it carried meaning of computation or calculation.⁴⁴ For that reason, various Western surveying instruments were introduced and employed in astronomical observational exercise and land-surveying practice, for instances:

watarante ワタランテ (Portuguese: quadrante; English: quadrant),

kuhadarantei クハダランテイ (Spanish: cuadrante; English: quadrant),

konpansu コンパス (Portuguese: compasso; Spanish: compass; English: compass),

isutarabiyo イスタラビヨ (イアトラヒ or 以亞太良比; Portuguese: astrolábio; Spanish: astrolabio; English: astrolabe)

kenban 量盤 (plane table).⁴⁵

As Higuchi Gonemon mastered the methods of Western surveying, Shimatani Sadanaga 島谷定重 (?-1690), Hirai Unsetsu 平井雲節, Yamasaki Kyuya 山崎休也 and Kanazawa Gyobuzaemon 金澤刑部左衛門 were studied under him.⁴⁶ Because of its wonderful ingenuity and convenience, many common people of Japan began to regard the methods of Western surveying as superstitious magic and as a result

崎利雄(1933-), *Edo jidai no sokuryojutsu* 江戸時代の測量術 (Surveying of the Edo Period) [Tokyo: Sogo Kagaku Shuppan, 1979], pp.5-12; Kawamura Hirota 川村博忠 (1935-), *Kinsei ezu to sokuryojutsu* 近世絵図と測量術 (Maps and Surveying in Pre-modern Japan) [Tokyo: Kokon Shoin, 1992], pp.55-59; Kazutaka Unno 海野一隆, “Cartography in Japan”, in J. B. Harley and David Woodward eds., *The History of Cartography Vol.2 Book 2 : Cartography in the Traditional East and Southeast Asian Societies* (Chicago & London: The University of Chicago Press, 1994), pp. 356-359; idem, *Chizu no bunkashi : sekai to nihon* 地図の文化史 : 世界と日本 (A Cultural History of Maps and Charts: The World and Japan) [Tokyo: Yasakashobo, 2004], pp.121-127.

40 Mikami Yoshio, *Nihon sokuryo jutsu shi no kenkyu*, pp.1-2; Matsuzaki Toshio, *Edo jidai no sokuryojutsu*, pp.13-18; Kawamura Hirota, *Kinsei ezu to sokuryojutsu*, pp.59-62.

41 Pagès Léon (1814-1886), *Histoire de la religion chrétienne au Japon depuis 1598 jusqu'à 1651, comprenant les faits relatifs aux deux cent cinq martyrs béatifiés le 7 juillet 1867* (Paris: C. Douniol, 1869-1870), Tome 1, pp.389.

42 J. E. Moran, *The Japanese and the Jesuits: Alessandro Valignano in Sixteenth-century Japan*, p.145; Takamura Tsuko 中村土, “Edo jidai no Tenmon no sokuryo giki 江戸時代の天文の測量儀器” (Astronomical and Land-survey Instruments of the Edo Period), *Kagakushi Kenkyu* (Tokyo), Vol.44 (No.234) [Summer 2005], pp.102-105.

43 Mikami Yoshio, *Nihon sokuryo jutsu shi no kenkyu*, pp.10-11; Kawamura Hirota, *Kinsei ezu to sokuryojutsu*, pp.59-61; Kazutaka Unno, “Cartography in Japan”, in J. B. Harley and David Woodward eds., *The History of Cartography Vol.2 Book 2 : Cartography in the Traditional East and Southeast Asian Societies*, pp. 393-394; Wolfgang Michel, *Von Leipzig nach Japan : der Chirurg und Handelsmann Caspar Schamberger (1623-1706)* [München: Iudicium, 1999], pp.45- 50.

44 Kawamura Hirota, *Kinsei ezu to sokuryojutsu*, p.63.

45 Kawamura Hirota, *Kinsei ezu to sokuryojutsu*, pp.87-88.

46 Mikami Yoshio, *Nihon sokuryo jutsu shi no kenkyu*, p.16.

teaching Western surveying publicly was prohibited by the Shogunate.⁴⁷ From then on, transmission of Western surveying among Japanese intellectuals became in secret and its development was considerably hindered. Kanazawa Gyobuzaemon handed down methods of Western surveying to his son, Kanazawa Seizaemon 金澤清左衛門(1624-1684) who taught it to his younger brother Kanazawa Kanemon 金澤勘右衛門(?-1691), and he in turn passed it on to Shimizu Sadanori 清水貞徳(1645-1717).⁴⁸ It is worthy to note here that after a great and destructive fire in Edo in the period of 18th –19th of the first month of the third year of Meireki 明曆 Era (1657), a large portion of the Edo City was greatly damaged and a large number of people lost their lives, Kanazawa Seizaemon was then summoned by the Shogunate and appointed as the chief surveyor to head the mapping campaign of Edo City.⁴⁹ Obviously, Kanazawa Seizaemon applied the methods of Western surveying in making the *Edo zu* 江戸圖 (Cartographical Map of Edo) which was completed in the later part of the second month of the same year.⁵⁰

As Shimizu Sadanori worried the “*kiku jutsu*” might be entirely lost because of the governmental prohibition, he intentionally arranged and enlarged what he had studied from Kanazawa Kanemon, and prepared a series of treatises entitled *Kiku genpo honden* 規矩元法本傳 (The Main Course of the Original Method of Western surveying), *Kiku genpo betsuden* 規矩元法別傳 (The Supplementary Course of the Original Method of Western surveying) and *Kiku genpo betsudeninka no maki* 規矩元法別傳自發の卷 (Further Elaboration of the Supplementary Course of the Original Method of Western surveying) respectively for his disciples.[Figure 3] Consequently, Shimizu Sadanori founded the “Shimizu School of Western surveying” in 1712. His methods of surveying employed different kinds of instruments such as compass, protractor, quadrant, measuring-pole, measuring-rope and leveling instrument.

Another mathematician Murai Masahiro 村井昌弘 (fl.1729-1754), also a follower of Higuchi’s surveying system but holding different views with Shimizu Sadanori, who published *Ryochi shinan* 量地指南 (A Guidance of Land Surveying) in 1733. In this volume, a number of instruments are illustrated, for instances, a plane table to be employed for both horizontal and vertical positions, a straight rule to be used as a simple alidade, a plumb for horizontal and vertical tests, a measuring-rope, a measuring-pole, a sign-post, a telescope, drawing-compasses and protractor for map-making.⁵¹ Murai Masahiro further summarized different schools of surveying can be characterized by techniques falling into five fundamental operational categories, namely, *kenban jutsu* 量(見)盤術, *banshin jutsu* 盤針術, *konbasu jutsu* 渾發術, *sankan jutsu* 算勘術, *kiten jutsu* 機轉術.⁵² Of these, the major two may be described as follows:

The first is known as *kenban jutsu* (lit., plane-table method) which used a board mounted on a tripod, to measure proximate and medial distances. This practical approach involved producing a scaled cartographical representation at the surveying site to deduce the actual distances. The other, known as *banshin jutsu* (lit., azimuth board-and-compass method), was based on an ancient Chinese tradition, utilizing magnets and an azimuth table. It was generally used for measuring distances of one *ri* 里 or more and for mapping relatively extensive areas.⁵³

5. Concluding Remarks

In the 17th -18th centuries, Chinese scholar-officials faced various challenges from the West. In order to strengthen the content of traditional Chinese science, Chinese scholar-officials made use of traditional thought arising from the *Zhoubi suanjing*, *Zuozhuan* and *Yijing* to reinforce their theoretical or philosophical fundamentals. Since Western surveying performed “practical values”, researches on surveying became a prevailing subject of the “Practical Learning” among Chinese scholar-officials. Apart from the afore-mentioned intellectuals, Mei Wending 梅文鼎 (1633-1721) and his friend Yang

47 Kawamura Hirotada, *Kinsei ezu to sokuryojutsu*, p.60.

48 Mikami Yoshio, *Nihon sokuryo jutsu shi no kenkyu*, p. 16; Kawamura Hirotada, *Kinsei ezu to sokuryojutsu*, p. 60.

49 Kawamura Hirotada, *Edo Bakufu sen kuni ezu no kenkyu* 江戸幕府撰國繪圖の研究 (A Study of Provincial Maps and General Maps of Edo Japan) [Tokyo: Kokon Shoin, 1984], pp.23-45.

50 Mikami Yoshio, *Nihon sokuryo jutsu shi no kenkyu*, pp. 37-41.

51 Murai Masahiro, *Ryochi shinan* 量地指南 (Oya Shinichi ed.; Tokyo: Kowa Shuppan, 1978 facsimile edition), maki 卷1, pp.13b-20b; Oya Shinichi, “sokuryo 測量”, in Yajima Suketoshi ed., *Nihon kagaku gijutsushi*, pp. 289-300.

52 Murai Masahiro, *Ryochi shinan*, Preface, pp.4a-4b.

53 Kawamura Hirotada, *Kinsei ezu to sokuryojutsu*, pp. 83-106.

Zuomei 楊作枚 had also made significant discourses on quadrant or geometric square surveying in their works. Evidential scholars like Li Rui 李銳(1769-1817), who was well-known in the *Yijing* learning and calendrical- mathematical sciences, had worked on surveying and problems related to isoperimetre figures in his unpublished diary *Guanmiao-ju ri ji* 觀妙居日記.⁵⁴

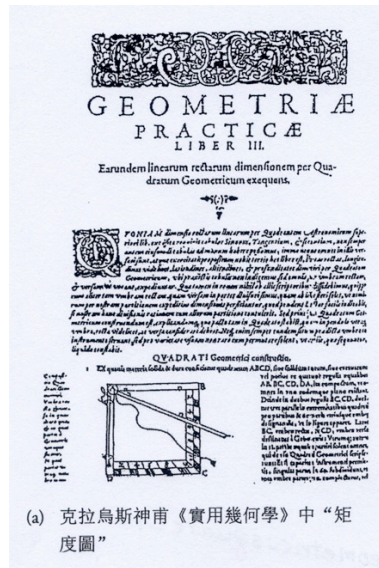
Likewise, Japanese Confucian mathematicians discussed the relation between the traditional Confucian six arts arising from the *Zhouli* 周禮 and the significance of learning mathematics. Some of them such as Hosoi Kotaku responded actively to the Western surveying and acknowledged the surveying techniques they had mastered had their roots in European navigation, of which was seldom addressed by their Chinese contemporaries. Other scholars such as Miyake Shosai 三宅尚齋(1662-1741) attempted to synthesize or elaborate the idea of Confucian value of learning mathematics with *Oranda-ryu chokenjutsu* 阿蘭陀流町見術 (Dutch School of surveying) and he even quoted Hosoi Kotaku's surveying work *Hiden chiiki zuho daizen* 秘傳地域圖法大全 (Complete Book of the Secret Art of Surveying and Mapping, 1717).⁵⁵

54 K. W. Fung, "Qianjia shiqi lisuanxuejia Lirui di shengping ji qi *Guanmiao-ju ri ji* 乾嘉時期曆算學家李銳(1769-1817)的生平及其《觀妙居日記》(A Confucian Mathematician :Li Rui's [1769-1817] Life and His Unpublished Diary *Guanmiao-ju ri ji*), *Zhongguo wen hua yan jiu suo xue bao* 中國文化研究所學報(Journal of Chinese Studies) [New Series], No.8 (1999), pp. 269-286.

55 Sato Kenichi 佐藤賢一, "Miyake Shosai to Oranda-ryu chokenjutsu 三宅尚齋と阿蘭陀流町見術" (Miyake Shosai and Dutch School of surveying), *Yogakushi Kenkyu* 洋学史研究 (Journal of the History of Western Learning), Vol.18 (2001), pp.23-51.

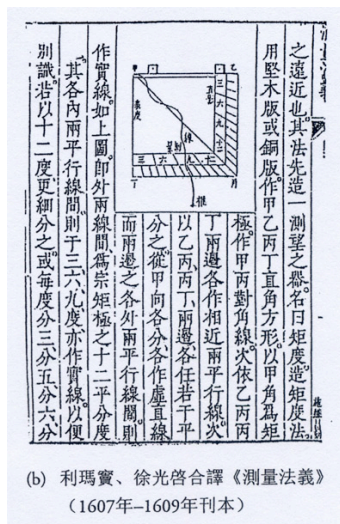
6. Figures

Fig.1: *Judu* 矩度 (Western geometric-square or quadrant) as seen amongst Western and Chinese works in the 17th centuries.



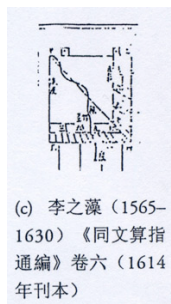
(a) 克拉烏斯神甫《實用幾何學》中“矩度圖”

Fig.1a: Clavius's *Geometriae Practicae* (Romae, 1604; Pé-táng Library , No.1300)



(b) 利瑪竇、徐光啓合譯《測量法義》 (1607年-1609年刊本)

Fig.1b: *Celiang fayi* 測量法義 (Principles of Mensuration; co-translated by Matteo Ricci and Xu Guangqi, and published by Shi Jichang 施繼昌 in the years between 1608 and 1609)



(c) 李之藻 (1565-1630) 《同文算指通編》卷六 (1614年刊本)

Fig.1c: Li Zhizao's *Tongwen suanzhi tongbian* 同文算指通編 (Extended Chapters for the Integrated Manual of Calculation, 1614)

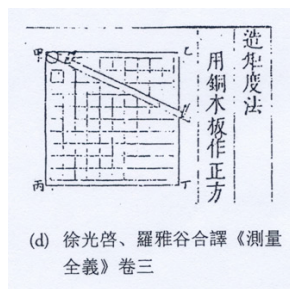


Fig.1d: *Celiang quanyi* 測量全義
 (Complete Meanings of [Western] Surveying; co- translated by Xu Guangqi and Jacobus Rho in 1630)

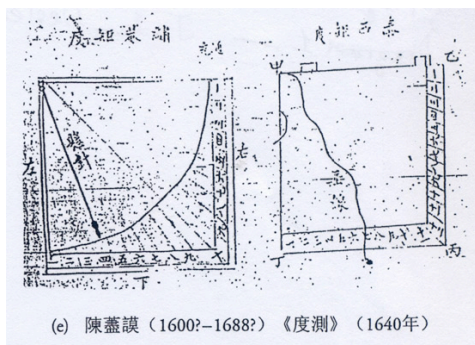


Fig.1e: Chen Jinmo's *Duce* 度測
 (Investigation on [Shadow] Scales, 1640)

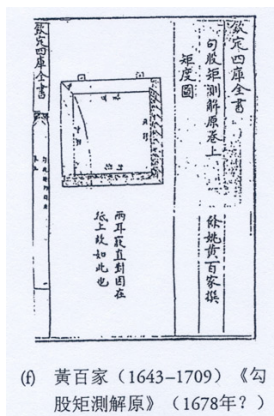


Fig.1f: Huang Baijia's *Gougu juce jieyuan* 勾股矩測解原
 (Explanations Illustrating Problems on Gougu and Judu Mensuration, 1678?)

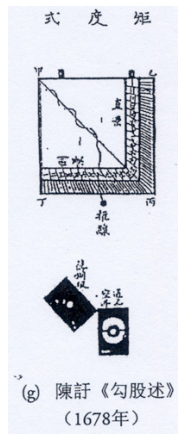


Fig.1g: Chen Xu's *Gougu shu* 句股述
(Narration on Gougu [Mensuration], 1678)

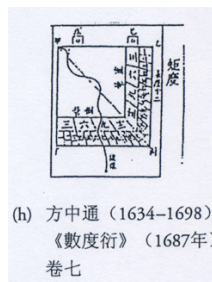


Fig 1h: Fang Zhongtong's *Shudu yan* 數度衍
(Development of Calculations and Measure, 1687)

Fig.2: An illustrative account of “The conversion of *daojing* into *zhijing*” (*daojing bian zhijing tushuo* 倒景變直景圖說) as seen amongst Western and Chinese works in the 17th centuries.

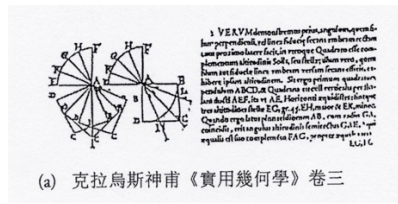


Fig.2a: Clavius’s *Geometriae Practicae* (Romae, 1604; Pé-táng Library , No.1300)

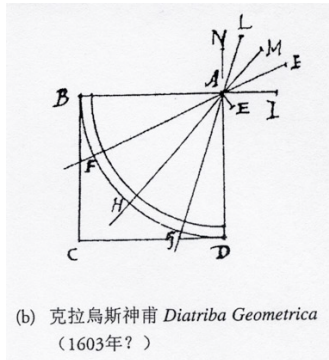


Fig.2b: Clavius’s *Diatriba Geometrica* (Romae, 1603)

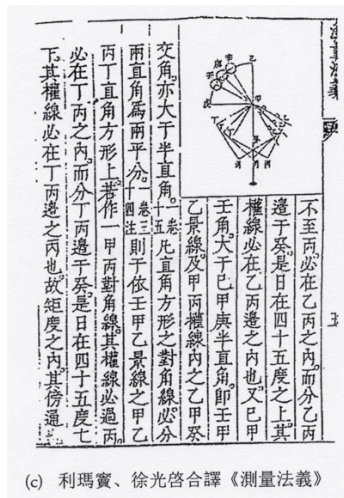


Fig.2c: *Celiang fayi* 測量法義 (Principles of Mensuration; co-translated by Matteo Ricci and Xu Guangqi)

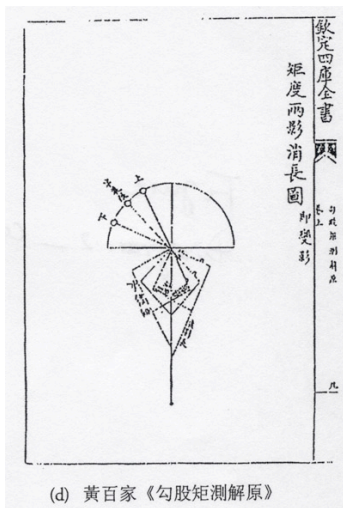


Fig.2d: Huang Baijia's *Gougu juce jieyuan* 句股矩測解原 (Explanations Illustrating Problems on *Gougu* and *Judu* Mensuration, 1678?)

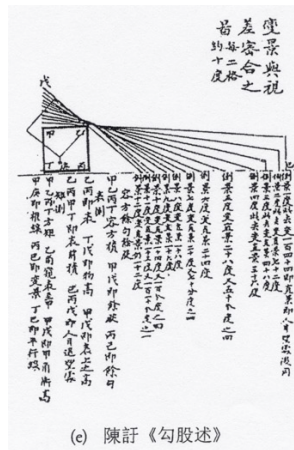


Fig.2e: Chen Xu's *Gougu shu* 句股述 (Narration on *Gougu* [Mensuration], 1678)

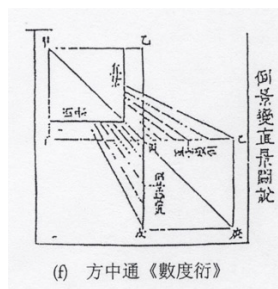
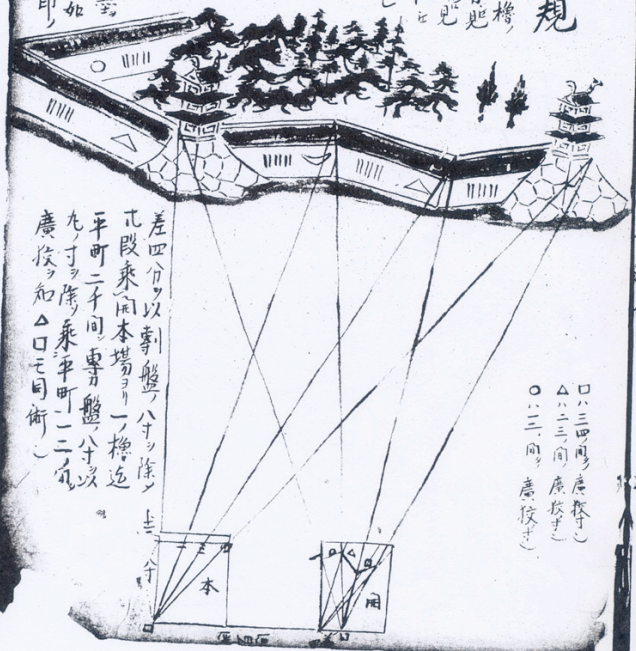


Fig 2f: Fang Zhongtong's *Shudu yan* 數度衍 (Development of Calculations and Measure, 1687)

城廣校扇規

如圖本塔盤居一塔
 角入赤二曲角也
 入墨引三曲角也
 入墨引四曲角也
 見入墨引盤
 二十九如圖割盤
 卷引一引
 盤八寸九真規
 展百回同塔
 發中二目返
 差四分也
 見返三見返四
 引二引合所未
 南校引口口印



差四分以割盤八寸陰
 正段乘同本塔引一引返
 平町二千回東引盤八寸以
 九寸乘平町二千
 廣校知口口同術

口三四同廣校
 △八三同廣校
 ○二三同廣校

The Knowledge about the Lever in 18th Century Chinese Mathematics

XIAO Yunhong

1. Introduction

In ancient Chinese mathematical works, the mechanical knowledge usually appeared in the form of computational problems. Starting with *Suanfa tongzong* 算法統宗 [General Source of Algorithmic], first published in 1592, computational problems concerning the equilibrium of the lever were included in the collections of problems in mathematical works. *Suanfa tongzong* 算法統宗 contains two problems of this kind. The first deals with a pig that is too heavy to be weighed by the available steelyard. Then, in the problem, another sliding weight is added to the original one and the pig is weighed again. The problem is to calculate the actual weight of the pig. In the second problem the sliding weight of a steelyard is lost and it is necessary to buy a new one of the same weight as the original, which is, however, unknown. Then, in the problem, an arbitrary sliding weight is used to weigh an object of known weight. The problem is to calculate the value of the original sliding weight from the result of the procedure.

From the 17th century on, Chinese theoretical knowledge about the lever can be attributed to two distinct traditions. The first is an indigenous Chinese tradition. It is represented by computational problems concerning the equilibrium of the lever that are found in mathematical works; the second is a Western tradition, represented by knowledge about the lever mainly introduced by *Yuanxi qiqi tushuo luzui* 遠西奇器圖說錄最 [The best Selection of Diagrams and Explanations of the Wonderful Machines of the Far West], first published in 1627. On the whole, the two traditions remained separate.

In the 18th century, Western knowledge concerning the lever again entered China. Chinese mathematicians began to pay attention to and study this knowledge. *Shuli jingyun* 數理精蘊 [Collected Basic Principles of Mathematics], first published in 1723, is a typical example.

Shuli jingyun 數理精蘊 was compiled and published during the reign of Emperor Kangxi 康熙. This work contains fifty three chapters (*juan* 卷). It occupies an important position in the history of mathematics during the Qing Dynasty. This work contains some important mechanical knowledge, including the knowledge concerning the lever. The characteristics of the knowledge of the lever in *Shuli jingyun* reflects the interaction of Chinese and Western theoretical knowledge on this topic. Although some modern scholars have paid attention to the mechanical knowledge in *Shuli jingyun*, the interaction between Chinese and Western knowledge in this work has remained largely unexplored.

2. Research Questions

In this paper, I address the following questions.

What knowledge about the lever is contained in *Shuli jingyun*?

What was the nature of the interaction between Western and traditional Chinese knowledge of the lever that is reflected in *Shuli jingyun*?

What was the influence of this interaction on the subsequent Chinese tradition?

3. The Knowledge about the Lever in *Shuli Jingyun*

The knowledge of the lever in *Shuli jingyun* appears in the form of computational problems concerning the equilibrium of the lever. There are seven problems of this kind in this work. The original texts of these problems and their translations are as follows:

1) (下編 卷九 綫部七 疊借互徵)

設如有石兩塊，大小不等，俱不知重數。只有銅條一根，重十二兩，互換稱之，而得二石之各重幾何。

法：先將銅條分作十二分，每分又作十分。用一繩繫於第五分之上（繫於五分者隨便取一數也），乃以五分加一倍。與十二分相較，餘二分，折半得一分。與五分相加為六分。乃以五分為一率，六分為二率，餘二分作二兩為三率（因銅條重十二兩，分為十二分，今二分故為二兩也），得四率二兩四錢（此四率是先將銅條之五分處取均平之法。蓋提繫在五分上，必於五分之端加二兩四錢，乃與七分相平也）

一率	五分
二率	六分
三率	二兩
四率	二兩四錢

爰以銅條作秤杆，將大石挂在銅條一頭，離提繫五分。而以小石作錘稱之，今離提繫得六分始平，記之（如前圖）。又將小石挂在銅條一頭，離提繫五分。而以大石作錘稱之，今離提繫得四分始平，亦記之（如後圖）。乃先借二十六兩四錢為大石衰數，與前所得二兩四錢相減，餘二十四兩（內減二兩四錢者，因銅條之五分一邊必加二兩四錢始平。今於借衰中減去者，所以補足均平之數，然後較物之輕重也）。用六分為一率（即小石在六分之數），五分為二率（即大石在五分之數），二十四兩為三率（即大石衰中減去二兩四錢所餘之數），得四率二十兩，為小石之衰數（此四率是以大石衰數求小石衰數）

一率	六分
二率	五分
三率	二十四兩
四率	二十兩

因以小石衰數二十兩與二兩四錢相減，餘十七兩六錢（此亦減去二兩四錢，因小石移在五分之一邊，補足均平之數也）。用四分為一率（即大石在四分之數），五分為二率（即小石在五分之數），十七兩六錢為三率（即小石衰中減去二兩四錢所餘之數），得四率二十二兩（此第二四率又以小石衰數轉求大石衰數，試其合否也）。

一率	四分
二率	五分
三率	十七兩六錢
四率	二十二兩

與所借大石衰數二十六兩四錢相較，則少四兩四錢。再借三十二兩四錢為大石衰數，與二兩四錢相減，餘三十兩。用六分為一率，五分為二率，三十兩為三率，得四率二十五兩，為小石之衰數。

一率	六分
二率	五分
三率	三十兩
四率	二十五兩

因以小石衰數二十五兩與二兩四錢相減，餘二十二兩六錢。用四分為一率，五分為二率，二十二兩六錢為三率，得四率二十八兩二錢五分。

一率	四分
二率	五分
三率	二十二兩六錢
四率	二十八兩二錢五分

與所借大石衰數三十二兩四錢相較，則少四兩一錢五分。乃將前借數二十六兩四錢，少四兩四錢書於右，後借數三十二兩四錢，少四兩一錢五分書於左。用兩不足法算之。於是兩少數相減，余二錢五分為一率。兩借數相減，餘六兩為二率。前借數與大石衰數相較之，少四兩四錢為三率，得四率一百零五兩六錢，加前借數二十六兩四錢，共一百三十二兩，即大石之重數。

三二四〇	三二四〇 二六四〇	二六四〇
	〇六〇〇	
少 四一五	四四〇 四一五	少 四四〇
	〇二五	

一率 二錢五分
 二率 六兩
 三率 四兩四錢
 四率 一百零五兩六錢

又於大石重數內減去二兩四錢，餘一百二十九兩六錢。用六分爲一率，五分爲二率（即前以大石衰數求小石衰數之法。既有大石真數，故仍以前法求小石真數），一百二十九兩六錢爲三率，得四率一百零八兩，爲小石之重數也。

一率 六分
 二率 五分
 三率 一百二十九兩六錢
 四率 一百零八兩

如以四分爲一率，五分爲二率（即前以小石求大石之重法）。於小石重數一百零八兩內減去二兩四錢，餘一百零五兩六錢爲三率，得四率一百三十二兩，爲大石之重數，亦合前數也。

一率 四分
 二率 五分
 三率 一百零五兩六錢
 四率 一百三十二兩

此法蓋因銅條重十二兩，而分作十二分。設如作一甲乙綫爲銅條，分作十二分，每分重一兩。提繫在丙處，甲丙與丙丁等，則其重亦必等。如以甲丁與甲乙相減，則余丁乙，即丙乙多於甲丙之二分也。既多二分，必重二兩。如以二兩重物掛於乙丁中間之戊處，則丙乙自重於甲丙也。今欲以物趨之，使其兩平。則以甲丙五分爲一率，丙戊六分爲二率，二兩爲三率，得四率二兩四錢。

一率 五分
 二率 六分
 三率 二兩
 四率 二兩四錢

是將二兩四錢之物加於甲處，始得兩平。其以丙戊六分爲二率者，何也？蓋丙丁與甲丙等，而重者止在丁乙一段。而戊爲丁乙之中。戊去丙遠，甲去丙近。惟近故加重而後可以勝遠之輕。若於甲接長二分，則於二分之一中，施二兩之物，即秤平矣。故以二兩四錢加於甲處，始能趨平丁乙之二分也。此法數層加減，幾用比例，頗覺繁瑣。而用方程算之，微覺簡明。但係疊借本法，故兩收之。收入疊借者，所以存其理。而收入方程者，所以取其簡也。

(1) (The Second Part Chapter IX Linear Section VII *diejie huzheng* 疊借互徵)

Suppose there are two lumps of stones unequal in size. The amount of both weights is unknown. There is only a copper bar, whose weight is twelve *liang* 兩.¹ (With the bar) (one) weighs one stone against the other and vice versa. Then (one) finds out how much the respective weight of the two stones is.

1 *Liang* 兩, *qian* 錢 and *fen* 分 are units of weight in ancient China. 1 *liang* is equal to 10 *qian*, and 1 *qian* is equal to 10 *fen*.

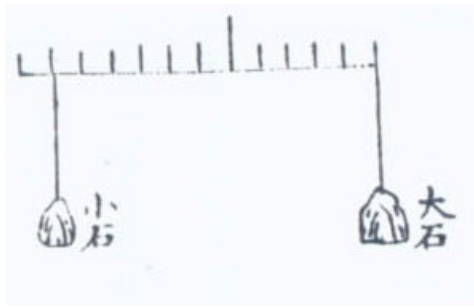


Figure 1
(Both figures taken from *Yu Zhi Shu Li Jing Yun*, Siku Quanshu edition)

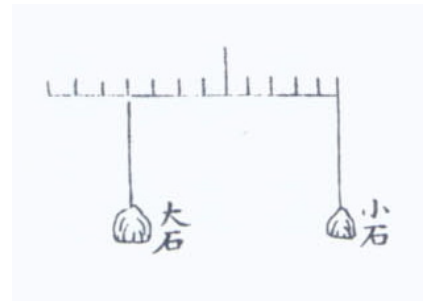


Figure 2

Method:

Firstly, divide the copper bar into twelve sections. Each section is furthermore divided into ten sections. Use a rope to be fastened (to the bar) at the fifth section (the fifth section at which the rope is fastened is an arbitrarily selected number). Then multiply five sections by two. Subtract (the product) from twelve sections, remaining two sections. (The remainder) reduced by half, getting one section. (One section) plus five sections are six sections. And then take five sections for the first proportional and six sections for the second proportional. The remainder two sections are treated as two *liang* and take it for the third proportional (As copper bar weighs twelve *liang* and is divided into twelve sections, now two sections are treated as two *liang*). (Then) get the fourth proportional of two *liang* four *qian* 錢 (Here the fourth proportional is for the time being a method for balance adopted at the fifth section on the copper bar. Because the lifting cord is at the fifth section, only by being added two *liang* four *qian* can the end of five sections be on a level with (the end) of seven sections).

The first proportional	five parts
The second proportional	six parts
The third proportional	two <i>liang</i>
The fourth proportional	two <i>liang</i> four <i>qian</i>

Then take the copper bar as the beam of a steelyard. Hang the big stone from the end of the copper bar that is five sections from the lifting cord. And take the small stone as sliding weight to weigh the big one. Now when (the small stone) is six sections from the lifting cord, (the bar) becomes level, which is indicated as the former figure. Furthermore, hang the small stone from the end of the copper bar that is five sections from the lifting cord. And take the big stone as sliding weight to weigh the small one. Now when (the big stone) is six sections from the lifting cord, (the bar) becomes level, which is also indicated as the latter figure. Then first borrow twenty-six *liang* four *qian* and take it as the *cuishu* 衰數 of the big stone. Subtract the previous result two *liang* four *qian* from it and the remainder is twenty-four *liang*. (Subtract two *liang* four *qian* from it because only by being added two *liang* four *qian* can the end of five sections be on a level (with the other end). Now what is subtracted from the borrowed *cui* makes up the number for balance, and then measure the weight of objects). Take six sections (namely six sections where the small stone is) for the first proportional; five sections (namely five sections where the big stone is) for the second proportional; and twenty-four *liang* (namely the remainder gotten when two *liang* four *qian* is subtracted from the *cui* of the big stone) for the third proportional. (Then) get the fourth proportional of twenty *liang*, which is the *cuishu* of the small stone (four proportionals are for finding the *cuishu* of the small stone by that of the big stone).

The first proportional	five parts
The second proportional	six parts
The third proportional	twenty-four <i>liang</i>
The fourth proportional	twenty <i>liang</i>

And then subtract two *liang* four *qian* from twenty *liang*, the *cuishu* of the small stone, and the remainder is seventeen *liang* six *qian*. (Here also subtract two *liang* four *qian* because the small stone is shifted to the side of five sections. (It is) the number to be made up for balance.) Take four sections (namely four sections where the big stone is) for the first proportional; five sections (namely five sections where the small stone is) for the second proportional; and seventeen *liang* six *qian* (namely the remainder gotten when two *liang* four *qian* is subtracted from the *cui* of the small stone) for the third proportional. (Then) get the fourth proportional of twenty-two *liang* (here the latter four proportionals are furthermore for conversely finding the *cuishu* of the big stone by that of the small stone and for checking up whether (they) are identical).

The first proportional	four parts
The second proportional	five parts
The third proportional	seventeen <i>liang</i> six <i>qian</i>
The fourth proportional	twenty-two <i>liang</i>

If (it is) compared to the borrowed *cuishu* of twenty-six *liang* four *qian* of the big stone, there is a deficit of four *liang* four *qian*. Once more borrow thirty-two *liang* four *qian* and take it as the *cuishu* of the big stone. Subtract two *liang* four *qian* from it and the remainder is thirty *liang*. Take six sections for the first proportional; five sections for the second proportional; and thirty *liang* for the third proportional. (Then) get the fourth proportional of twenty-five *liang*, which is the *cuishu* of the small stone.

The first proportional	six parts
The second proportional	five parts
The third proportional	thirty <i>liang</i>
The fourth proportional	twenty-five <i>liang</i>

And then subtract two *liang* four *qian* from the *cuishu* of twenty-five *liang* of the small stone and the remainder is twenty-two *liang* six *qian*. Take four sections for the first proportional; five sections for the second proportional; and twenty-two *liang* six *qian* for the third proportional. (Then) get the fourth proportional of twenty-eight *liang* two *qian* five *fen*.

The first proportional	four parts
The second proportional	five parts
The third proportional	twenty-two <i>liang</i> six <i>qian</i>
The fourth proportional	twenty-eight <i>liang</i> two <i>qian</i> five <i>fen</i>

If (the fourth proportional) compared to the borrowed *cuishu* of thirty-two *liang* four *qian* of the big stone, there is a deficit of four *liang* one *qian* five *fen*. Then write the former borrowed number of twenty-six *liang* four *qian* and the deficient number of four *liang* four *qian* on the right; the latter borrowed number of thirty-two *liang* four *qian* and the deficient number of four *liang* one *qian* five *fen* on the left. Apply the method of two deficiencies to calculate it. And then operate subtraction between the two deficient numbers, the remainder of two *qian* five *fen* taken for the first proportional; operate subtraction between the two borrowed numbers, the remainder of six *liang* taken for the second proportional. Compare the former borrowed number to the *cuishu* of the big stone, the deficit of four *liang* four *qian* taken for the third proportional. (Then) get the fourth proportional of one hundred and five *liang* six *qian*. Adding it to the former borrowed number of twenty-six *liang* four *qian*, that is, one hundred and thirty-two *liang* in all, namely the weight of the big stone.

3240	3240	2640
	2640	
<hr/>		
	600	
deficit		deficit
415	440	440
	415	
<hr/>		
	25	

The first proportional	two <i>qian</i> five <i>fen</i>
The second proportional	six <i>qian</i>
The third proportional	four <i>liang</i> four <i>qian</i>
The fourth proportional	one hundred and five <i>liang</i> six <i>qian</i>

Furthermore, subtract two *liang* four *qian* from the weight of the big stone and the remainder is one hundred and twenty-nine *liang* six *qian*. Take six sections for the first proportional; five sections for the second proportional (namely the former method of finding the *cuishu* of the small stone by that of the big stone. Since there is already the true number of the big stone, so still apply the former method to finding the true number of the small stone); (Take) one hundred and twenty-nine *liang* six *qian* for the third proportional. (Then) get the fourth proportional of one hundred and eight *liang*, which is the weight of the small stone.

The first proportional	six parts
The second proportional	five parts
The third proportional	one hundred and twenty-nine <i>liang</i> six <i>qian</i>
The fourth proportional	one hundred and eight <i>liang</i>

If taking four sections for the first proportional, five sections for the second proportional (namely the former method of finding the weight of the big stone by (that) of the small stone). From the weight of one hundred and eight *liang* of the small stone subtract two *liang* four *qian*, remaining one hundred and five *liang* six *qian* taken for the third proportional, (we) get the fourth of one hundred and thirty-two *liang*, which is the weight of the big stone. (It) also accords with the former number.

The first proportional	four parts
The second proportional	five parts
The third proportional	one hundred and five <i>liang</i> six <i>qian</i>
The fourth proportional	one hundred and thirty-two <i>liang</i>

This method (is applied) because the copper bar weighs twelve *liang* and is divided into twelve sections. Supposedly draw a line AB denoting the copper bar and divide it into twelve sections, the weight of each section being one *liang*. The lifting cord is at C and AC and CD are equal, so their weights are also equal. If operating subtraction between AD and AB, the remainder is DB, namely the two sections that CB is more than AC. Since (it is) two sections more, certainly (it is) two *liang* heavier. If a heavy object of two *liang* is hung on E between B and D, CB is of course heavier than AC. Now you want to match it with an object and make the two ends level. Then take AC of five sections for the first proportional; CE of six sections for the second proportional; two *liang* for the third proportional. (Then) get the fourth proportional of two *liang* four *qian*.

The first proportional	five parts
The second proportional	six parts
The third proportional	two <i>liang</i>
The fourth proportional	two <i>liang</i> four <i>qian</i>

It is when an object of two *liang* four *qian* is put on A that the two ends get level.

Take CE of six sections for the second proportional, why? It is because that (the weight) of CD is equal to (that) of AC and the heavier is only on the segment DB. E is the middle point of DB. E is far from C and A is close to C. It is because (A) is close that a weight is put on (it) and then it can bear the light of being further. If you extend two sections at A, when on the middle of (the segment) of two sections an object of two *liang* is set, the steelyard becomes level. Therefore, it is when two *liang* four *qian* are added to A that it can match with two sections of DB. This method needs layer upon layer of addition and subtraction and to apply proportion several times, it is felt rather overelaborate. However, if applying the equation to calculate it, it will be felt simple and clear slightly. But it is the original method of *diejie* 疊借, so both are collected. To collect *diejie* is to preserve its reason and to collect equation is to adopt its simplicity.

(2) (下篇 卷十 綫部八 方程)

設如有石二塊，大小不等，不知重數。只有銅條一根，重十二兩。均分十二分，以繩繫於第五分之上。一頭五分，一頭七分。將大石掛於銅條一頭，離提繫五分。而以小石作砣稱之，離提繫得六分始平。又將小石掛在銅條一頭，離提繫五分。而以大石作砣稱之，離提繫得四分始平。問大小二石各重幾何。

法：先以五分加一倍。與十二分相較，餘二分，折半得一分。與五分相加為六分。乃以五分為一率，六分為二率。餘二分作二兩為三率。得四率二兩四錢。即五分之端加二兩四錢始與七分相平也。爰將二兩四錢以大石離提繫五分因之，得十二兩，為五大石比六小石所多之數（大石離提繫五分小石離提繫六分而平，是大石重六分小石重五分也。若五大石六小石，則各得三十分其重始等。然五分之一端，應加二兩四錢，是大石重六分，尚多二兩四錢也。若五大石，則多十二兩矣。故為五大石比六小石多十二兩也）。

一率	五分
二率	六分
三率	二兩
四率	二兩四錢

又將二兩四錢，以小石離提繫五分因之，亦得十二兩，為四大石比五小石所少之數（小石離提繫五分，大石離提繫四分而平，是小石重四分，大石重五分也。若五小石四大石，則各得二十分其重始等。然五分之一端應加二兩四錢，是小石重四分尚多二兩四錢也。若五小石則多十二兩矣。故為五小石比四大石多十二兩。因以大石為首，故變為四大石比五小石少十二兩也）。

因作較數方程法算之。以大石五為正，小石六為負，重多十二兩為正，列於上。又大石四為正，小石五為負，重少十二兩為負列於下。乃以上大石五遍乘下大石四、小石五、少十二兩，得大石二十、小石二十五、少六十兩。又以下大石四遍乘上大石五、小石六、多十二兩，得大石二十、小石二十四、多四十八兩。兩下相較，則大石各二十，彼此減盡。小石兩層皆負，故相減餘一。重少六十兩與多四十八兩相加，得一百零八兩，即為一小石之重數。

大	小	重
五正	六負	一二正
四正	五負	一二負
二〇正	二五負	六〇負
二〇正	二四負	四八正

〇〇	〇一	一〇八
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以小石六因之，得六百四十八兩，為六小石之共重數。加五大石所多十二兩，得六百六十兩，為五大石之共重數。以五歸之，得一百三十二兩，即為一大石之重數也。此本豐借互徵之法。而以方程算之，稍為簡易焉。

(2) (The Second Part Chapter X Linear Section VIII Equation)

Suppose there are two lumps of stone unequal in size. The amount of (both) weights is unknown. There is only a copper bar, whose weight is twelve *liang*. Equally divide (the copper bar) into twelve sections. Use a rope to be fastened to the bar at the fifth section. One end is five sections and the other is seven sections. Hang the big stone on the end of the bar that is five sections from the lifting cord and take the small stone as a sliding weight to weigh the big stone. When (the small stone) is six sections from the lifting cord, (the bar) gets level. Furthermore, hang the small stone on the end that is five sections from the lifting cord and take the big stone as a sliding weight to weigh the small stone. When (the big stone) is four sections from the lifting cord, (the bar) gets level. Ask: how much do the big and small stones weigh respectively?

Method: Firstly multiply five sections by two. Subtract (the product) from twelve sections, remaining two sections. (The remainder) reduced by half, getting one section. (One section) plus five sections are six sections. And then take five sections for the first proportional and six sections for the second proportional. The remainder two sections are treated as two *liang* and take it for the third proportional. (Then) get the fourth proportional of two *liang* four *qian*. That is, only being increased by two *liang* four *qian* the end of five sections is on the same level with (the end of) seven sections. Then multiply two *liang* four *qian* by five sections that the big stone is from the lifting cord, and get twelve *liang*, which is the number of the five big stones more than the six small stones. (The big stone five sections from the lifting cord balances with the small stone six sections from the lifting cord, because the weight of the big stone is six portions and the weight of the small stone is five portions. If there are five big stones and six small stones, when they have thirty portions respectively, their weight is equal. However, the end of five sections should be increased by two *liang* four *qian*, because the weight of (one) big stone is two *liang* four *qian* more than six portions. Then five big stones are twelve *liang* more. Therefore, five big stones are twelve *liang* more than six small stones.)

The first proportional	five parts
The second proportional	six parts
The third proportional	two <i>liang</i>
The fourth proportional	two <i>liang</i> four <i>qian</i>

Furthermore, multiply two *liang* four *qian* by five sections that the small stone is from the lifting cord, and also get twelve *liang*, which is the number of four big stones less than five small stones. (The small stone five sections from the lifting cord balances with the big stone four sections from the lifting cord because the weight of the small stone is four portions and the weight of the big stone is five portions. If there are five small stones and four big stones, when they have twenty portions respectively, their weight is equal. However, the end of five sections should be increased by two *liang* four *qian*, because the weight of one small stone is two *liang* four *qian* more than four portions. Then five small stones are twelve *liang* more. Therefore, five small stones are twelve *liang* more than four big stones. Because the big stone is at the head, it is changed into that four big stones are twelve *liang* less than five small stones.)

Thus make *jiaoshu equation* (*jiaoshu fangcheng* 較數方程) to calculate it. Take five (of big stones) as the positive, six (of small stones) as the negative, weight twelve *liang* (more) as positive. (All of them) are listed above. Further, take four (of big stones) as the positive, five (of small stones) as the negative and weight twelve *liang* (less) as the negative. (All of them) are listed below. Then multiply by the above five (of big stones) individually the below four (of big stones), five (of small stones) and twelve *liang* (less), resulting in respectively twenty (of big stones), twenty-five (of small stones) and sixty *liang* (less). Once more, multiply by the below four (of big stones) individually the above five (of big stones), six (of small stones) and twelve *liang* (more), resulting in respectively twenty (of big stones), twenty-four (of small stones) and forty-eight *liang* (more). Operate subtraction between the two below (rows). Then, (as) big stones (in the two rows) are twenty respectively, the difference between them is 0. Small stones in both rows are negative. So, operating subtraction between them the remainder is 1. The weight 60 *liang* (less) plus the weight 48 *liang* (more) is 108 *liang*, namely the weight of one small stone.

the big stone	the small stone	weight
5 (positive)	6 (negative)	12 (positive)
4 (positive)	5 (negative)	12 (negative)
20 (positive)	25 (negative)	60 (negative)
20 (positive)	24 (negative)	48 (positive)

0	1	108
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Multiply it by six (of small stones), resulting in 648 *liang*, which is the total weight of six small stones. This plus 12 *liang* that six small stones are more results in 660 *liang*, which is the total weight of five big stones. Divide it by 5, resulting in 132 *liang*, namely the weight of one big stone. Originally it is (solved by) the method of *diejie huzheng* 疊借互徵 (*the method of double-false-position*). However, it is slightly simpler to solve it by equation (*fangcheng* 方程).

(3) (下篇 卷三十四 末部四 借根方比例 綫類)

設如有大小二石，不知重數。有銅條一根，重十二兩，均分十二分，以繩繫於第五分之上，一頭五分一頭七分。將大石掛於銅條之端，離提繫五分。而以小石作砵稱之，離提繫六分始平。又將小石掛於銅條之端，離提繫五分。而以大石作砵稱之，離提繫四分始平。問二石各重若干。

法：先以五分加一倍，與十二分相減，餘二分折半得一分，與五分相加為六分。乃以五分為一率，六分為二率，餘二分之重二兩為三率，求得四率二兩四錢。即五分之端加二兩四錢始與七分相平也。

一率	五分
二率	六分
三率	二兩
四率	二兩四錢

今大石離提繫五分，小石離提繫六分而平。是大石重六分，小石重五分，而大石多二兩四錢，則小石為大石六分之五而少二兩也（銅條五分之端應加二兩四錢而平。今大石在五分之一頭，是大石多二兩四錢也。將二兩四錢以大石之六分除之，每分得四錢，是大石比小石每分多四錢。以小石五分計之，則大石比小石多二兩。故小石為大石之六分之五而少二兩也）。又小石離提繫五分，大石離提繫四分而平，是小石重四分大石重五分。而小石多二兩四錢，則小石為大石五分之四而多二兩四錢也（銅條五分之端應加二兩四錢而平。今小石在五分之一頭，是小石多二兩四錢也。將二兩四錢以小石之四分除之，每分得六錢，是小石比大石每分多六錢。以小石四分計之，則小石比大石多二兩四錢。故小石為大石之五分之四而多二兩四錢也）。乃借三十根（六分五分相乘之數）為大石之重數，以小石為大石六分之五而少二兩計之，則小石之重為二十五根少二兩。以小石為大石五分之四而多二兩四錢計之，則小石之重又為二十四根多二兩四錢，此兩數為相等。兩邊各加二兩，得二十五根與二十四根多四兩四錢相等。兩邊再各減去二十四根，餘一根與四兩四錢相等。一根既與四兩四錢相等，則三十根必與一百三十二兩相等。即大石之重數六歸之，得二十二兩，五因之得一百一十兩，減去二兩得一百零八兩，即小石之重數。或以大石之重數五歸之，得二十六兩四錢，四因之得一百零五兩六錢，加二兩四錢，亦得一百零八兩，為小石之重數也（此疊借互徵法，用方程法算之亦可）。

(3) (The Second Part Chapter XXX IV Last Section IV *jiegenfang* 借根方 Proportion Line Class)

Suppose there are two stones, one is big and the other is small. Their weights are unknown. There is a copper bar, whose weight is twelve *liang*. Equally divide (the copper bar) into twelve sections. Use a rope to be fastened to the bar at the fifth section. One end is five sections and the other is seven sections. Hang the big stone on the end of the bar that is five sections from the lifting cord. And take the small stone as a sliding weight to weigh the big stone. When (the small stone) is six sections from the lifting cord, (the bar) gets level. Furthermore, hang the small stone on the end that is five sections from the lifting cord. And take the big stone as a sliding weight to weigh the small stone. When (the big stone) is four sections from the lifting cord, (the bar) gets level. Ask: how much do the big and small stones weigh respectively?

Method: Firstly multiply five sections by two. Subtract (the product) from twelve sections, remaining two sections. (The remainder) reduced by half, getting one section. (One section) plus five sections are six sections. And then take five sections for the first proportional, six sections for the second proportional, and the weight two *liang* of remainder two sections for the third proportional. And find the fourth proportional of two *liang* four *qian*. That is, only being increased by two *liang* four *qian* the end of five sections is on the same level with (the end of) seven sections

The first proportional	five parts
The second proportional	six parts
The third proportional	two <i>liang</i>
The fourth proportional	two <i>liang</i> four <i>qian</i>

Now the big stone that is five sections from the lifting cord balances with the small stone that is six sections from the lifting cord, so the weight of the big stone is six portions and the weight of the small stone is five portions. Nevertheless, the big stone is two *liang* four *qian* more (than six portions), so the small stone is two *liang* less than five-sixths of the big stone. (Only if the end of five sections of the copper bar is increased by two *liang* four *qian*, can the bar be level. Now the big stone is at the end of five sections, so the big stone is two *liang* four *qian* more. Divide two *liang* four *qian* by six portions of the big stone. Each portion gets four *qian*, which means each portion of the big stone is four *qian* more than each portion of the small stone. Calculating with five portions of the small stone, the big stone is two *liang* more than the small stone. Thus the small stone is two *liang* less than five-sixths of the big stone.) Furthermore, the small stone that is five sections from the lifting cord balances with the big stone that is four sections from the lifting cord, so the weight of the small stone is four portions and the weight of the big stone is five portions. Nevertheless, the small stone is two *liang* four *qian* more (than four sections), so the small stone is two *liang* four *qian* more than four-fifths of the big stone. (Only if the end of five sections of the copper bar is increased by two *liang* four *qian*, can the bar be level. Now the small stone is at the end of five sections, so the small stone is two *liang* four *qian* more. Divide two *liang* four *qian* by four portions of the small stone. Each portion gets six *qian*, which means each portion of the small stone is six *qian* more than each portion of the big stone. Calculating with four portions of the small stone, the small stone is two *liang* four *qian* more than the big stone. Thus the small stone is two *liang* four *qian* more than four-fifths of the big stone.) Then borrow thirty unknowns (the product of six sections and five sections) as the weight of the big stone. Calculating according to that the small stone is two *liang* less than five-sixths of the big stone, the weight of the small stone is two *liang* less than twenty-five unknowns. Calculating according to that the small stone is two *liang* four *qian* more than four-fifths of the big stone, the weight of the small stone is two *liang* four *qian* more than twenty-four unknowns. These two numbers are equal. Two *liang* is added to two sides respectively, then twenty-five unknowns are equal to the sum of twenty-four unknowns and two *liang* four *qian*. Furthermore, subtract twenty-four unknowns in two sides respectively. Then the remainders, one unknown and four *liang* four *qian*, are equal. Get the fourth proportional 8 *qian*, namely the weight of the original sliding weight. Since one unknown is equal to four *liang* four *qian*, thirty unknowns should be equal to one hundred and thirty-two *liang*, namely the weight of the big stone. Dividing it by six, get twenty-two *liang*. Multiplying it by five, get one hundred and ten *liang*. Subtracting two *liang*, get one hundred and eight *liang*, namely the weight of the small stone. Or dividing the weight of the big stone by five, get twenty-six *liang* four *qian*. Multiplying it by four, get one hundred and five *liang* and six *qian*. Plus two *liang* four *qian*, also get one hundred and eight *liang*, namely the weight of the small stone (this is the method of *diejie huzheng*. It may be solved by the method of *fangcheng*)

(4) (下篇 卷三十七 末部七 難題)

設如有一大石，不知其重，但知一小石重四兩。求大石重幾何。

法：用一木杆，結繫於中，兩端令平。乃以大石掛於一端，以小石作砵稱之。如大石距提繫一寸，小石距提繫六寸得平。則以一寸為一率，小石重四兩為二率，六寸為三率，求得四率二十四兩即大石之重也。如圖，甲乙為大石距提繫一寸，甲丙為小石距提繫六寸。丁為大石，戊為小石。戊小石之重即甲乙之分，丁大石之重即甲丙之分。甲乙與戊小石之比同於甲丙與丁大石之比也。

(4) (The Second Part Chapter XXXVII Last Section VII Difficult Problems)

Suppose there is a big stone, whose weight is unknown. But it is known that the weight of a small stone is four *liang*. Find how much the big stone weighs.

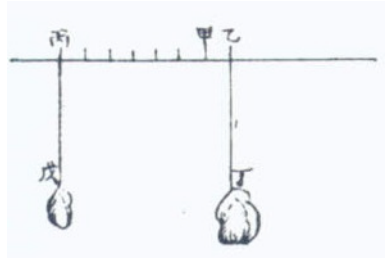


Figure 3
(Figure taken from *Yu Zhi Shu Li Jing Yun*, Siku Quanshu edition)

Method: Use a wooden pole, in the middle of which a knot is fastened. Make the two ends (of the pole) level. Then hang the big stone from one end and take the small stone for the sliding weight to weigh it. If the big stone is one *cun* 寸² from the lifting cord and the small stone is six *cun* from the lifting cord, the pole gets level. (Then) take one *cun* for the first proportional, the weight of the small stone four *liang* for the second proportional and six *cun* for the third proportional. Get the fourth proportional twenty-four *liang*, which is the weight of the big stone. See the figure, AB is one *cun* from the big stone to the lifting cord and AC is six *cun* from the small stone to the lifting cord. D is the big stone and E is the small stone. The weight of the small stone E is the sections of AB and the weight of the big stone D is the sections of AC. The ratio between AB and the small stone E is the same as the ratio between AC and the big stone D.

(5) (下篇 卷三十七 末部七 難題)

設如有銀大小二錠，共重十五兩。求大小錠各重幾何。

法：用一木杆，結繫於中，兩端令平。乃以大錠小錠各挂一端。如大錠距提繫四寸，小錠距提繫六寸得平。則以四寸六寸相加，得十寸為一率，共重十五兩為二率，大錠距提繫四寸為三率，得四率六兩，即小錠之重。如以小錠距提繫六寸為三率，則得四率九兩，即大錠之重也。如圖，甲乙為大錠距提繫四寸，甲丙為小錠距提繫六寸。故以甲乙甲丙共分與丁戊共重之比同於甲乙與戊小錠之比，亦同於甲丙與丁大錠之比也。

(5) (The Second Part Chapter XXXVII Last Section VII Difficult Problems)

Suppose there are two ingots of silver, one big and the other small. The weight altogether is fifteen *liang*. Find how much the big and small ingots weigh respectively.

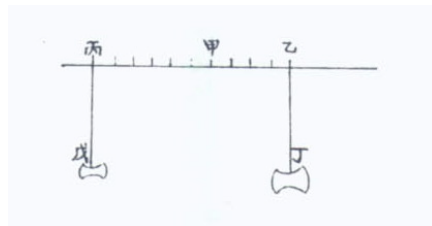


Figure 4
(Figure taken from *Yu Zhi Shu Li Jing Yun*, Siku Quanshu edition)

Method: Use a wooden pole, in the middle of which a knot is fastened. Make the two ends (of the pole) level. Then hang the big and small ingots separately from the two ends of (the pole). If, when the big ingot is four *cun* from the lifting cord and the small ingot is six *cun* from the lifting cord, the pole gets level, four *cun* plus six *cun* is ten *cun*. (Take it) for the first proportional, the weight altogether fifteen *liang* for the second proportional and four *cun* that the big ingot is from the lifting cord for the third proportional. Get the fourth proportional six *liang*, namely the weight of the small ingot. If taking six *cun* that the small ingot is from the lifting cord for the third proportional, then get the fourth proportional nine *liang*, namely the weight of the big ingot. See the figure, AB is four *cun* from the big stone to the lifting cord and AC is six *cun* from the small stone to the lifting cord. Therefore, the ratio between the sum of sections of AB and AC and the weight altogether of D and E

2 *Cun* 寸 is a unit of length in ancient China.

is the same as the ratio between AB and the small ingot E, and is also the same as the ratio between AC and the big ingot D.

(6) (下篇 卷三十七 末部七 難題)

設如以戥稱銀，戥數不足，將砵上加四兩，稱之得二百兩。原砵重八兩，問銀實重幾何。

法：以原砵重八兩爲一率，又以原砵八兩與加四兩相并得十二兩爲二率，以今稱二百兩爲三率，得四率三百兩爲原銀之重數也。如圖，甲乙爲二百兩之分，丙爲砵，重十二兩。試將甲乙戥衡引長至丁。甲丁爲三百兩之分。戊爲原砵，重八兩。甲乙乘丙砵即與甲丁乘戊砵之數等。故以戊砵與甲乙之比，同於丙砵與甲丁之比。爲轉比例四率也。

(6) (The Second Part Chapter XXXVII Last Section VII Difficult Problems)

Suppose (someone) weighs silver with a small steelyard, but the scale of the small steelyard is not (long) enough. Add four *liang* to the sliding weight. Weighing the silver, get two hundred *liang*. The original sliding weight is eight *liang*. Ask: how much is the true weight of the silver?

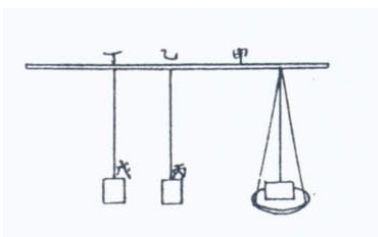


Figure 5
(Figure taken from *Yu Zhi Shu Li Jing Yun*, Siku Quanshu edition)

Method: take the weight eight *liang* of the original sliding weight for the first proportional. Furthermore, the weight eight *liang* of the original sliding weight plus four *liang* is twelve *liang*. Take (the sum) for the second proportional and the present result two hundred *liang* for the third proportional. Get the fourth proportional three hundred *liang*, which is the original weight of the silver. See the figure, AB is the section corresponding to two hundred *liang* and C is the sliding weight, whose weight is twelve *liang*. For the moment, extend the arm AB of the small steelyard to D. AD is the section corresponding to three hundred *liang*. E is the original sliding weight, whose weight is eight *liang*. The product of AB and the sliding weight C is equal to that of AD and the sliding weight E. Therefore, the ratio between the sliding weight E and AB is the same as that between the sliding weight C and AD. This is (the theory of) the four ratios of *zhuanbili* 轉比例.

(7) (下篇 卷三十七 末部七 難題)

設如戥子失去墜砵，欲配一砵，不知輕重。以重三兩之物，用六錢之砵稱之得四兩，問原砵重幾何。

法：以原重三兩爲一率，今稱得四兩爲二率，今砵重六錢爲三率，求得四率八錢即原砵之重也。如圖，甲乙爲戥盤距提繫之分，丙爲物重。甲丁爲三兩之分。戊爲原砵，甲己爲四兩之分，庚爲今砵。以比例論之。甲乙與戊砵之比，同於甲丁與丙重之比。又甲乙與庚砵之比，同於甲己與丙重之比。是甲丁乘戊砵即與甲己乘庚砵之數等。故以甲丁與庚砵之比，即同於甲己與戊砵之比，爲轉比例四率也。

(7) (The Second Part Chapter XXXVII Last Section VII Difficult Problems)

Suppose a small steelyard lost its sliding weight. (Someone) wants to have a sliding weight made to fit the steelyard, but its weight is unknown. Using a sliding weight of 6 *qian* to weigh an object of 3 *liang*, get 4 *liang*. Ask: how much is the weight of the original sliding weight?

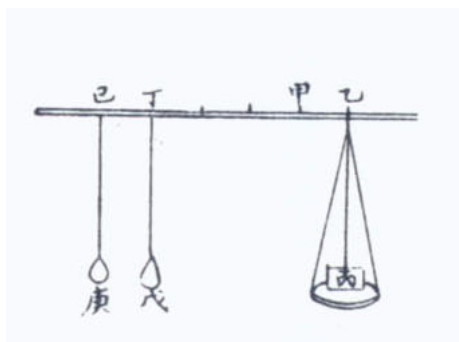


Figure 6
(Figure taken from *Yu Zhi Shu Li Jing Yun*, Siku Quanshu edition)

Method: Take the original weight 3 *liang* for the first proportional; the present result 4 *liang* for the second proportional; the weight 6 *qian* of the present sliding weight for the third proportional. Get the fourth proportional 8 *qian*, namely the weight of the original sliding weight. See the figure, AB is the section from the pan of the small steelyard to the lifting cord; C is the object; AD is the section (corresponding to) 3 *liang*; E is the original sliding weight; AF is the section (corresponding to) 4 *liang*; G is the present sliding weight. Discuss it by the theory of proportions. The ratio between AB and the sliding weight E is the same as that between AD and the weight of C. In addition, the ratio between AB and the sliding weight G is the same as that between AF and the weight of C, that is, the product of AD and the sliding weight E is equal to that of AF and the sliding weight G. Therefore, the ratio between AD and the sliding weight G is the same as that AF and the sliding weight E. This is (the theory of) the four ratios of *zhuanbili* 轉比例.

4. Analysis of Sources

4.1. Origin of the Problems

Shuli Jingyun is a compilation of Western mathematical knowledge prepared during the reign of Kangxi. Many previously translated and edited manuscripts in the Palace served as the basis for this compilation, one of which was *Suanfa zuanyao zonggang* 算法纂要總綱 [*General Principles of Algorithmic*], probably written between 1689 and 1695.

Textual comparison has suggested that *Suanfa zuanyao zonggang* 算法纂要總綱 was translated and edited by the Belgian missionary Antoine Thomas (1644-1709), who made use of his own *Synopsis Mathematica*, first published in 1685.³ By comparing *Suanfa zuanyao zonggang*⁴ and *Shuli jingyun*, we have found that some computational problems related to equilibrium of the lever in *Shuli jingyun* are directly taken from *Suanfa zuayao zonggang*. But among them, some had been provided with new solutions and in some the numerical data had been changed. In addition, some problems have no counterpart in *Suanfa zuayao zonggang*.

The seven problems quoted above and discussed here turned out to come from different origins. The first three problems ask for the answer to the same question posed in each of the three cases in a slightly different way. The basic question is taken from *Suanfa zuanyao zonggang*. The first of the three problems has the original solution, the other two provide new solutions to the problem. The fourth and fifth problems are not taken from *Suanfa zuanyao zonggang*. The fourth problem is similar to the thirtieth section (第三十款) of chapter two (卷二) of *Qiqi tushuo*. The fifth problem seems to be an adaptation of related problems in *Qiqi tushuo*. The sixth and the seventh problems stem directly from *Suanfa tongzong*.

3 See: Han Qi and Catherine Jami, "The Circulation of Western Mathematics at the Court during the Kangxi Period-A Case Study of the Compilation of the *Suanfa zuanyao zonggang*." *Studies in the History of Natural Sciences*, 2003(2): 145-156.

4 According to statistics by Han Qi and Catherine Jami, *Suanfa zuanyao zonggang* has nine editions, eight in Chinese and one in Manchu. I made use of the Chinese edition of the *Anletang* collection at the Institute for the History of Natural Sciences, Chinese Academy of Sciences.

4.2. The Types of Problems and the Structures of Their Solutions

All seven problems discussed here are composed of two parts: the question (*ti* 題) and the method (*fa* 法). This is, in fact, the general pattern of problems in the Chinese mathematical tradition.

As for the types of problem, the common question of the first three problems deals with two stones of unknown weight and a copper bar of known weight. The question is how to determine the weights of the two stones. The fourth problem is to find the unknown weight of an object from the known weight of another object by applying the law of the lever. The fifth problem asks for the unknown weights of two objects by applying the law of the lever, if the sum of their weights is given. The sixth and the seventh problems were both Chinese traditional computational problems of steelyard equilibrium which follows from the fact that they appear already in *Suanfa tongzong*. The first, the fourth and the fifth problem differ from traditional Chinese problems in that they are what may be termed “heuristic problems”,⁵ i.e., they ask in a more general way to find a method of solution. Their common characteristic is that the numerical values of the data are partly left unstated in the question, and the method to solve the problem is left open.

As for the structures of solution, the methods of these problems not only consist of the algorithms for the solution but are mingled with explanations or even proofs of the algorithms. This is different from both the former Western and the former Chinese mathematical tradition. In the former Western mathematical tradition, algorithm and proof are separate. And in the former Chinese mathematical tradition, often only the algorithm is given, but the proof, if there was any, is concealed.

4.3. The Solutions

4.3.1. The Solution of the First Problem

The solution of the first problem is based on the *method of double-false-position* (*diejie huzheng fa* 疊借互徵法, literally the “method of doubly borrowing and mutually comparing”). The Western *method of double-false-position* was first introduced into China by *Tongwen suanzhi* 同文算指 [A Guide to Arithmetic in Common Language], first published in 1613. Essentially, this method corresponds to the traditional Chinese *method of surplus and deficit* (*yingbuzu fa* 盈不足法), which was introduced already in the Chinese mathematical classic work *Jiuzhang suanshu* 九章算術 [Nine Chapters of Mathematical Procedures], probably written between 50 and 100 A.D. The way the Western *method of double-false-position* could be reduced in the *Shuli jingyun* to the traditional Chinese *method of surplus and deficit* although the given problem does not involve a surplus and a deficit, is by introducing a surplus and a deficit by two times assuming arbitrary numbers (*jie* 借, literally “borrowing” [originally: from the pile of calculation rods]) and then solving the problem by the *method of surplus and deficit*.

The first problem is solved in the following way:

First, the copper bar of known weight is equally divided into several sections, and each of the sections is further subdivided. Preferably, the number of sections and subsections corresponds to the weight of the bar in the Chinese system of weight units. A suspension cord is fastened to the bar at a suitable point dividing the bar into a longer and a short part, thus turning it into the beam of a steelyard.

Second, implicitly using the law of the lever, the weight which would compensate the greater weight of the longer arm of the bar, i.e., the size of a weight which would balance the suspended bar if it were attached to its shorter end is calculated.

Third, the big stone is actually attached to this end, the small stone is used as a sliding weight to bring the bar into equilibrium, and its position at the bar is determined. Then the small stone is attached to this end, the big stone is used as sliding weight to bring the bar into equilibrium, and again the position at the bar is determined.

Fourth, the weights of the two stones are calculated from the two positions of equilibrium by using the *method of double-false-position*.

5 This term is borrowed from George Polya. See: George Polya, *How to solve it*, 2nd ed., Princeton University Press, 1957.

In *Shuli jingyun*, the copper bar is assumed to weigh 12 liang and the bar is divided into 12 sections each of which is further subdivided into 10 subsections corresponding to the division of the *liang* into 10 *qian*. The suspension cord is assumed to be attached at the fifth section. The size of the weight which attached at this shorter end would bring the bar into equilibrium is calculated as 2 *liang* 4 *qian*.

For the two situations in which the suspended bar is actually brought into equilibrium, the missing values are now introduced. In the first situation, in which the small stone is used as sliding weight and the big stone is attached to the shorter end, the missing value of the position of the small stone is given as the sixth section counted from the suspension point. Similarly, in the second situation, in which the big stone is used as sliding weight and the small stone is attached to the shorter end, the also missing value of the position of the big stone is given as the fourth section counted from the suspension point.

Then, follows the application of the *method of double-false-position*.

First, 26 *liang* 4 *qian* is tentatively assumed to be the weight of the big stone, so that after subtraction of the weight which would compensate the greater weight of the longer side of the bar the remaining weight disturbing the equilibrium is 24 *liang*.

The corresponding weight of the small stone is then calculated by regarding the first situation of the balance and applying the law of the lever. The result is 20 *liang*. Now this value of the weight of the small stone is assumed to be given and the corresponding weight of the big stone is calculated by regarding the second situation of the balance and applying the law of the lever. The result is 22 *liang*.

This calculated value for the weight of the big stone differs from the value of 26 *liang* 4 *qian* tentatively assumed at the outset by 4 *liang* 4 *qian*.

Second, the same procedure is performed starting with a tentatively assumed value of 32 *liang* 4 *qian* for the weight of the big stone. The resulting value of the weight of the small stone is 25 *liang* from which a value of 28 *liang* 2 *qian* 5 *fen* is calculated for the weight of the big stone which in this case differs from the tentatively assumed value of 32 *liang* 4 *qian* by 4 *liang* 1 *qian* 5 *fen*.

The problem has thus been transformed into a form which turns out to be the Chinese surplus and deficit problem as it occurs in many Chinese mathematical texts beginning with the *Jiuzhang suanshu*, albeit usually applied to quite different problems. If one inserts the figures of the present problem into a typical problem on which the method is applied, one obtains a problem of the following kind:

“Now some persons buy an article together. If each person contributes 26 *liang* 4 *qian*, the total money has a deficit of 4 *liang* 4 *qian*; If each person contributes 32 *liang* 4 *qian*, the total money has a deficit of 4 *liang* 1 *qian* 5 *fen*. The posed problem is to find out how much money each person should contribute.”

The method by which the Chinese would solve this problem is the special case of the *method of surplus and deficit* in which two deficits are given. It is therefore called the *method of two deficits* (*liangbuzu fa* 两不足法). The Chinese solution is based on the proportionality of the individual and the total deficit, i.e., that the ratio between the total deficit and the deficit of each person is equal in both cases. This is so because the total deficit is in both cases the deficit of each person multiplied by the number of persons involved.

To evaluate such proportions is the subject of the ancient Chinese *method of proportion* (*jinyou shu* 今有術), which is similar to the way such proportions were treated in the Western mathematical tradition. The Chinese solution in the present case is based on the fact that if two ratios of magnitudes are equal, the differences between the corresponding magnitudes are also in the same ratio. These two differences can easily be calculated in the present case. The two total deficits are given in the problem so that their difference can be calculated. The deficits of each person are the differences between what they have to pay and what they actually paid in both cases. Since what they really have to pay is equal in both cases, the difference between the deficits of each person is equal to the difference of what they actually paid. Again these values are given in the problem.

Based on these given values, the solution of the problem consists in first calculating from the known proportionality the deficit of each person in the first case as 105 *liang* 6 *qian*. Added to the actually paid amount of 26 *liang* 4 *qian*, the amount each person really has to pay turns out to be 132 *liang*.

The calculation of the weight of the stones in the original problem follows precisely the procedure of the paradigmatic example, thus leading to a weight of the big stone of 132 *liang*. Finally, based on the law of the lever, the weight of the small stone is calculated from the weight of the big stone as 108 *liang*. This application of the *method of surplus and deficit* can, of course, be applied to the original problem only because this problem is a linear problem as it is in the discussed paradigmatic problem of this method.

4.3.2. The Solution of the Second Problem

As mentioned before, the second problem presents a different solution to answer the same posed question as that of the first problem. Here, the problem is solved by applying the theory of systems of linear algebraic equations. Generally, this theory as it is applied in *Shuli jingyun* is adapted from *Fangcheng lun* 方程論 [On Equations], written by Mei Wending 梅文鼎 (1633-1721), first published in 1672.⁶ In fact, with regard to the definition, the classification and the solution of systems of linear algebraic equations (*fangcheng* 方程), both works are very similar. *Shuli jingyun* differs slightly from Mei Wending's *Fangcheng lun* with regard to the form equations are represented. In the latter, equations are represented in the form of columns while in *Shuli jingyun* they are represented by horizontal lines. The main source of *Shuli jingyun*, *Suanfa zuanyao zonggang*, does not introduce the theory of system of linear algebraic equations at all. The solution of the second problem must therefore have been introduced by the compilers of the present work.

The main difficulty in solving the given problem consists of finding the correct equations. Once the system of linear equations is formulated, the solution follows a standard procedure.

Similar to the solution of the first problem, the *method of proportion* is applied to find at first the size of a weight that is able to bring the copper bar into equilibrium when it is suspended at the end of the fifth of the 12 sections into which it is divided. The result of 2 *liang* 4 *qian* is, of course, the same as in the first problem. In the first situation in which the big stone is attached to the shorter end and the small stone is used as sliding weight the bar is in equilibrium when the position of the small stone is at the end of the sixth section counted from the suspension point. If the weight of the bar is disregarded it follows from the law of the lever that the weight of 5 big stones equals the weight of 6 small stones. Taking the weight of the bar into account, the weight of the 5 big stones must exceed the weight of 6 small stones by 5 times the calculated weight compensating the greater weight of the longer arm, i.e. by 5 times 2 *liang* 4 *qian* or 12 *liang*.

In the second situation in which the small stone is attached to the shorter end, the big stone is used as sliding weight and the bar is in equilibrium when the position of the big stone is at the end of the fourth section counted from the suspension point, the weight of 5 small stones must exceed the weight of 4 big stones by 12 *liang*, which follows by the same argument. On the basis of these two conditions, a system of two linear algebraic equations can be set up, which can then be solved by Mei Wending's method mentioned above.

Using modern notation, this method can be explained as follows. Suppose that the weight of the big stone is x and that of the small stone is y . Then the two conditions described above can be represented as

$$5x - 6y = 12 \quad (3)$$

$$4x - 5y = -12 \quad (4)$$

Then (4) and (3) are multiplied by 5 and 4 respectively, giving

$$20x - 25y = -60 \quad (5)$$

$$20x - 24y = 48 \quad (6)$$

⁶ See: Qian Baocong (ed. in chief), *A History of Chinese Mathematics*, Beijing: Science Press, 1964, p. 271.

subtract (5) from (6), then $y = 108$.

Substituted into (3), then $x = 120$.

Therefore, the big stone is 120 *liang* and the small stone is 108 *liang*.

4.3.3. The Solution of the Third Problem

The method used for solving the third problem, i.e. the third way of answering the common question of the first three problems, is called *borrowing root and power* (*jiegenfang* 借根方). It is a kind of algebraic method introduced into China during the reign of Emperor Kangxi (reg. 1661 - 1722). In *Shuli jingyun*, the word “proportion” (*bili* 比例) is sometimes attached to the name of this method, thus being called the *proportion of borrowing root and power* (*jiegenfang bili* 借根方比例).

In the first paragraph of Chapter 31 of the second part of *Shuli jingyun*, the compilers emphasize the advantage of this method to have a wider range of possible applications. They say that this method is applicable also to quadratic and the cubic problems, whereas another method called *borrowing the rates and mutually comparing* (*jiemui huzheng* 借衰互徵) as well as the *method of double false position* (*diejie huzheng* 疊借互徵) discussed above can be used only to solve linear problems.

In the same paragraph, the *method of borrowing root and power* is furthermore explained. This explanation may be summarized as follows. First, there is an unknown number which is called *root* (*gen* 根). Second, an equation is formulated according to the proportional relations between given and unknown numbers occurring in the problem. Finally, the equation is solved.

Chapters 34 through 36 contain problems in whose solutions the *method of borrowing root and power* is applied. It has been shown that the content of these chapters was adapted from *Jiegenfang suanfa jieyao* 借根方算法節要 [A Summary of the Algorithm of Borrowing Root and Power].⁷ The problem discussed here is contained in chapter 34, but the copy of *Jiegenfang suanfa jieyao* preserved in Beijing Library does not contain this problem. So, the solution to this problem may have been added by the compilers of *Shuli jingyun*.

In the *Shuli jingyun* the problem is solved by a sequence of equations formulated in everyday language in the following way.

First, the size of the weight to compensate the greater weight of the longer arm of the copper bar is calculated along the same lines as in the first and the second problem, yielding 2 *liang* 4 *qian*.

Second, the following equations are formulated on the basis of the relations given in the problem:

$$\text{the small stone} = \frac{5}{6} \text{ the big stone} - 2 \text{ liang} \quad (7)$$

$$\text{the small stone} = \frac{4}{5} \text{ the big stone} + 2 \text{ liang } 4 \text{ qian} \quad (8)$$

Third, the unknown (*root*) is introduced in the following way.

assume the big stone = 30 *roots*, then

$$\text{the small stone} = 25 \text{ roots} - 2 \text{ liang} \quad (9)$$

$$\text{the small stone} = 24 \text{ roots} + 2 \text{ liang } 4 \text{ qian} \quad (10)$$

Therefore,

$$25 \text{ roots} - 2 \text{ liang} = 24 \text{ roots} + 2 \text{ liang } 4 \text{ qian}$$

$$1 \text{ root} = 4 \text{ liang } 4 \text{ qian}$$

$$\text{Therefore, the big stone} = 30 \text{ roots} = 132 \text{ liang}$$

Fourth, 132 *liang* is substituted for “the big stone” in (7) or (8), then obtain

$$\text{the small stone} = \frac{5}{6} * 132 \text{ liang} - 2 \text{ liang} = 108 \text{ liang}$$

⁷ See: Han Qi, “A Summary of Shuli jingyun,” in *Zhongguo kexue jishu dianji tonghui*, shuxue juan, Henan jiaoyu Chubanshe, 1993.

or

$$\text{the small stone} = 4/5 * 132 \text{ liang} + 2 \text{ liang} 4 \text{ qian} = 108 \text{ liang}$$

Again, the main difficulty in solving the given problem consists of finding the correct equations formulated in the second step. We take (7) as an example and analyze the train of thought in the text. Equation (7) represents the situation that the beam is in equilibrium if the big stone hangs at the shorter end of five parts and the small stone used as a sliding weight hangs at the sixth part of the longer end consisting of seven parts. If the weight of the copper bar is not taken into account, since “the big stone that is five parts from the lifting cord balances with the small stone that is six parts from the lifting cord,” “the weight of the big stone is six parts and the weight of the small stone is five parts.” However, if the weight of the copper bar is taken into account, the big stone must be two *liang* four *qian* more than six parts while the small stone is five parts. If the big stone is still taken as six parts and the small stone is still taken as five parts, then one part of the big stone should be 4 *qian* more than one part of the small stone. Here we may take a part of the big stone and a part of the small stone as two different units of weight, for the sake of differentiation, being denoted “G” and “g”. Then, the weight of the big stone is 6G and the weight of the small stone is 5g, where G is 4 *qian* more than g or g is 4 *qian* less than G. If g is converted into G, the weight of the big stone is 6G and the weight of the small stone is 5g = 5 (G - 4 *qian*) = 5G - 2 *liang*. Since G is one-sixth of the weight of the big stone, so “the small stone is two *liang* less than five-sixths of the big stone”, which is (7) if being expressed in formula. Along a similar reasoning (8) can be obtained.

4.3.4. The Solution of the Fourth Problem

The fourth problem is to find the unknown weight of a big stone from the known weight of a small stone by using a wooden pole suspended in the middle and applying the law of the lever. This solution is based on the *method of proportion*, which, using modern notation, can be expressed as follows.

Suppose the weight of the big stone is W and the distance between the big stone and the point of suspension is L; the weight of the small stone is w and the distance between the small stone and the point of suspension is l. From the law of the lever, the following proportion is obtained.

$$L : w = l : W$$

Therefore,

$$W = w l / L$$

Replacing the variables by the given data, i.e.

$$L = 1 \text{ cun}, \quad w = 4 \text{ liang}, \quad l = 6 \text{ cun}$$

this formula yields the procedure used in the text, resulting in the answer:

$$W = 24 \text{ liang}.$$

4.3.5. The Solution of the Fifth Problem

The fifth problem asks for the unknown weight of two silver ingots, one big and the other small, if the sum is given. The problem is solved by using a pole suspended in the middle and applying the law of the lever.

The solution of this problem is based on the *method of the proportion of the total number* (*heshu bili* 和數比例). By applying this method a proportion is obtained in the following way: if a total number needs to be divided into several parts and the ratio of each part to the sum of the parts is known, then the ratio between the sum of the parts and the total number is equal to the ratio between each part and corresponding number to be determined. This method is equivalent to the *art of distributing according to proportions* (*cui fen shu* 衰分術), in the *Jiuzhang suanshu*, which is mainly used to solve distribution problems.

In the present problem, since the pole is in equilibrium if the distance L of the big ingot from the point of suspension is four *cun*, and the distance l of the small ingot from the point of suspension is six *cun*, it

follows from the law of the lever that the big ingot is six parts and the small ingot is four parts of the total weight. Then, applying the method of the proportion of the total number, the following proportional expression involving the weight of the big and the small ingots W and w , respectively, can be obtained.

$$(L + l) : (W + w) = L : w$$

or

$$(L + l) : (W + w) = 1 : W$$

The total weight of the big and small ingots is given as 15 *liang*. Inserting the numbers into the two formulas yields

$$(4+6) : 15 = 4 : w$$

or

$$(4+6) : 15 = 6 : W$$

From these proportions it follows that the weights of the small and big ingots are 6 *liang* and 9 *liang* respectively.

4.3.6. The Solution of the Sixth and Seventh Problems

It is mentioned above that the sixth and seventh problems are both Chinese traditional problems concerning the equilibrium of steelyards. When they first appeared in *Suanfa tongzong*, they were solved by the ancient Chinese method called *jinyou shu* (今有術), which corresponds to the Western rule of three and is essentially equivalent to the use of proportions. Later, the same two problems were included in the *Tongwen suanzhi*, now with a solution using proportions, a method adopted from the West. The solutions to the sixth and seventh problems are the same as those used in the *Tongwen suanzhi*.

4.4. The Characteristics of the Knowledge Concerning the Lever in *Shuli Jingyun*

The knowledge concerning the lever in *Shuli jingyun* has two distinct characteristics. First, the law of the lever adopted from the West is for the first time applied to steelyards. This type of balance was used in China already since ancient times, however, no theoretical law such as the law of the lever is known from ancient Chinese sources. The production and use of these balances was probably based merely on empirical knowledge derive the Chinese experiential formula for steelyards. An indication for such empirical knowledge may be the formula given in the 17th century by the Chinese mathematician Fu Guozhu 傅國柱 about whom nearly nothing is known. Fu Guozhu's 傅國柱 formula indicates that whenever the sliding weight of a steelyard is replaced by another one with different weight, the product of the weight of the sliding weight and the value on the scale of the steelyard after having moved the sliding weight so that equilibrium is obtained does not change.

In *Shuli jingyun*, the solutions of the seven computational problems dealing with the lever are all consistent with the law of the lever. Especially the sixth and seventh problems are remarkable in this respect. Although they continue to use Chinese traditional methods of calculation, the solutions are directly based on the law of the lever. This is not found in earlier works of the Chinese tradition.

The second characteristic of the knowledge concerning the lever in *Shuli jingyun* is the way in which the dead weight of a material beam is taken into account. In this text, the method of dealing with the dead weight of the beam of the lever is called the *method of balancing* (*junping zhifa* 均平之法).

This method becomes clear from the solution of the first problem discussed above. The symmetric parts in both arms of the copper bar suspended at a point outside of its center are cancelled out. The surplus weight of the longer arm is considered to be compensated by a weight attached to the end of the shorter arm of the bar. If the copper bar is used to determine the weight of an object attached to this end of the shorter arm, part of its weight has to be considered as representing this compensating weight.

In contrast, the *Qiqi tushuo* deals with the dead weight of a material beam in a different way. This weight is imagined to be concentrated in the middle of the beam. This corresponds to imagining an object attached to the beam at its center of gravity, an object whose weight is equal to the deadweight of the beam. On this basis problems involving material beams can be solved by a direct application of the law of the lever.

Comparing the two ways of dealing with the dead weight of a material beam in *Shuli jingyun* and *Qiqi tushuo*, it turns out that there are both similarities and differences between them. The similarity consists in the fact that in both cases the dead weight of the beam is converted into the weight of an object. The difference is that in *Shuli jingyun* a suitable weight is considered to be added to the end of the short arm of the beam, while in *Qiqi tushuo* an object whose weight is equal to the dead weight of the beam is considered to be attached at the center of gravity of the beam so that the object and the dead weight have the same effect on balancing the beam.

5. Conclusions

From the above analysis, we can come to the following conclusions. In the 18th century, Chinese mathematicians had given attention to and studied Western knowledge concerning the lever. In the mathematical work *Shuli jingyun*, Western knowledge concerning the lever and Chinese traditional knowledge interacted with each other. The results suggest that, on the one hand, the law of the lever adopted from the West was applied to reconceptualize Chinese empirical knowledge about the steelyard. On the other hand, Chinese traditional mathematical methods were applied to solve problems concerning the equilibrium of the lever adopted from Western sources.

It needs to be pointed out that this interaction had a great influence on the subsequent work of Chinese mathematicians. For example, He Mengyao 何夢瑤 (1693-1764) in his *Suandi* 算迪 [A Guide to Arithmetic], written around 1732, changed Chinese traditional computational problems of steelyard. He absorbed the ideas concerning the treatment of the dead weight of a material beam taken from *Shuli jingyun*.

Suandi contains eight computational problems concerning the equilibrium of the lever. Among them, four problems are identical to those in *Shuli jingyun*, two other problems are also the same as those in *Shuli jingyun*, but the data are different. Moreover, the latter two problems are supplemented with a comment, which emphasizes the influence of the dead weight of a steelyard's beam on the results of weighing. The further two problems dealing with steelyard equilibrium are largely different from those in previous mathematical works. They make the additional supposition explicit that "the head of the beam of a steelyard is four *qian* lighter than the end" so that the influence of the dead weight of a steelyard on equilibrium has to be taken into account.

6. References

Shuli jingyun 數理精蘊 [Collected Basic Principles of Mathematics]. Published in 1723. Collected in *Siku quanshu* 四庫全書 1781.

Li Di, "The Historical Materials of Physics in 'Shu Li Jing Yun'." *Collection of papers on the History of Chinese Science and Technology* (1). Inner Mongolia Educational press, 1991.

Jürgen Renn, Zhang Baichun and Matthias Schemmel, *The Method of Chinese Steelyard-making and Related Mechanical and Mathematical Tradition*.

Wang Xieshan, "The Solution for Problems of Lever Mechanics in China Between the Late Ming Dynasty and the Early Qing Dynasty," *China Historical Materials of Science and Technology*, Vol.12, No.1, 1991.

Xiao Yunhong and Zhang Baichun, "A Preliminary Study of the Mechanical Knowledge about Lever in 'Qiqi Tushuo'," *Journal of Natural Dialectic*, No.5, 2004.

Selected Bibliography of Chinese Sources

Ai Rulue 艾儒略(G. Aleni, 1582-1649) and Yang Tingjun 楊廷筠(1562-1627), *Zhifang waiji* 職方外紀 [*Record of the places outside the jurisdiction of office of geography*]

Anonymous author, *Suanfa zuanyao zonggang* 算法纂要總綱 [*General Principles of Algorithmic*]. Probably written between 1689 and 1695. (Probable authors: An Duo 安多 (Antoine Thomas, 1644-1709), Nian Xiyao 年希堯)

Anonymous author, *Heguan zi* 鶡冠子 [*Master Heguan*]. Written in the third century B.C. *Zhuzi baijia congshu* 諸子百家叢書 [*Series of Various Schools*]. Shanghai: Shanghai guji chubanshe, 1990. [Reprint]

Anonymous author, *Jiegenfang suanfa jieyao* 借根方算法節要 [*Extract of the Algorithm of Borrowing Root and Power*]. Probably written between and 1695. Microcopy of manuscript at National library of China.

Anonymous author, *Jiuzhang suanshu* 九章算術 [*Nine Chapters of Mathematical Procedures*]. Probably compiled in the first century B.C. Beijing: *Wuying dian juzhen* edition, 1774.

Anonymous author, *Mozi* 墨子 [*Master Mo*]. Written during 5th-3rd century BC. In Zhengtong daoang 正統道藏 (*Depository of Taoist Scriptures*) preserved in baiyun guan 白雲觀, which was block-printed in 1445 and was repaired during 1845-1874. Photocopied by Shanghai Hanfenlou, 1924. Abbreviated as *Daozang* edition.

Anonymous author, *Yuzhi shuli jingyun* 禦制數理精蘊 [*Collected Basic Principles of Mathematics in the Name of the Emperor*]. Published firstly in 1723. The photocopy of Neifu block-printed edition in: Guo Shuchun (ed.) *Zhongguo kexue jishu dianji tonghui*, Shuxue juan 3 中國科學技術典籍通彙, 數學卷三 [*General Collection of Works on Science and Technology in Ancient China, Mathematics Vol.3*]. Zhengzhou: Henan jiaoyu chubanshe 1993. [Reprint]

Anonymous author, *Yuzhi shuli jingyun* 禦制數理精蘊 [*Collected Basic Principles of Mathematics in the Name of the Emperor*]. Published first in 1723. *Yingyin wenyuange siku quan shu* 景印文淵閣四庫全書, Vol. 799-800. Taipei: Shangwu yinshuguan, 1983.

Anonymous author, *Wuzi* 吳子 [*Master Zi*]. Written in the 4th-3rd century B.C. In *Zhuzi jicheng* 諸子集成 [*Collected Books of Various Schools*]. Published by Shijie Shuju in 1937. Photocopied by Shanghai Shudian. 1991.

Anonymous author, *Xici* 繫辭 in *Yi* 易 (or: *Yi jing* 易經) [*Appendix to the Book of Changes*]. The edition of *Zixia yi zhuan* 子夏易傳 [*Commentaries on the Book of Changes*]. Compiled by Zi Xia in the 5th century B.C. *Yingyin wenyuange siku quan shu* 景印文淵閣四庫全書, Taipei: Shangwu yinshuguan, 1983.

Anonymous author, *Zhoubi Suanjing* 周髀算經 [*Mathematical Classic of Zhoubi*]. Beijing: *Wuying dian Juzheng* edition, 1774.

Ban Gu 班固 (32-92), *Han shu* 漢書 [*Historical Book on Han Dynasty*]. *Hanshu buzhu* 漢書補注 [*Supplementary Commentary of Han shu*], compiled by Wang Xianqian 王先謙 (1842-1917), *xushoutang* 虛受堂, 1900.

Bi Yuan 畢沅 (1730-1797), *Mozi (zhu)* 墨子(注) [*Mozi (with Commentary)*]. Shanghai: Shanghai guji chubanshe, 1989.

Chen Qiyong 陳奇猷 (1917-), *Han fei zi xin jiaozhu* 韓非子新校注 [*The New Collation and Commentary of Han fei zi*]. Shanghai: Shanghai guji chubanshe, 2000. (*Han fei zi* 韓非子 was written in the 3rd century B.C.)

Chen Qiyong 陳奇猷, *Lǔshì chūnqiū jiāoshì* 呂氏春秋校釋 [*The Collation and Explanation of Lü's Spring and Autumn Annals*]. Shanghai: Xuelin chubanshe, 1984. (*Lǔshì chūnqiū* was written in 239 B.C.)

- Cheng Dawei 程大位, *Suanfa tongzong* 算法統宗 [General Source of Algorithmic]. 1592. In: Guo Shuchun (ed.). *Zhongguo kexue jishu dianji tonghui*, Shuxue juan 2 中國科學技術典籍通彙, 數學卷二 [General Collection of Works on Science and Technology in Ancient China, Mathematics Vol. 2]. Zhengzhou: Henan jiaoyu chubanshe, 1993, pp. 1217-1421.
- Deng Yuhan 鄧玉函 (Johann (Schreck) Terrenz, 1576-1630), Wang Zheng 王徵 (1571-1644), *Yuanxi qiqi tushuo luzui* 遠西奇器圖說錄最 [The Record of the Best Illustrations and Descriptions of Extraordinary Devices of the Far West], Accomplished in 1627. Published first in 1628. Chendu: Lai lu tang edition, 1830.
- Du Zhigeng 杜知耕, *Shuxue yao* 數學鑰 [The Key to Mathematics]. Published first in 1681. *Yingyin wenyuange siku quan shu* 景印文淵閣四庫全書, Vol. 802. Taipei: Shangwu yinshuguan, 1983.
- Duan Yucai 段玉裁 (1735-1815), *Shuowen jiezi zhu* 說文解字注 [The Commentary on Shuowen jiezi]. Shanghai: Shanghai guji chubanshe, 1981.
- Fang Xuanling 房玄齡 [et al], *Jin shu* 晉書 [Book on the Jin Dynasty]. Written in the Tang Dynasty. *Yingyin wenyuange siku quan shu* 景印文淵閣四庫全書, Taipei: Shangwu yinshuguan, 1983.
- Fang Zhongtong 方中通, *Shu du yan* 數度衍 [Development of Number and Measure]. Published first in 1661. *Yingyin wenyuange siku quan shu* 景印文淵閣四庫全書, Vol. 802. Taipei: Shangwu yinshuguan, 1983.
- Fu Guozhu 傅國柱, *Suanfa tongzong guangfa* 算法統宗廣法 [The Method of Extension to Suanfa ongzong]. Transcript preserved in National Library of China.
- Fu Guozhu 傅國柱, *Suanfa tongzong shiyi* 算法統宗釋義 [A Paraphrase of Suanfa ongzong]. Transcript preserved in National Library of China.
- Guo Qingfan 郭慶藩 (1844-1896), *Zhuang zi jishi* 莊子集釋 [The Collected Explanations of the Zhuang zi]. In *Zhuzi jicheng* 諸子集成 [Collected Books of Various Schools], published by Shijie shuju in 1937. Photocopied by Shanghai shudian, 1991.
- Guo Shuchun 郭書春, *Jiuzhang suanshu huijiao* 《九章算術》彙校 [The Convergent Collation of Jiuzhang suanshu]. Shenyang: Liaoning jiaoyu chubanshe, 1990.
- Han Ying 韓嬰 (2nd century B.C.), *Hanshi waizhuan* 韓詩外傳 [Han's Extra Explanations of the Classic of Poetry]. Collected in *Han Wei Congshu* 漢魏叢書 [Series of Books From Han Dynasty to Wei Dynasty]. Changchun: Jilin daxue chubanshe, 1992.
- He Mengyao 何夢瑤, *Suandi* 算迪 [A Guide to Calculation]. Written around 1732. Wu's block-printed edition in 1845, *Zhongguo lidai suanxue jicheng* (Vol. 3) 中國歷代算學集成(下), [The Collection of Chinese Mathematical Works in the Past Dynasties] edited by Jing Yushu 靖玉樹. Jinan: Shandong renmin chubanshe, 1994, pp. 3485-3791.
- He Rubin 何汝賓, *Bing lu* 兵錄 [The Records of Military Art]. Published in 1628. In Hua Jueming (ed.) *Zhongguo kexue jishu dianji tonghui*, jishu juan 5 中國科學技術典籍通彙, 技術卷五 [General Collection of Works on Science and Technology in Ancient China. Technology Vol. 5]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint in part]
- Huang Huaixin 黃懷信, *Yizhoushu jiaobu zhuyi* 逸周書校補注譯 [The Collation, Complementarity, Commentary and Translation of the Extra Books on Zhou Dynasty]. Xi'an: Xibei daxue chubanshe, 1996.
- Huang Hui 黃暉, *Lunheng jiaoshi* 論衡校釋 [The Collation and Explanation of the Discourses Weighed in the Balance]. Beijing: Zhonghua Shuju, 1995. *Lunheng* was written by Wang Chong in the second half of the first century A.D.
- Huang Zongxi 黃宗羲 (1610 – 1695), *Gaoping uenan* 高平學案 [Scholarly Biography of Gaoping]. In *Songyuan uenan* 宋元學案 [Scholarly Biographies in the Song and Yuan Dynasties]. First printed in 1846. Ch. 3, Hangzhou: Zhejiang guji chubanshe, 1994.

- Jiang Lihong 蔣禮鴻 (1916-1995), *Shangjun shu zhuzhi* 商君書錐指 [*The Explanation of the Book of Lord Shang*]. Beijing: Zhonghua shuju, 1986.
- Jin Nige 金尼閣 (Nicolas Trigault, 1577-1628), *Xiru ermu zi* 西儒耳目資 [*An Aid to the Ear and the Eye of Western Scholars*]. Published first in Hangzhou in 1626. Beijing: Wenzhi gaige chubanshe, 1957. [Reprinted]
- Li Jie 李誠 (?~1110), *Yingzao fashi* 營造法式 [*Architectural Methods and Patterns*]. Published first in 1103. First edition. In: Hua Jueming (ed.) *Zhongguo kexue jishu dianji tonghui*, Jishu juan 2 中國科學技術典籍通彙, 技術卷二 [*General Collection of Works on Science and Technology in Ancient China, Technology Vol. 2*]. Zhengzhou: Henan jiaoyu chubanshe 1994, pp. 7-309. [Reprint]
- Li Madou 利瑪竇 (Matteo Ricci, 1552-1610) and Xu Guangqi 徐光啓 (1562-1633), *Jihe yuanben* 幾何原本 [*Elements*]. Published first in 1607. *Tianxue Chuhan* 天學初函 [*First collectanea of heavenly studies*]. Vol. 4. Published first in 1628. Taipei: Taiwan xuesheng shuju, 1965. [Reprinted]
- Li Madou 利瑪竇 (Matteo Ricci), *Tianzhu shiyi* 天主實義 [*True Meaning of the Lord of Heaven*]. Written in 1603. *Tianxue Chuhan* 天學初函 [*First collectanea of heavenly studies*]. Vol.1, pp.351-636. Published first in 1628. Taipei: Taiwan xuesheng shuju, 1965. [Reprinted]
- Li Madou 利瑪竇 (Matteo Ricci) and Li Zhizao 李之藻 (1565-1630), *Tongwen suanzhitongbian* 同文算指通編 [*Extended Chapters for the Guide to Arithmetic in Common Language*] (an edited translation of Clavius's *Epitome of Practical Arithmetic*), published in 1613. Haishan xian guan edition, 1849. In: Guo Shuchun (ed.). *Zhongguo kexue jishu dianji tonghui*, Shuxue juan 4 中國科學技術典籍通彙, 數學卷四 [*General Collection of Works on Science and Technology in Ancient China, Mathematics Vol. 4*]. Zhengzhou: Henan jiaoyu chubanshe, 1993, pp. 113-267.
- Li Zhiqing 李之勤 (ed.), *Wang Zheng yizhu* 王徵遺著 [*Wang Zheng's Posthumous Writings*]. Xi'an: Shanxi renmin chubanshe, 1984.
- Liang Qichao 梁啟超 (1873-1929), *Mojing jiaoshi* 墨經校釋 [*The Collation and Explanation of the Mohist Canons*]. Published first in 1921. Collected in *Yinbinshi zhuanji* 飲冰室專集 [*Specialized Collection of Yinbinshi*]. Beijing: Zhonghua shuju, 1989.
- Liang Qixiong 梁啟雄 (1900-1965), *Xun zi jianshi* 荀子簡釋 [*The Concise Explanation of Xun zi*]. Beijing: Zhonghua shuju, 1983.
- Liu Hui 劉徽 (third century A.D.), Xu 序 [preface], in *Jiuazhang suanshu* 九章算術 [*Nine Chapters on Mathematical Procedures*]. Beijing: Wuyingdian juzheng edition, 1774.
- Lu Jiuyuan 陸九淵(1139-1193), *Lu Jiuyuan ji* 陸九淵集 [*Collection of Lu Jiuyuan*]. Beijing: Zhonghua shuju, 1980, p. 482.
- Luo Shilin 羅士琳 (1789-1853), *Bili huitong* 比例彙通 [*Integration (of traditional mathematics) with proportion*]. Written in 1818. Guan wo shengshi huigao 觀我生室彙稿 edition, 1821.
- Mei Wending 梅文鼎 (1633-1721), *Fangcheng lun* 方程論 [*On Equations*]. Accomplished in 1672. Meishi congshu jiyao edition, 1759. In: Guo Shuchun (ed.), *Zhongguo kexue jishu dianji tonghui*, Shuxue juan 2 中國科學技術典籍通彙, 數學卷二 [*General Collection of Works on Science and Technology in Ancient China, Mathematics Vol. 4*]. Zhengzhou: Henan jiaoyu chubanshe, 1993, pp. 323-408.
- Nan Huairan 南懷仁 (Ferdinand Verbiest, 1623-1688), *Xinzhi Lingtai Yixiang Zhi* 新製靈臺儀象志 [*A Record of Newly-built Astronomical Instruments at the Observatory of Beijing*]. Published in 1674. In Bo Shuren (ed.) *Zhongguo kexue jishu dianji tonghui*, Tianwen juan 7 中國科學技術典籍通彙, 天文卷七 [*General Collection of Sources on Science and Technology in Ancient China, Astronomy, Vol.7*], edited by Bo Shuren 薄樹人. Zhengzhou: Henan jiaoyu chubanshe, 1994. [Reprint]
- Nan Huairan 南懷仁 (Ferdinand Verbiest, 1623-1688), *Qiongli xue* 窮理學 [*The Learning of Making a Thorough Inquiry into Principles*]. Published in 1683. The incomplete book is in the library of Beijing University.

- Nan Huairen 南懷仁 (Ferdinand Verbiest, 1623-1688), *Jincheng qiongli xue shu zou* 進呈窮理學書奏 [*The Respectful Dedication of the Book Qiongli xue*]. Written in 1683. Xu Zongze 徐宗澤 (1886-1947) (ed.), *Mingqing jian yesu huishi yizhu tiyao* 明清間耶穌會士譯著提要 [*Summary of the Translation and Works of Jesuits in the Ming and Qing Dynasties*]. Beijing: Zhonghua shuju 1989. [Reprint]
- Pang Diwo 龐迪我 (Diego de Pantoja, 1571-1618), *Qike* 七克 [*The Seven Victories*]. Published first in 1614. *Tianxue chuhan* 天學初函 [*First collectanea of heavenly studies*], Vol. 2, pp.689-1126. Published first in 1628. Taipei: Taiwan xuesheng shuju, 1965. [Reprinted]
- Qi Jiguang 戚繼光 (1528-1587), *Lianbing shiji zaji* 練兵實紀雜集 [*Miscellaneous Records concerning Military Training*]. Published in 1571. In Hua Jueming (ed.) *Zhongguo kexue jishu dianji tonghui*, Jishu juan 5 中國科學技術典籍通彙, 技術卷五 [*General Collection of Works on Science and Technology in Ancient China*, Technology Vol. 5]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint]
- Qu Zengfa 屈曾發, *Shuxue jingxiang* 數學精詳 [*Accurate and Detailed Mathematics*]. Also named as *Jiushu tongkao* 九數通考 [*Encyclopedia of Nine Branches of Mathematics*], 1772. Block-printed edition of the 10th year of Tongzhi reign, 1871.
- Ruan Yuan 阮元, *Chouren zhuan* 疇人傳 [*Biographies of Mathematicians*], chapter 29. Published first in 1799. Republished in *Wenxuan lou congshu* in 1842. In Lin Wenzhao (ed.) *Zhongguo kexue jishu dianji tonghui*, Zonghe juan 7 中國科學技術典籍通彙, 綜合卷七 [*General Collection of Works on Science and Technology in Ancient China*, Synthesis Vol. 7]. Zhengzhou: Henan jiaoyu chubanshe 1994. pp.137-788, [Reprint]
- Sima Qian 司馬遷 (145 or 135 B.C. - early period of the first century B.C.), *Shi ji* 史記 [*Records of History*]. Written in the early 1st century. Beijing: Zhonghua shuju, 1985.
- Song Yingxing 宋應星 (1587-1666?), *Tiangong kaiwu* 天工開物 [*Natural Skills Exploiting Things*]. Published in 1637. Beijing: Zhonghua shuju, 1959. [Reprint]
- Sun Xingyan 孫星衍 (1753-1818), *Shangshu jin gu wen zhushu* 尚書今古文注疏 [*The Commentary on the New and Ancient Texts of Shangshu*]. First published in 1815. Beijing: Zhonghua shuju, 1986. (A new collated edition based on the first edition. The “Ancient Text” means the pre-Qin text rephrased in Han Dynasty, the “New Text” means the text written in Han Dynasty.)
- Sun Xingyan 孫星衍 (1753-1818), Huang Yizhou 黃以周 (1828-1890) (collated.), *Yan zi chunqiu* 晏子春秋 [*The Story about Master Yan*]. Shanghai: Shanghai guji chubanshe, 1989. *Yan zi chunqiu* was originally written during the Warring States (475-221 B.C.).
- Sun Yirang 孫詒讓 (1848-1908), *Mozi jiangou* 墨子間詁 [*The Detailed Explanation of the Mozi*]. *Zhuzi jicheng* 諸子集成 [*Collected Books of Various Schools*]. Published by Shijie shuju in 1937. Photocopied by Shanghai shudian, 1992.
- Sun Yirang 孫詒讓 (1848-1908), *Zhouli zhengyi* 周禮正義 [*The Proper Meanings of the Rites and Institutions of Zhou Dynasty*]. Published first in 1905. Beijing: Zhonghua shuju, 1987. (A new collated edition based on the first edition.)
- Sun Yuanhua 孫元化 (1581-1633), *Xifa shenji* 西法神機 [*Western Masterpieces of Firearms*]. Published in 1662.
- Sun Zi 孫子, *Sun zi suanjing* 孫子算經 [*Mathematical Classics of Master Sun*]. Completed around 400 A.D. Beijing: Wuyingdian juzhen edition, 1774.
- Tan Jiefu 譚戒甫 (1887-1974), *Mobian fawei* 墨辯發微 [*The Discovery of the Hidden Meanings of the Mohist Disputation*]. Beijing: Zhonghua Shuju, 1964, reprinted in 1987.
- Tu Wenyi 屠文漪, *Jiuzhang luyao* 九章錄要 [*The Record of Essentials of Jiuzhang*]. Completed in the period from 1662 to 1722. *Yingyin wenyuange siku quan shu* 景印文淵閣四庫全書, Vol. 802. Taipei: Shangwu yinshuguan, 1983.
- Wang Siyuan 王泗原, *Chuci jiaoshi* 楚辭校釋 [*The Collation and Explanation of the Poetry of Chu*]. Beijing: Renmin jiaoyu chubanshe, 1995. The *Chuci* was accomplished in the first century B.C.

- Wang Wensu 王文素 (ca.1465-?), *Suanxue baojian* 算學寶鑒 [*Precious Mirror of Learning of Mathematics*]. Accomplished in 1524. Transcription preserved in National Library of China.
- Wang Zheng 王徵, *Liangli lüe* 兩理略 [*An Outline of Two Administrations*]. Published first in 1637. In Li Zhiqing 李之勤 (ed.), *Wang Zheng yizhu* 王徵遺著 [*Wang Zheng's Posthumous Writings*]. Xi'an: Shanxi renmin chubanshe, 1984, pp. 12-131.
- Wang Zheng 王徵, *Xinzhi zhuqi tushuo* 新制諸器圖說 [*Illustrations and Descriptions of Newly-built Devices*]. Accomplished in 1626. Published first in 1628. Chendu: Lai lu tang edition, 1830.
- Wang Zheng 王徵, *Ela jiya youzao zhuqi tushuo ziji* 額辣濟亞牖造諸器圖說自記 [*Record of the illustration and description of devices made with the blessing of the Gratla*]. Accomplished in 1640. In Li Zhiqing 李之勤 (ed.), *Wang Zheng yizhu* 王徵遺著 [*Wang Zheng's Posthumous Writings*]. Xi'an: Shanxi renmin chubanshe, 1984, pp. 229-232.
- Wen Renjun 聞人軍, *Kao gong ji yizhu* 考工記譯注 [*Translation and Commentary of the Kao Gong Ji*]. Shanghai: Shanghai guji chubanshe, 1993. The *Kao gong ji* was accomplished probably in the 5th-4th centuries B.C.
- Wu Yujiang 吳毓江 (1898-1977), *Mozi jiaozhu* 墨子校注 [*Collation and Commentary of the Mozi*]. Chongqing: Xi'an shifan daxue chubanshe, 1992.
- Xiong Mingyu 熊明遇 (1579-1649). *Qike yin* 七克引 [*Introduction to Qike*]. In *Qike, Tianxue chuhan* 天學初函 [*First collectanea of heavenly studies*], Vol. 2, 1628, pp. 697-700.
- Xiong Sanba 熊三拔 (Sabbathine de Ursis, 1575-1620), Xu Guangqi 徐光啟, *Taixi shuifa* 泰西水法 [*Hydraulic Methods of the Great West*]. Published in 1612. In *Tianxue chuhan* 天學初函 [*First collectanea of heavenly studies*], Vol. 3, 1628, pp. 1505-1710.
- Xu Guangqi 徐光啟 [et al]. *Xinfa Suanshu* 新法算書 [*Mathematical Books on New Methods*], 1629-1643. *Yingyin wenyuange siku quan shu* 景印文淵閣四庫全書, Vol. 802. Taipei: Shangwu yinshuguan, 1983. This series of books was accomplished first in 1629-1635, the original name of which was *Chongzhen lishu* 崇禎曆書 [*Chongzhen reign-period manual of mathematical astronomy*], 1629-1635, and was recompiled by Adam Schall von Bell (1592-1666) in 1643.
- Xu Shen 許慎 (c.58-c.125 AD), *Shuowen jiezi* 說文解字 [*Explaining Radicals and Analyzing Compound Characters*]. Finished between 100 and 121 A.D. Chen Changzhi block-printed edition, 1873. Beijing: Zhonghua shuju, 1978. [Reprinted]
- Yang Bojun 楊伯峻 (1909-1992), *Meng zi yizhu* 孟子譯注 [*Translation and Commentary on the Mencius*]. Beijing: Zhonghua shuju, 1962. The *Mencius* was accomplished in the third century B.C.
- Yang Bojun 楊伯峻, *Lie zi jishi* 列子集釋 [*The Collected Explanations on the Lie zi*]. Beijing: Zhonghua shuju, 1991. The *Lie zi* was accomplished in the 4rd to 3rd centuries B.C., and re-edited in the 4rd century A.D.
- Yang Bojun 楊伯峻, *Chunqiu zuozhuan zhu (xiuding ben)* 春秋左傳注(修訂本) [*The Annotation of the Zuo's Commentary to the Spring and Autumn Annals (Revised Edition)*]. Beijing: Zhonghua shuju, 1995. The *Zuo's Annals of the Spring and Autumn* was accomplished in the 5th century B.C.
- Yang Bojun 楊伯峻, *Lunyu yizhu* 論語譯注 [*Translation and Commentary on the Analects*]. Beijing: Zhonghua shuju, 1962. *Lunyu* was accomplished in the 5th to 4th century B.C.
- Yuan Ke 袁珂, *Shanhai jing jiaozhu* 山海經校注 [*Collated and Annotated Classic of the Mountains and Rivers*]. Shanghai: Shanghai guji chubanshe, 1980. *Shanhai jing* was written before Christ.
- Yuan Mei 袁梅, *Shijing yizhu (guofeng bufen)* 詩經譯注(國風部分) [*Translation and Commentary on the Classic of Poetry (the Part of the Songs of Various States)*]. Ji'nan: Qilu shushe, 1980. *Shijing* was accomplished in the 6th century B.C.
- Zeng Gongliang 曾公亮 (999-1078) (ed.), *Wujing zongyao* 武經總要 [*Collection of the Most Important Military Techniques*]. Accomplished in the 1040s. Republished in 1510. In Hua Jueming (ed.) *Zhongguo kexue jishu dianji tonghui*, Jishu juan 5 中國科學技術典籍通彙, 技術卷五 [*General*

- Collection of Works on Science and Technology in Ancient China, Technology, Vol. 5*. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint in part]
- Zhang Huiyan 張惠言 (1761-1802), *Mozi jing shuo jie* 墨子經說解 [*Explanation of the Chapters Canons and Canon Explanations in the Mozi*]. Collected in the *Congshu jicheng xubian* 叢書集成續編 [*The Second Collection of Series of Books*]. Shanghai: Shanghai shudian chubanshe, 1994.
- Zhang Qingchang 張清常 (1915-1998), Wang Yandong 王延棟, *Zhanguo ce jianzhu* 戰國策箋注 [*Commentary on the Strategies of Warring States*]. Tianjin: Nankai daxue chubanshe, 1993.
- Zhang Shuangdi 張雙棣, *Huainan zi jiaoshi* 淮南子校釋 [*The Collation and Explanation of the Huainan zi*]. Beijing: Beijing daxue chubanshe, 1997. The *Huainan zi* was accomplished in 139 B.C.
- Zhang Shuangdi 張雙棣, Zhang Wanbin 張萬彬, Yin Guoguang 殷國光, Chen Tao 陳濤, *Liushi Chunqiu yizhu* 呂氏春秋譯注 [*Translation and Commentary on the Liushi chunqiu*]. Changchun: Jilin wenshi chubanshe, 1987.
- Zhao Shizhen 趙士楨 (1553?-1611?), *Shenqi pu* 神器譜 [*Treatise on Marvellous Weapons*]. Published first in 1598. Republished by Xuan lan tang congshu 玄覽堂叢書 1941. In: Hua Jueming (ed.) *Zhongguo kexue jishu dianji tonghui*, Jishu juan 5 中國科學技術典籍通彙, 技術卷五 [*General Collection of Works on Science and Technology in Ancient China, Technology, Vol. 5*]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint in all]
- Zhao Shouzheng 趙守正, *Guan zi zhuyi* 管子注譯 [*Commentary and Translation of the Guan zi*], Nanning: Guangxi remin chubanshe, 1987. *Guan zi* was accomplished in ca. 5th to 3rd century B.C.
- Zheng Ruozeng 鄭若曾 (1503-1570), *Chouhai tubian* 籌海圖編 [*The Illustrations Seaboard Strategy and Tactics*]. Published in 1562. In Hua Jueming (ed.) *Zhongguo kexue jishu dianji tonghui*, Jishu juan 5 中國科學技術典籍通彙, 技術卷五 [*General Collection of Works on Science and Technology in Ancient China, Technology, Vol. 5*]. Zhengzhou: Henan jiaoyu chubanshe 1994. [Reprint in part]